# University of Pennsylvania Department of Electrical Engineering 

# A VLSI COMPUTATIONAL SENSOR FOR THE DETECTION OF IMAGE FEATURES 

Masatoshi Nishimura

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Masatoshi Nishimura<br>A DISSERTATION<br>in<br>Electrical Engineering

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Jan Van der Spiegel
Supervisor of Dissertation

Kenneth R. Laker
Graduate Group Chairperson

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2001

Dedicated to Kazuko, Taku, and Mizuki

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#### Abstract

\title{ A VLSI computational sensor for the detection of image features }

Masatoshi Nishimura

Jan Van der Spiegel

A new type of VLSI computational sensor for the detection of image features is presented. The purpose of the proposed sensor is to overcome some of the limitations of pattern recognition systems, which is due to the inherent separation of image capture and analysis. It takes a fairly long time to transfer a huge amount of image data from an image sensor to a host CPU, which makes it difficult for the system to be used in real time applications requiring high speed. The proposed sensor detects important image features such as corners, junctions (T-type, X-type, Y-type) and linestops in a discriminative fashion. The on-chip detection of these features significantly reduces the data amount and hence facilitates the subsequent processing of pattern recognition.

The thesis deals specifically with: (1) development of an algorithm for the feature detection based on template matching; (2) implementation of the algorithm onto VLSI hardware; (3) investigation of a systematic design procedure based on transistor mismatch. The proposed algorithm is inspired by hierarchical integration of features found in the biological vision system. The algorithm first decomposes an input image into a set of line segments in four orientations, and then detects the features based on the interaction between these decomposed line segments. The algorithm is realized in the form of a CMOS sensor. An analog/digital mixed-mode approach is employed to achieve both compact implementation of neighborhood interaction in the analog domain and design flexibility in the digital domain. Experimental results demonstrate the sensor operates at fairly high speed. The circuit design is performed systematically using the proposed procedure to satisfy both accuracy and speed requirement.


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## Chapter 1 Introduction

### 1.1 Research background and motivation

Vision provides us invaluable information in every part of our life. The information obtained from an eye is much more direct and appealing than the information obtained from other sensory organs. This appealing nature is a natural consequence of the tremendous amount of information obtained as a result of the number of photoreceptors in the retina. There are two types of photoreceptors, the rods and the cones. The number of rods is 100 million while the number of cones is 5 million. It is surprising that such a huge amount of information is processed almost instantaneously to understand the scene surroundings us. It is the researcher's dream to build a machine that captures and instantaneously recognizes the image in order to solve real-world problems.

The goal of building such a machine has been approached from two separate sides: development of a high quality image sensor and investigation of an algorithm for pattern recognition. This is just the result of the extreme simplification of treating the retina as an input device and treating the brain as a processing device. The image sensor, which consists of an array of photoreceptors, performs phototransduction at each pixel to capture an image. The number of pixels is increasing year by year, reaching several millions these days, which is almost comparable to the number of cones. The obtained visual information is then passed onto a processing system, where various vision algorithms can be performed for pattern recognition.

The above conventional approach for pattern recognition, which consists of image capture and analysis, has two time consuming steps: the time required for image transfer and the time required for analysis. The former is determined by the video rate $(1 / 33 \mathrm{msec})$; the latter is determined by the amount of data and the algorithm employed. These two time-consuming
steps make it difficult for the conventional approach to be used for real-time pattern recognition such as high-speed product inspection on an assembly line and autonomous navigation. The problem is the inherent separation between image capture and analysis. It is instructive to look at the visual information processing system in biology, where the captured information is processed in an efficient way to recognize the image. A better understanding of the visual system should be investigated to extract the essence of processing so that it can be mapped onto hardware in some form.

The essence of the biological vision system is hierarchical integration of features. Along the visual pathway from the retina to the cortical area, cells in each layer in the hierarchy function as a detector for a certain image feature. One of the examples is the simple cell and the complex cell present in the visual cortex. These cells have orientation selectivity and respond to lines and edges aligned in a particular orientation. There are other cells known to respond to linestops, corners, and junctions. Features detected in a lower level are integrated in the nexthigher level to represent a more complicated feature. An object is represented as a set of these features in the final recognition layer. This model of pattern recognition based on hierarchical feature integration has the following advantages: (1) significant amount of data reduction is possible, (2) processing is fast due to massive parallelism, (3) recognition is robust to image deformation.

The above observations lead to new type of image sensors which incorporate some processing element at each pixel. They can be roughly divided into two categories, analog and digital, from the point of the type of implementation. The analog implementation is usually dedicated for a particular purpose while the digital implementation is more general purpose oriented. What is common for both implementations is the exploitation of parallelism, which is realized by placing a processing element in each pixel. The on-chip function realized by the additional processing element extracts relevant information for subsequent processing of pattern


Figure 1.1 Various shapes of letter "A". Even though the shape is different, all these letter are categorized as "A".
recognition, resulting in a reduced amount of data.
The features detected by the analog implementation are primarily edges and orientations. These are relatively low level features corresponding to those detected at early stages in the visual pathway. Although the features that can be detected in the digital implementation depend on the architecture and the programming flexibility, no attempt has been made so far to detect higher level features such as corners and junctions. Existing software algorithms for these features are too complicated to be mapped on these digital implementations.

