Quantitative Analysis of Milks and Wheats
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Abstract
The problem of modeling economic exchange on networks is one that has traditionally been examined through solving for an equilibrium set of prices. However, this method falls short in practice as human behavior often deviates from the assumptions that are generally made in order to predict an equilibrium. This paper examines the results of a study by Professor Kearns which augments equilibrium analysis by examining human interaction on networks through behavioral experiments.

Specifically, we examine human subjects conducting exchange on various network structures, attempting to maximize their individual wealth. Experimenting on network structures with high equilibrium wealth inequality tended to result in high wealth inequality between participants. Additionally, such networks produced lower social wealth than networks with low equilibrium wealth inequality.

Introduction
The evolution of the study of economic exchange on networks can be broken up into three phases, each building on the previous one: equilibrium analysis, social network theory, and behavioral network science. Developments in each phase have been driven by the desire to understand how agents interact in real-world networks, with each level adding sophistication to the models we use to predict such interactions.

Although economists Arrow and Debreu (1954) established long ago that there is always a set of equilibrium prices in an economy with perfect competition and rational players, their framework was not successfully applied to economic interaction on networks until a half a century later by Kakade, Kearns, and Ortiz (2004a). Their result allowed for the application of equilibrium analysis in situations where the assumption that each party can trade with all other parties is not valid. In reality, this is often the case, as parties choose to exchange with each other based on their knowledge or trust of each other (in the case of individuals) or on their trade status (in the case of nations). Kakade, Kearns, and Ortiz thus accounted for this by describing a network where the vertices represented consumers and edges represented a potential trading relationship between consumers.

As the study of economic exchange began to draw more heavily on the understanding of networks, it eventually collided with social network theory. According to Kakade, Kearns, Ortiz, Pemantle, and Suri, (2004b) this is “the study of apparently ‘universal’ properties of natural networks . . . and statistical generative models
that explain such properties” (p. 1). Researchers realized that, in order to predict economic interactions in real-world markets, it was not sufficient to have a theory of economic exchange on networks, but also an understanding of which types of networks are worth modeling because of their recurrence in the real world. Kakade et al. (2004b) state that some of the characteristics of natural networks are “small diameter, local clustering of edges, and heavy-tailed degree distribution” (p. 2). In applying equilibrium analysis to networks generated to reflect these real world properties, they found that a vertex’s positioning in a network can have significant influence on its wealth, adding a previously unconsidered variable to the factors used to predict an individual’s economic standing.

The most recent development in this strain of research has been to apply an insight that has previously been applied in economics and finance - that human behavior often differs significantly from what might be predicted in an equilibrium analysis, for reasons ranging from irrationality to lack of information. Thus, researchers have started to supplement theoretical research on network science with experimentation on human subjects. In this vein, Michael Kearns and Stephen Judd recently ran a behavioral experiment on 36 Penn students based on the “Milks and Wheats” network model, which I will briefly describe.

Each instance of “Milks and Wheats” has the following properties. It is a bipartite graph, similar to Figure 1, with the vertices being divided into two sets; edges are only allowed to connect vertices in one set to vertices in the other. An agent is assigned to each vertex and is allotted an endowment of one unit of milk (for the first set) or one unit of wheat (for the second set); the agent is assigned a preference of zero for the good that it is allotted, and a non-zero preference for the other good. Therefore, the agent’s objective is to obtain as much of the other good as possible. This class of games was examined by Kakade et al. (2004b), who described the equilibrium characteristics of such a network.

In adapting the “Milks and Wheats” model to an experimental setting, with two crucial differences from the traditional model, Professor Kearns made two significant changes that addressed some of the concerns raised by behavioral economists. First, the participants could not see the entire network structure, only who their neighbors were - this helped to better model the degree of information an agent might reasonably be expected to know. Second, the price negotiations happened in real time - players would make an offer consisting of price and quantity, and could see their neighbors’ offers as well as their neighbors’ remaining endowment. Thus, unlike in the equilibrium situation, players could exchange their goods at more than one price and time. Additionally, there was a time limit to the negotiation, so pressure was created for a player to unload his goods before a competitor made a better offer or time ran out - again, this helped to better model the real world than a static negotiation setting. To incentivize the players, they were offered a cash reward proportional
to the amount of wealth they accumulated throughout the trials.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Diagram of a simple instance of “Milks and Wheats,” with equilibrium wealths of each node. From Michael Kearns’ lecture slides, Networked Life, Spring 2007.}
\end{figure}

\section*{Analysis}
\subsection*{Preliminaries}

In order to facilitate the discussion, a number of definitions are in order. Fair graphs are graphs where the equilibrium wealths are perfectly equal among all participants, for example the perfect matching and the dense preferential attachment graphs. Unfair graphs are graphs where the equilibrium wealths differ significantly between each other, such as in the unrewired clan graphs and the preferential attachment trees. Chicken regret is the case where two parties with the potential for exchange both conclude the experiment with some of their endowment remaining, meaning they could not settle on a suitable exchange price for at least some of their goods. Overpriced regret is the case where a party finishes the experiment with some of its initial endowment remaining, but their trading partners have all sold out their endowments. Underpriced regret is the opposite, where a party sells out all of its initial endowment but some of their trading partners still have endowment remaining at the end of the experiment.

