(*a*) Find $[\mathbf{T}]_{\mathcal{X}\mathcal{X}}$.
(*b*) Find the coordinate transformation $\mathbf{S}_{\mathcal{X}\mathcal{Y}}$.
(*c*) Use the answers to (*a*) and (*b*) to compute $[\mathbf{T}]_{\mathcal{Y}\mathcal{Y}}$ by means of a similarity transformation.

2.35 Multiplication by an invertible matrix can be interpreted either as a linear transformation or as a change of coordinates. Let $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2\}$ be a basis for a two-dimensional space $\mathcal{V}$ and $\mathbf{x}$ a vector in $\mathcal{V}$. Then $[\mathbf{x}_1]_{\mathcal{X}} = \begin{pmatrix}1\\0\end{pmatrix}$ and $[\mathbf{x}_2]_{\mathcal{X}} = \begin{pmatrix}0\\1\end{pmatrix}$. Let

$$[\mathbf{x}]_{\mathcal{X}} = \begin{pmatrix}2\\1\end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix}1 & -1\\1 & 0\end{pmatrix}$$

(*a*) Alias interpretation: assume $\mathbf{A}[\mathbf{x}]_{\mathcal{X}} = [\mathbf{x}]_{\mathcal{Y}}$, where $\mathcal{Y} = \{\mathbf{y}_1, \mathbf{y}_2\}$ is a second basis for $\mathcal{V}$. Find $[\mathbf{y}_1]_{\mathcal{X}}$ and $[\mathbf{y}_2]_{\mathcal{X}}$. Sketch $[\mathbf{x}_1]_{\mathcal{X}}$, $[\mathbf{x}_2]_{\mathcal{X}}$, $[\mathbf{x}]_{\mathcal{X}}$, $[\mathbf{y}_1]_{\mathcal{X}}$, and $[\mathbf{y}_2]_{\mathcal{X}}$ as arrows in a plane. What is the relationship between $[\mathbf{x}]_{\mathcal{X}}$ and the basis $\{[\mathbf{y}_1]_{\mathcal{X}}, [\mathbf{y}_2]_{\mathcal{X}}\}$; that is, what is meant by the notation $[\mathbf{x}]_{\mathcal{Y}}$?
(*b*) Alibi interpretation: assume $\mathbf{A}[\mathbf{x}]_{\mathcal{X}} = [\mathbf{Tx}]_{\mathcal{X}}$. Sketch $[\mathbf{x}_1]_{\mathcal{X}}$, $[\mathbf{x}_2]_{\mathcal{X}}$, $[\mathbf{x}]_{\mathcal{X}}$, and $[\mathbf{Tx}]_{\mathcal{X}}$ as arrows in a plane. What is the relationship between $[\mathbf{Tx}]_{\mathcal{X}}$ and the basis $\{[\mathbf{x}_1]_{\mathcal{X}}, [\mathbf{x}_2]_{\mathcal{X}}\}$; that is, what is meant by the notation $[\mathbf{Tx}]_{\mathcal{X}}$?

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