It should be mentioned at this point that while all those implementations incorporate parallel processing capability, the concept of hierarchical processing, which is another important characteristic in the biological vision system, is not taken into account. The realization of the hierarchy in some form may lead to a new type of sensor for the detection of higher level features. This is the motivation of the research presented in the thesis. The thesis proposes a new type of computational sensor for the detection of the following features: corners, T-type junctions, X-type junctions, Y-type junctions, and linestops. These are considered very important set of features characterizing an object. For example, think about various shapes of letter "A" shown in Figure 1.1. These four letters are all considered "A" despite its variation in shape. What is common for these letters is the presence of the corner pointing upward at the top, T-type junctions on both sides, and two linestops of the vertical line segments at the bottom.


Figure 1.2 Two types of pattern recognition systems. (a) Newly proposed pattern recognition system using the feature detection sensor as a front-end. (b) Conventional pattern recognition system. Note that the feature extraction carried out in a CPU in (b) is incorporated inside the sensor in (a).

Such a characterization is not possible simply by using lower level features consisting of edges and orientations. Therefore, detection of these features is important for the recognition of an object.

The configuration of the possible pattern recognition system using the proposed sensor is shown in Figure 1.2 (a). The sensor extracts the above-described features of an input image on chip using some form of hierarchical processing. Only extracted features are transferred to the host CPU, where the input image is classified as "A" based on the extracted features. The advantage of this approach is illustrated when compared to the conventional system shown in

Figure 1.2 (b), where all the pixel information is transferred to the CPU. The CPU first performs feature extraction for the subsequent recognition task. The difference between (a) and (b) is where preprocessing of feature detection is performed. Feature detection on chip is much faster than the preprocessing on the CPU since the processing is carried out in a hardware level in a parallel fashion. Feature detection reduces the data amount, which results in faster data transfer and facilitates the final recognition task.

Although the above example is specifically for the application of character recognition, it can be extended for various applications. Think about the case of autonomous navigation. An autonomous vehicle always has to detect where the road is in front of the vehicle. For this purpose, the vehicle probably has to detect both sides of the road as well as the centerline, which are important guidelines for navigation. The vehicle usually keeps following the centerline. When the vehicle approaches an intersection, it detects the corner on one of the two sides of the road and makes a turn based on the angle of the corner. For safe navigation, detection of the centerline with its orientation as well as the corner has to be carried out at high speed. The proposed sensor will best suit this type of applications. It also has the potential to have a significant impact on other applications that require high-speed pattern recognition.

### 1.2 Contribution of the thesis

The thesis examines several aspects concerning the design and implementation of an image sensor for the detection of the image features. The primary contributions of the thesis are: (1) development of an algorithm for the feature detection based on template matching; (2) implementation of the algorithm onto VLSI hardware, (3) investigation of a systematic design procedure based on transistor mismatch. Each of these contributions is briefly summarized below.
(1) An algorithm for the detection of image features for a binary image is proposed. The
detected features are corners, T-type junctions, X-type junctions, Y-type junctions, and linestops. No algorithm has been presented which extracts these features in a discriminative fashion. The proposed algorithm is inspired by the hierarchical integration of features found in biology. The algorithm first decomposes an input image into a set of line segments in four orientations, and then detects the features based on the interaction between these decomposed line segments. With final hardware implementation in mind, the algorithm performs a $3 \times 3$ template-matching operation in an iterative fashion.
(2) The algorithm is implemented in the form of a CMOS optical sensor. The sensor contains an array of $16 \times 16$ pixels, each measuring $150 \mu \mathrm{~m} \times 150 \mu \mathrm{~m}$, in a chip area of 3.2 $\mathrm{mm} \times 3.2 \mathrm{~mm}$. The analog/digital mixed-mode architecture is employed to achieve both compact implementation of $3 \times 3$ neighborhood interaction in the analog domain and design flexibility in the digital domain. The internal operation within a $3 \times 3$ neighborhood is carried out by the current distribution and thresholding operation. To speed up the operation of the sensor, a high speed charging/discharging mechanism and another mechanism for node-locking were implemented. The sensor is able to detect the image features on chip in about $50 \mu \mathrm{sec}$.
(3) For the determination of the transistor dimension and the reference current, a systematic design procedure for current-mode circuits is proposed. The proposed procedure converts the requirement for accuracy, speed, and the operating region to the specifications for the transistor dimensions and the reference current. The design parameters are chosen to satisfy all these three requirements. This procedure is based on the formulation of the current variation due to transistor mismatch.

The organization of the thesis is as follows. Chapter 2 describes the algorithm for the feature detection. This chapter includes the survey of biological findings, the mathematical formulation of the algorithm, and the evaluation of the algorithm. Chapter 3 presents the
architectural design of the sensor. Based on the past surveys, an analog/digital mixed mode architecture is selected. Chapter 4 presents the systematic design procedure for current-mode processing circuits. Transistor mismatch is analyzed and modeled for the formulation of the design procedure. Chapter 5 shows implementation details and experimental results obtained using the prototype sensor to characterize the sensor performance. Finally, Chapter 6 concludes the thesis.