In discussing the results of the experiments with Professors Kearns and Judd, I received some guidance on which areas of the experiment had already been analyzed. Among other things, they had looked at the “price of anarchy”, the degree to which the total wealth of participants falls short of what it would be in equilibrium, and found that the price of anarchy was much higher in the unfair graphs than in the fair graphs. Since this was one of the principal findings of the experiment, I wanted to set out to find what exactly caused this discrepancy in performance between the two types of graphs.
Disproving the “Ultimatum” Hypothesis

My first hypothesis was that the structure of the network forced some players to make offers that other players simply would not accept. I wanted to look at whether this was a similar phenomenon to the ultimatum game (where participants rejected offers they perceived to be unfair); which it seems would manifest itself in instances of “chicken regret” as the participants would fail to agree upon an exchange price. My hypothesis was that there would be some baseline rate of chicken regret for all experiments, and if there was a significantly greater rate of chicken regret on the unfair graphs (PA trees and unrewired clan graphs), where exchange prices were bound to deviate from 1 significantly due to the structure of the network, then some of the failure to achieve optimum social welfare could be attributed to instances where players rejected offers they perceived as unfair, even though it would have been economically rational for them to accept those offers.

To test the hypothesis, I compared the rates of chicken regret experienced on the PA tree and unrewired clan graphs to that experienced on the experiments as a whole. On all of the experiments the rate of chicken regret was .087, whereas for unrewired clan graphs it was .046 and for the PA trees it was .12 (by this, I mean the number of players experiencing that type of regret). From this I concluded that “ultimatum”-type rejections of offers could not account for any significant portion of the wealth that was left on the table in the high-regret graphs. Figure 2 gives a breakdown of the data from this part of the analysis.

Figure 2: The left chart demonstrates instances of regret throughout all experiments, while the right two charts demonstrate instances of regret in the unfair graphs. An instance of regret means that, in a given trial, a player experienced that type of regret (they form a partition).

Examination of Opening Offers

Having established that “ultimatum” offer rejections were not to blame for the low social welfare in the unfair graphs, I began looking at the evolution of offers that were made during a trial on these graphs. I noticed that regardless of what a players
initial degree was, the player would generally offer a 1-to-1 exchange to start the game. The players' offers would then often move monotonically downward (if the player was disadvantaged as far as the network structure) or monotonically upward (if the player was advantaged). Likewise, the deals that were executed during a trial would generally start at a 1-to-1 exchange and become more uneven as time went on. The graphs in figure 3, for a trial on the clan graph, no rewiring are indicative of what I observed in the trials on the “unfair” graphs.

The phenomenon demonstrated in figure 3 seems to be analogous to the psychological phenomenon of “anchoring” players start off the trial offering a price they perceive to be fair (1-to-1), and it takes more evidence than just seeing their local network structure to change their mind about this. An alternate view of this phenomenon can be seen in figure 4, which offers further confirmation of the fact that players do not place a great deal of emphasis on their local network structure in making their initial offer, rather choosing to stick to an offer close to 1-to-1.

**Figure 3:** The first three charts demonstrate the evolution throughout time of the offers made by participants in the experiment. Offers made by the disadvantaged players (degree 4) decreased, and offers made by advantaged players (degrees 8 and 18) increased. This manifested itself in more unequal deals throughout time, seen in the last chart.

**Establishing the Importance of Opening Offers**

So at this point, what we have is that players generally start off with offers of 1-to-1 that may deviate significantly from what the equilibrium dictates they should
Figure 4: This figure shows the opening offers in each of the unrewired clan graph trials, where the opening offers are averaged over groups of players that have the same degree. Note that the equilibrium wealths were 0.5 for the players with valence 4, and 2.0 for players with valence 8 and 18, so that the opening offers differ significantly from what the equilibrium dictates. There were always a few opportunistic players for each degree set that would start off making a pricey offer, so that the median opening offers were even closer to 1-to-1 than the means were.

offer. Of course, this is inconsequential if no trades are executed early in the game, when prices are roughly even. However, as figure 5 demonstrates, players exchange a large portion of their wealth early in the game; it is often the case that around half of all the wealth that will be exchanged in a trial is exchanged within the first 30 seconds of the trial. Now it seems we're getting close to the source of the low social wealth in unfair graphs. It is clear than in unfair graphs, 1-to-1 exchange benefits one group disadvantage players that are able to execute 1-to-1 deals. Meanwhile, 1-to-1 exchange hurts two groups: advantaged players, who experience underpriced regret, and disadvantaged players who were not able to execute 1-to-1 deals, who experience overpriced regret because the advantaged players who they should have been able to trade with traded away all of their goods to the other disadvantaged players. Thus, there is a bit of irony in that in the ultimatum game, low social welfare is caused by offers perceived as unfair (and thus rejected), whereas in this game low social welfare is caused by players making offers that they deem are fair but are actually too generous.