## Chapter 2 Algorithm for feature detection

In this chapter, an algorithm for feature detection is described. First, the objective of the algorithm is defined as the discriminative detection of corners and three types of junctions (Ttype, X-type, Y-type). Then biological findings concerning the visual pathway are surveyed to confirm the presence of feature detectors and to understand the mechanism of hierarchical integration for pattern recognition. Existing algorithms for feature detection are also reviewed. From these observations, the algorithm for feature detection is proposed at a conceptual level. Then the mathematical framework of template matching in a $3 \times 3$ neighborhood is presented to formulate the algorithm. The relationship between template matching and other image processing methods is also discussed. Based on the framework of template matching, the detail of the algorithm is described as a sequence of processing flow. The proposed algorithm is applied to printed and handwritten characters, as well as several patterns, to demonstrate its performance. Finally, pros and cons of the algorithm are discussed to conclude this chapter.

### 2.1 Problem definition

It is well known that corners and junctions carry rich information about the structure of an object [1]. This is obvious in the example shown in Figure 1.1, where a set of these geometrical features, together with linestops, characterizes the letter "A". Another example demonstrating the significance of these geometrical features for pattern recognition is shown in Figure 2.1. The three-dimensional view of the two cubes shown in (a) contains corners and junctions of the Y-type and T-type. Corners and Y-type junctions always exist in the image of a solid body viewed from a certain angle. The presence of T-type junctions indicates the occlusion between multiple objects: the rear cube is occluded by the front one. The occlusion is


Figure 2.1 Schematic explaining how different types of junctions are generated in different configurations of objects. For each image, junctions are superimposed on an edge-extracted image shown below the corresponding original image. (a) Three-dimensional view of two cubes. The front one occludes the rear one. (b) Two sheets in which an opaque one is placed on top of the other. (c) Two sheets in which a transparent one is placed on top of the other. (d) Checkerboard pattern.
also found in (b) where the top sheet occludes the bottom one. The image shown in (c) also contains two sheets. However, the top one is transparent and hence X-type junctions are generated. T-type junctions and X-type junctions are important cues for occlusion and transparency, respectively. These junctions are not only found in images including several objects but also found in textures. For example, the checkerboard pattern in (d) produces Xtype junctions. These examples demonstrate the usefulness of these features for object understanding and pattern recognition.

The detection of these features is very important from a practical point of view. They are robust with respect to changes in perspective or small distortions. In addition, they are zerodimensional and do not cause an aperture problem. Therefore, they can be used to make correspondence between multiple images obtained from different perspectives (see ref. [2], for


Figure 2.2 Features of interest. (a) Corner. (b) T-type junction. (c) X-type junction. (d) Y-type junction. (e) Linestop.
example). Although corner detection in real-world problems should deal with a gray image, once edges are properly extracted in a preprocessing stage, the problem can be simplified as the detection of these features for an edge extracted image. Then the situation becomes somewhat similar to that shown in Figure 1.1, where the features are detected for characters, which are basically line drawings. For this reason, the thesis focuses on binary images, i.e, black and white images, especially characters, which are considered good examples of line drawings. Characters are also suitable to test the performance of the algorithm since there are many variations and deformations. The features we would like to identify are listed in Figure 2.2. Note that linestops are included in this feature set. It is important to be able to detect these features for line drawings with any line width.

To summarize, the objective of the algorithm is to detect and locate these features (corners, T-type, X-type, and Y-type junctions, linestops) in a discriminative fashion for a binary image. It is also important for the algorithm to be relatively simple so that it can be eventually implemented in hardware.

### 2.2 Feature detection and pattern recognition in biology

Since our primary objective is the detection of corners and junctions, it should be helpful and suggestive to investigate if there are detectors for these features in biological vision system. If that is the case, how is an object recognized based on these features? In the following, several biological findings are described to introduce different levels of feature detectors present
in the visual pathway and their possible role in pattern recognition.
The visual information processing starts at the retina where the intensity of an incoming light is converted to an electrical signal. Phototransduction is carried out by two types of photoreceptor cells, the cone and the rod. The cone is responsible for day vision while the rod is responsible for night vision. The number of the cone is 5 million while the number of the rod is 100 million [3]. In addition to these two cells, there are four other types of cells in the retina: the horizontal cell, the bipolar cell, the amacrine cell, and the ganglion cell. These cells belong to one of the two layers, inner plexiform layer and outer plexiform layer, depending on their physical location. The outer plexiform layer consists of the photoreceptors (rods and cones), the horizontal cells, and the bipolar cells, while the inner plexiform layer consists of the amacrine cells and the ganglion cells.

In the outer plexiform layer, the cones and the rods are connected to the horizontal cells as well as to the bipolar cells. The horizontal cells, which are mutually connected in a lateral direction, spread the input signal from the photoreceptors and hence produce a spatially smoothed version of the incoming signal. The bipolar cell receives excitatory inputs from the photoreceptors and inhibitory inputs from the horizontal cells, and thus produces an output that is equal to the difference between the photoreceptor signal and the horizontal cell signal. This results in a concentric ON-center type receptive field ${ }^{1}$. The ON-center receptive field indicates that the light that falls onto the center area of the receptive field excites the bipolar cell while the light that falls onto the periphery inhibits the bipolar cell. Already at this level in the visual information pathway, a preliminary level of feature detection is performed: the bipolar cell responds to edges of an object where intensity changes sharply, while it responds poorly to uniform illumination [4].

[^0]

Figure 2.3 Orientation selectivity of the cell response. (a) Response of the simple cell to stimuli of different orientations. (b) Hypercolumn structure.

Further signal modification and integration are carried out in the inner plexiform layer to produce an output at the ganglion cell, which serves as the final station of the retinal processing. The activity of the ganglion cell seems to be made up by the total contribution of the other four cell types. The receptive field of the ganglion cell takes an ON-center shape, which is almost similar to that of the bipolar cell. The difference is that the output is represented by the firing frequency at the ganglion cell while the output is represented simply by the level of the potential at the bipolar cell.

From the ganglion cells runs the optic nerve through lateral geniculate nucleus (LGN) to the cortical area V1, where three types of cells, the simple cell, the complex cell, and the hypercomplex cell, exist. The simple cell responds to a bar of light aligned in a certain orientation as shown in Figure 2.3 (a). For a receptive field that is vertically oriented, the bar stimulus in the vertical orientation elicits a largest number of pulse sequences. As the bar is
rotated from this preferential orientation, the response decreases quickly. This is quite different from the response of the ganglion cell, which does not have any orientation selectivity. The complex cell, like the simple cell, also responds to stimulus aligned in a certain orientation. However, the demand for precise positioning found in the simple cell is relaxed in the complex cell. As long as a properly oriented stimulus falls within the boundary of the receptive field, the complex cell responds. The third type of cell, the hypercomplex cell, needs more refined shape of stimulus for excitation. They require that the stimulus have discontinuity. The simple line stimulus, even if it is aligned in the preferential orientation, does not excite the cell completely. Consequently the best stimulus results in linestops and corners (they are also called the endstopped cell for this reason). This is one of the feature detectors that is interest to us.

The receptive fields of these cells in area V1 are not arranged in a random order: there is a clear retinotopic mapping present such that adjacent cells have adjacent receptive field positions in the retina. Orientation selectivity also has a nonrandom arrangement: there are vertical columns through the thickness of the cortical sheet containing cells with similar orientation preferences. Each column is about $30-100 \mu \mathrm{~m}$ wide and 2 mm deep. The preferential orientation shifts from one column to the next column by about $10^{\circ}$. A set of these columns, which covers the entire orientation, is termed hypercolumn by its discoverer Hubel and Wiesel [5] (see Figure 2.3 (b)). Each local area in the retinotopic map has a corresponding hypercolumn. It should be also noted that the cortical cells in area V1 do not respond to uniform illumination. Therefore, in area V1, these cortical cells extract edges or contour of an object and represent it as a set of line segments in different orientations.

The extracted information is further transferred from area V1 to V4, thereafter to the posterior part of inferotemporal cortex (PIT), and finally to the anterior part of inferotemporal cortex (AIT) through the ventral pathway, which is believed to be responsible for object


Figure 2.4 Shape of stimulus which excites the cell at each stage along the visual pathway.
recognition. The complexity of the stimulus necessary for cell excitation increases along this pathway. At the AIT, cells which respond to T-type junctions were reported by Fujita et al. [16]. This is another type of feature detector of interest. Actually the typical pattern for the cell excitation ranges from relatively simple features such as T-type junctions or star-shaped stimulus, to a rather complicated one defined by a combination of several objects such as dark circular area above a slightly larger white circular region.

Like in area V1, columnar organization is also found in this area. Cells belonging to one column, whose size is about $400 \mu \mathrm{~m}$, respond to similar patterns, while cells belonging to different columns respond to totally different pattern [6]. However, there is variation in the cell response even in the same column. The following case was reported. Two cells belonging to the same column respond to a vertical bar protruding from the base structure. The response is different between these two cells for different intersection angles. Cell 1 responded best to right angles and maintained $50-60 \%$ of the maximum activation to other angles, while cell 2 responded only to right angles. As this example indicates, the clustering of cells with overlapping but slightly different selectivity works as a buffer to absorb the small variation of
the input pattern [7]. These observations lead to the following hypothesis: an object is probably represented in the AIT by a set of activated columns, each corresponding to different patterns, with some variations allowed within each column.

As explained above, the complexity of the stimulus for cell excitation increases toward the higher levels in the hierarchy, which is schematically shown in Figure 2.4. The size of the receptive field also increases as the complexity of the stimulus increases. The increase in the complexity is achieved by combining the output form several cells having different selectivity at earlier stages in the visual pathway. For example, the orientation selectivity of the simple cell can be obtained by combining the output of the ganglion cell aligned in a certain orientation. The response of the complex cell can be obtained as the combination of the simple cells having the same orientation selectivity but with different spatial positions. During the process of feature integration, the position of a particular feature necessary to excite the cell in the next layer is allowed to shift to some extent. The positional tolerance at each level in the hierarchy leads to a large variation in the input pattern for cell excitation in the final recognition stage. This is how the cell in the AIT achieves shift invariance. The response of the cell is essentially constant throughout its receptive field, which is much larger than that in the earlier stages in the visual pathway.