It would seem to be beneficial for one group (the advantaged players) to hold off on exchange in the early game, which one would think might imply the speed of exchange would decrease as players became more used to the game; however this turns out not to be the case, as the above graphs demonstrate. This does not mean that the players are irrational, however. It is possible that players are wary of being “locked out” (having overpriced regret) if they have experienced it before, and even if they have a high degree place some significant probability on this happening if they do not unload their goods early.
Figure 5: This figure shows the cumulative wealth exchanged at any point in time during the experiment. The three graphs depict the three trials that were conducted on the preferential attachment trees; these are representative of the trials conducted on the unfair graphs in general. Note the rapid exchange of wealth occurring in the first 30 seconds of the experiment.

Placing the Blame for Low Social Welfare on Advantaged Nodes

So if we take our analysis above, that early exchange helps some disadvantaged nodes while hurting other disadvantaged nodes and hurting advantaged nodes, we get the testable prediction that disadvantaged nodes will perform about as well as they are expected to in equilibrium, while advantaged nodes will perform worse than they are expected to in equilibrium. This is exactly what is demonstrated in figure 6 for the unre wired clan graphs, which in this respect are representative of all of the unfair graphs. Thus, it is clear from the above figures as well as figure 6 that the “blame” of low social welfare may be placed on the advantaged nodes for underpricing their goods in the early game; one cannot fault opportunistic disadvantaged nodes for capitalizing on these low prices. This mispricing does not hurt the disadvantaged nodes in the aggregate, the advantaged nodes are only bringing down the social wealth by hurting themselves.

Figure 6: This figure demonstrates average performance of each class of nodes. Average performance is calculated by taking the sum of the wealths of all nodes of a certain degree, and dividing it by the sum of equilibrium wealths for nodes of that degree.

Of course, as is to be expected, high-degree players perform better than low-degree players when all players are expected to obtain the same wealth in equilibrium (this
is the case for the dense PA and .1 rewired clan graphs); this result is demonstrated in figure 7. It would appear that this is because the high-degree players simply have more offers to choose from, and because the network structure is not fully known these players can take advantage of their status. This also explains why the 18-degree vertices do better than the 8-degree vertices in figure 6, as they are both expected to achieve the same equilibrium wealth.

Figure 7: This figure also demonstrates average performance of each class of nodes, only this time for fair graphs. Just as unrewired clan graphs were representative of unfair graphs in figure 6, dense PA graphs are representative of fair graphs as far as this measure is concerned.

Inconclusive Analyses

I also did a few analyses that, while informative, produced no conclusive results. The first one was my analysis of offer size as a function of player degree my prediction was that lower-degree players would make larger offers, as they recognize that they generally get better deals than they should and will want to capitalize by unloading quickly (and of course vice versa for the higher-degree players). The data were not all that supportive of this hypothesis, with only trial 22 out of all of the unfair graph trials displaying this trend. The correlation between offer size and degree throughout all of the trials was approximately 0. This is another case where reviewing the player responses may reveal some information that the data are not conclusive on.

Figure 8: This figure demonstrates average offer size for each class of nodes. As is apparent, my prediction that offer size will decrease with degree is only borne out by the third graph, which seems to be enough evidence to reject the hypothesis or at least table it pending any further evidence.
I also examined if there was any relationship between whether a person's status as a price taker/price maker and other characteristics, specifically degree and final wealth. I thought it was possible that if a person was often a price taker with respect to the deals he executed during the trial, he would be at an advantage as opposed to if he was often a price maker. I could find no evidence that this was the case.

Finally, I looked at how average offer size evolved as the trials went on, as I thought there may be two influences pulling in the opposite direction. People may start making smaller offers if they considered underpriced regret a concern since they may be trying to test out the market more in trying to find a good price for their goods. Conversely, people who were concerned about overpriced regret may start making larger offers as they hope to sell out their goods before their trading partners do. In the aggregate, there seemed to be no clear trend as to how average offer size evolved as the trials went on.

Conclusion

After conducting this analysis, it seems to be apparent that humans are not as good at conducting exchange in environments which call for unequal exchange as they are at conducting exchange in more fair environments. Specifically, the well-positioned nodes tend to leave money on the table, which not only hurts themselves but locks some poorly-positioned nodes out of the market entirely, decreasing social welfare. In my second paper on this topic, I introduce a model that studies how one might combat this problem, by augmenting naturally-occurring networks (which are often unfair networks similar to preferential attachment trees) through allowing players to purchase edges.
References