The above idea of hierarchical feature integration is clearly explained in Fukushima's paper where he proposed neocognitron model for pattern recognition [8]. Figure 2.5 shows the idea of pattern recognition taking letter "A" as an example. Suppose that there are three processing layers. The input image in layer $U_{0}$ is first mapped onto layer $U_{1}$. The cell in layer $\mathrm{U}_{1}$ has a corresponding receptive field in layer $\mathrm{U}_{0}$ and fires when the stimulus pattern that falls onto its receptive field resembles the reference pattern. The cell shown at the top responds to the presence of $\mathrm{a}^{\wedge}$-shaped corner. Another cell specifically responds to slanted T-type junctions while the other cell responds to vertical line segments that stop within its receptive field. These


Figure 2.5 Neocognitron model of pattern recognition (adapted from Fukushima [8] and slightly modified).
features are further integrated in layer $\mathrm{U}_{2}$. The cell at the top responds to the stimulus which has a combination of a ${ }^{\wedge}$-shaped corner at the top and two slanted T-type junctions with opposite directions at the bottom. Likewise, there exist cells in layer $\mathrm{U}_{2}$, which detect a combination of slanted T-shape junction and a linestop of the vertical line. In the final recognition layer $\mathrm{U}_{3}$, features are further integrated to generate a cell that is excited by the presence of all three features in the preceding layer $U_{2}$. The resultant requirement to excite the cell in layer $U_{3}$ is that there is a ${ }^{\wedge}$-shaped corner at the top, two slanted T-type junctions on both sides, and two linestops of a vertical line segments at the bottom, which extracts the essence of letter "A". In other words, the letter "A" is characterized by a set of these features.

Representation of an object by a set of features results in a significant amount of data compression. In addition, by allowing some positional tolerance for a feature at each level in the hierarchy, the final layer is able to recognize deformed letters "A". It should be also noted that the whole processing is carried out in a massively parallel fashion, resulting in an enormous amount of computational power even it is performed by slowly operating neurons.

To summarize this section, the biological vision system is characterized by hierarchical
integration of features. Detectors for corners and junctions do exist in this hierarchy. These features are formed from lower level features such as edges or linestops in different orientations, and they are further integrated to higher level features to finally represent an object as a set of these higher level features. These observations not only confirm the importance of the detection of corners and junctions for pattern recognition, but also give some clue for the implementation of these features.

### 2.3 Survey of the algorithms for corner and junction detection

Having understood the importance of corners and junctions in pattern recognition and the presence of detectors for these features, we are ready to implement these feature detectors borrowing some ideas from biology. Before going into detail, however, several algorithms for detection of these features, some of which are biology based and some of which are not, are briefly discussed below.

Fukushima implicitly implemented detectors for corners and junctions in his later work [9], where he applied the neocognitron concept to the character recognition problem in the form of a five-layered neural network. To form the receptive field of different complexities in different layers, appropriate patterns are used to train each layer. The second layer is trained to respond to line segments in eight orientations $\left(0^{\circ}, 22.5^{\circ}, 45^{\circ}, \cdots, 157.5^{\circ}\right)$. The third layer is trained to respond to different combinations of the line segments detected in the second layer, which represents corners with different angles and junctions of different types (T-type, X-type, Y-type). The trained network was able to correctly classify input patterns even with its position shifted or with its shape deformed. Although this work demonstrates the excellent example of the neural network architecture for pattern recognition, these types of feature detectors do not explicitly detect and locate the position of features and do not fit our interest.

Actually within the framework of the neural network, explicit implementation of feature detectors other than oriented line segments and linestops have not been reported (for example, see refs. [10], [11], [12]). This is because the direct detection of corners and junctions is not as simple as it first looks, which is explained in Section 2.5.

Heitger et al. have proposed an "endstopped operator", which mimics the function of the endstopped cell in the visual system [13]. They first convolved an input image with even and odd symmetrical orientation selective filters and combined the output of these two filters to compute a local energy measure. The output image corresponds to the output of the complex cell, where edges and lines are enhanced in a preferential orientation. Then the differentiation (the first derivative and the second derivative) of this output image is calculated along the preferential orientation. The local maxima of combined endstopped operators for all orientations indicate the position of what they call "key-points". The simulation result showed that the endstopped operator was able to detect linestops, corners, and T-type junctions. The detector would be able to detect Y-type junctions with less specificity. However, it would be difficult to detect X-type junctions because they do not produce a local maximum at these points.

Freeman also applied the similar approach for the detection of corners, T-type junctions and X-type junctions [14]. His method computes the local energy along multiple orientations to find two dominant orientations along which edges are represented. Then the stopped-ness is calculated as the derivative along these two orientations. If stopped-ness is high in two orientations, the junction is detected as the corner. If stopped-ness is low in one of the two orientations, it is classified as the T-type junction. If stopped-ness is not low in both orientations, the junction is classified as the X-type junction. In other words, this method keeps the orientation information and defines the type of junctions based on how the components in two orientations interact with each other. The only restriction for this method is that the number
of dominant orientations is limited to two, which would make it difficult to find Y-type junctions.

Dobbins et al. have modeled the output of the hypercomplex (endstopped) cell as the difference of two simple cells with a different size of the receptive field. This operation results in the receptive field that has the excitatory center zone and the inhibitory zone at both ends, which is similar to the result of the second derivative along the preferential orientation. The modeled endstopped cell showed the curvature dependent response [15] [16]. Manjunath et al. have proposed an almost similar approach and applied their method for practical applications such as face recognition, image registration, and motion estimation [2].

Apart from these biology-based approaches, there are two popular methods for corner detection. One is what is known as Plessey detector proposed by Harris and Stephens [17] while the other is known as SUSAN detector proposed by Smith and Brady [18]. The Plessey detector calculates the derivative of image intensities and defines the average squared gradient matrix. The feature is detected as the point where a certain measure computed from the average squared gradient matrix takes a local maximum in the image plane. Although this algorithm can detect corners and three types of junctions (T-type, X-type, Y-type), the detected point does not exactly coincide with the true location of these features. The performance of the SUSAN detector is superior in this sense. The SUSAN algorithm defines a circular area for each point with the center of the circle located on that point and counts the number of pixels within the circle which has the same intensity as the center. Note that no derivative is computed in this algorithm, which gives superior noise immunity [18]. The counted number takes a maximum value in the uniform region and decreases to its half at straight edge points and decreases even more at corners and junctions. Thus, these features are detected by finding local minima of the counted number. One of the possible problems of this method is that it does not seem to work for binary edge images. Also, note that no orientation information is attached to the detected
feature points. Consequently, the determination of the junction type is difficult.
All the methods so far described perform computation locally. For line drawings, there are other methods proposed which need searching neighborhood pixels along the line segment. See refs. [19], [20], and [21], for example. These methods require lots of computation but still cannot detect junctions very well.

### 2.4 Proposed approach

Each algorithm surveyed in the previous section has its advantages and disadvantages. The only method to partly satisfy the demand of discriminative identification of corners and junctions is Freeman's method despite its expected difficulty for the detection of Y-type junctions. The other methods cannot discriminate corners and junctions. Freeman's method is biology based in the sense that orientation decomposition with simultaneous edge extraction is performed as preprocessing.

Having a look at Figure 2.2 again, it is obvious that these five features are defined as a result of interaction between line segments and linestops of different orientations. The corner is defined as the point where two line segments of different orientations meet at its linestop. The T-type junction is defined as the point where the linestop of a line segment meets another line segment of different orientation. The X-type junction is defined as the point where at least two line segments of different orientations meet. The Y-type junction is defined as the point where at least three line segments of different orientations meet at its linestop. This simple definition of these features leads to the conceptual processing flow of the proposed algorithm, which is shown in Figure 2.6. The algorithm finally produces the five features, which are shown in the boxes with a thick boundary line, after several steps of hierarchical processing based on orientation decomposition.

The first processing is the thinning of an input image. Thinning is a process of reducing


Figure 2.6 Algorithm for feature detection at a conceptual level. The definition of each feature is as follows. totalJCT: the point where at least two line segments in different orientations are present; totalLS: the point where at least one linestop in any orientation is present; totalC: the point where at least two linestops in different orientation planes are present; JCT-Y (Y-type junction): the point where at least three linestops in different orientations are present; JCT-XT: the point which belongs to totalJCT and does not belong to totalC; JCT-X (X-type junction): the point which belongs to totalJCT and does not belong to totalLS; trueLS (linestops): the point which belongs to totalLS and does not belong to totalJCT; trueC (corner): the point which belongs to totalC and does not belong to JCT-Y; JCT-T (T-type junction): the point which belongs to JCT-XT and does not belong to JCT-X. The five features shown in the boxes with a thick boundary, trueC, JCT-T, JCT-X, JCT-Y, trueLS, correspond to the features listed in Figure 2.2.
the line width to a single pixel to represent a structural shape of an object. This process can be skipped when dealing directly with a binary edge map, where the line has a single pixel width. However, for dealing with line drawings such as characters, thinning is necessary as preprocessing to normalize the line width to a single pixel so that the subsequent algorithm for feature detection is not influenced by the original line width. Note that the process of thinning is introduced from a purely engineering point of view: there is no evidence that thinning is performed in the biological visual processing.

The thinned pattern is decomposed into four orientation planes, i.e., $0^{\circ}, 45^{\circ}, 90^{\circ}$, and $135^{\circ}$ orientation planes, each of which contains line segments of the designated orientation. For example, letter "A" is decomposed into three line segments of different orientations, $0^{\circ}, 45^{\circ}$, and $135^{\circ}$ orientations. Note that there is orientation tolerance to some degree: each line segment is classified into one of the four orientations that is closest to its orientation. The orientation decomposition can be considered a simplified realization of the processing which takes place at the hypercolumn in the brain, although the resolution of orientation decomposition is $45^{\circ}$, which is much lower than that found in the hypercolumn.

The next processing after orientation decomposition is the detection of linestops for each line segment. This processing corresponds to that of the hypercomplex cell (endstopped cell) in biology. Then based on the oriented line segments and linestops, higher level features are computed as described below. totalJCT is defined as the point where at least two line segments in different orientations are present. The operation for this detection is designated as AND2 in the figure: AND2 operation sets the pixel value to 1 where at least two pixels whose value is 1 are present among the four orientations. For letter "A", totalJCT includes two junctions on both sides and the corner at the top. In contrast to totalJCT, which are derived from oriented line segments, the other three features, totalLS, totalC, and JCT-Y, are detected from linestops. totalLS is detected as the point where at least one linestop in any orientation plane is present, which is obtained by OR operation for four linestop images (LSa, LSb, LSc, LSd) as shown in the figure. Likewise, totalC is defined as the point where at least two linestops in different orientations are present, which is obtained by AND2 operation. JCT-Y is defined as the point where at least three linestops in different orientations are present, which is obtained by AND3 operation. JCT-Y is one of those finally detected five features (Y-type junction). Note that totalLS includes totalC as its subset, which further includes JCT-Y as its subset.


Figure 2.7 Venn diagram showing the relationship between the five features detected by the proposed algorithm.

From these features, trueC is detected as the point belonging to totalC but not belonging to JCT-Y. This operation is shown in the figure as receiving the excitatory input from totalC and the inhibitory input from JCT-Y to generate trueC. The X-type junction (JCT-X) is then detected as the point that belongs to totalJCT and does not belong to totalLS. Likewise, true linestops (trueLS) is detected as the point that belongs to totalLS but does not belong to totalJCT. Note that these features are detected as a result of excitatory and inhibitory interactions between several features. These interactions result in the increased level of selectivity for each of these resultant features, which is analogous to the formation of the receptive field of a cell from the receptive field of cells at lower levels. The T-type junction (JCT-T) is detected as a result of the excitatory input from JCT-XT, which is an intermediate result obtained by the excitatory input from totalJCT and the inhibitory input from totalC, and the inhibitory input from JCT-X. Thus the final five features shown in Figure 2.2, trueC, JCT-T, JCT-X, JCT-Y, and trueLS, are produced. The relationship between these features is represented in Figure 2.7 in the form of Venn diagram. The set relationship between several features should
be associated with the excitatory and inhibitory arrows shown in Figure 2.6.

### 2.5 Description of the template matching algorithm

In the previous section, an algorithm for feature detection is described at a conceptual level. To formulate the algorithm, the mathematical framework of template matching is given in this section.

### 2.5.1 Basic framework

The mathematical framework employed in the thesis to formulate the algorithm is template matching in a $3 \times 3$ window. For a given input image, the updated image is computed as follows. For every pixel in the input image, the updated status is set to 1 if its $3 \times 3$ neighbors match the given template with some specified tolerance, and is otherwise set to 0 . This procedure is mathematically represented as

$$
\begin{equation*}
x_{i j}(n+1)=f\left(\sum_{l=-1}^{1} \sum_{m=-1}^{1} x_{i+l, j+m}(n) r_{l m} ; I\right) \tag{2.1}
\end{equation*}
$$

where $x_{i j}(n)$ is the binary status of the pixel at the position $(i, j)$ at a discrete instant $n, r_{i j}$ is the element of the template, $f$ is the function to generate a binary output using the threshold $I$ given in the form below:

$$
f(x ; I)=\left\{\begin{array}{lll}
1 & \text { for } & x \geq I  \tag{2.2}\\
0 & \text { for } & x<I .
\end{array}\right.
$$

The template is represented as

$$
R=\left(\begin{array}{ccc}
r_{-1-1} & r_{-10} & r_{-11}  \tag{2.3}\\
r_{0-1} & r_{00} & r_{0-1} \\
r_{1-1} & r_{10} & r_{11}
\end{array}\right) .
$$

Each element of the template takes one of the following three values: $-1,0$, or 1 . Value of 1 imposes that the pixel value at the corresponding position should be 1 ; Value of -1 imposes
that the pixel value at the corresponding position should be 0 ; Value of 0 does not impose any restriction for the corresponding pixel position ("don't care"). For simplicity, let us denote the template matching using the template $R$ and the threshold $I$ as

$$
\begin{equation*}
X(n+1)=T_{R, I}(X(n)) \tag{2.4}
\end{equation*}
$$

where $X(n)$ represents the whole image plane, i.e., a whole set of $x_{i j}(n)$, at instant $n$.
The implication of the above computation is briefly explained below. Template matching is nothing but the calculation of correlation. First, at any position in the image plane, the correlation between its $3 \times 3$ neighbors and the template is calculated: each element is multiplied by the correspondong coefficient in the template $R$ and the result is accumulated. Then the accumulated result is compared to the threshold $I$ to digitize the output. Depending on how to construct the template and the threshold value, the strictness of the template matching can be controlled as shown in Figure 2.8.

The first example shown in (a) demonstrates the case of exact matching: the template does not contain 0 and the threshold is 4.5 , which is smaller than the number of pixels whose value is 1 by one half. Only the center of the cross on the top left corner in the image plane is kept in the updated image, since the correlation result is 5 and is greater than the threshold only at this point. Any other point in the image plane is not selected. The only possible pattern to satisfy the criteria for selection is the template itself with its -1 element interpreted as 0 .

The incorporation of 0 in the template gives "don't care" condition to the pixel value at the corresponding position as shown in Figure 2.8 (b). This is the first level of stringency control in template matching. Since the pixel value at the "don't care" position does not contribute to the correlation computation, the bottom left pattern in the image plane, is selected in the updated image. The second level of stringency control in template matching is realized by the decrease in the threshold value as shown in Figure 2.8 (c), where the threshold is set to


Figure 2.8 Example of template matching. The input image containing four patterns on the left is processed by the template and the threshold shown above the arrow to produce the image on the right. Patterns to be selected are shown below the arrow. A black pixel corresponds to 1 while a white pixel corresponds to 0 ; " $\times$ " mark indicates "don't care". (a) A pattern that exactly matches the template is selected. (b) The presence of 0 at the top right corner in the template gives tolerance for this position to select the left bottom pattern in the image plane. (c) The decreased threshold value gives even more tolerance to select all the four patterns.
3.5, smaller than 4.5 used above for exact matching. The lowered threshold value increases the number of allowable patterns from one to eight, which are shown below the arrow. Hence, the top right pattern in the image plane in which one of the elements constituting the cross is missing, is selected as well as the other two patterns at the bottom in which an additional pixel of 1 is included, since all these patterns produce the correlation value of 4 .

It is worthwhile at this point to investigate if template matching can be really used for the


Figure 2.9 Orientation detection using template matching. The rectangular input pattern is used instead of line drawing patterns to demonstrate the selectivity of the template for the top edge and the bottom edge.
three important operations shown in Figure 2.6. These operations include orientation decomposition, linestop detection, and processing based on interaction between several features. Figure 2.9 shows the example of orientation detection. The pattern defined in the template has directional selectivity for horizontal direction. It extracts the top edge of the rectangular input pattern and does not respond to uniform illumination as well as to the bottom edge. The template pattern is similar to the receptive field of the simple cell shown in Figure 2.3 although the pattern in the template corresponds to only half of the receptive field: the template does not extract the bottom $0^{\circ}$ edge for this reason. The receptive field similar to that of the simple cell is possible to implement by using the following template:

$$
R=\left(\begin{array}{ccc}
-1 & -1 & -1  \tag{2.5}\\
2 & 2 & 2 \\
-1 & -1 & -1
\end{array}\right)
$$



$$
\begin{aligned}
& R=\left(\begin{array}{ccc}
0 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 0
\end{array}\right) \quad I=0.5 \\
& R=\left(\begin{array}{ccc}
0 & -1 & 0 \\
-1 & 1 & -1 \\
0 & -1 & 0
\end{array}\right) \quad I=-0.5
\end{aligned}
$$



Figure 2.10 Linestop detection based on template matching. The combination of the template and the threshold shown above the arrow is equivalent to the combination of the template and the threshold shown below the arrow.
if the value of 2 is allowed for elements in the template. For dealing with the four orientations, it is necessary to allow the value of 2 for all elements in the template, which complicates the template construction in hardware. Therefore, the template corresponding to the top edge and the bottom edge is separately realized. Figure 2.9 also shows the effect of the threshold. All the points along the top edge are not selected with a threshold of 2.5 while all these points are selected with a threshold of 1.5 .

The second operation important for the feature detection is the detection of linestops. Figure 2.10 shows the application of template matching for this purpose. By using the template for which the value of 2 is allowed at the center with the threshold value of 0.5 enables the detection of linestops regardless of the orientation of line segments. The template selects pixels that have only one neighbor in either horizontal or vertical direction as linestops. Pixels located in the middle of the line, which has two neighbors, are deleted by the negative contribution from two neighbors. Note that the threshold value of 0.5 , which indicates lowered stringency, makes the implementation of feature detection scheme simple: a set of different templates can be represented by only one template.

The third operation important for the detection of higher level features is the realization of excitatory and inhibitory interaction between multiple input images. The three operations
shown in Figure 2.6, OR, AND2, and AND3, are defined as the operation to set the pixel value to 1 where at least one, at least two, and at least three line segments are present in different orientation planes, respectively. These operations are naturally represented as the special case of template matching as

$$
\begin{equation*}
X(n+1)=T_{E, I}\left(X_{a}(n)+X_{b}(n)+X_{c}(n)+X_{d}(n)\right) \tag{2.6}
\end{equation*}
$$

with

$$
\begin{gather*}
E=\left(\begin{array}{lll}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right),  \tag{2.7}\\
I=\left\{\begin{array}{lc}
0.5 & \text { for OR operation } \\
1.5 & \text { for AND2 operation } \\
2.5 & \text { for AND3 operation. }
\end{array}\right. \tag{2.8}
\end{gather*}
$$

where $X_{a}, X_{b}, X_{c}$, and $X_{d}$ designate the $0^{\circ}, 45^{\circ}, 90^{\circ}$, and $135^{\circ}$ orientation plane, respectively, and $E$ is the special template focusing only the present pixel. The plus sign which appears in eqn. (2.6) represents the numerical summation instead of logical OR.

As demonstrated above, the three important operations for the algorithm for feature detection were found possible to implement based on the framework of template matching. Next, let us consider the possibility of direct detection of corners. Figure 2.11 shows the difficulty of corner detection using a $3 \times 3$ template. No unique mask exists for the inverted Ltype corner. In cases (a) and (b), the corner is correctly detected with different templates. However, when the combination of the pattern and the template is changed, no corner is detected in case (c) while a spurious corner is detected in (d). This is due to the small window size of $3 \times 3$. The only way to circumvent this problem is the increase of the template size to cover all possible types of corners. However, an interaction area larger than $3 \times 3$ is not practically feasible from an implementation point of view. These observations again justify the


Figure 2.11 Schematic explaining the difficulty of corner detection using a $3 \times 3$ template. In (a) and (b), the corner is correctly detected with different templates. When the combination of the pattern and the template is changed as shown in (c) and (d), no corner is detected in (c) and misclassification occurs in (d).
approach of the proposed algorithm where the corner is defined as the intersection of linestops of lines in different orientations.

Before going into the detail of the algorithm, it is useful to mention the relationship between the template matching and the convolution operation. Eqn. (2.1) can be rewritten as

$$
\begin{equation*}
x_{i j}(n+1)=f\left(\sum_{l=-1}^{1} \sum_{m=-1}^{1} x_{i-l, j-m}(n) k_{l m}\right) \tag{2.9}
\end{equation*}
$$

where

$$
\begin{equation*}
k_{l m}=r_{-l-m} \tag{2.10}
\end{equation*}
$$

Eqn. (2.9) represents the convolution operation for image $X(n)$ with the kernel $K$, which is the flipped version of the template $R$ in both horizontal and vertical directions. In this representation, the kernel $K$ is better understood as an operator, or a filtering element, rather than as a template: the original image $X(n)$ is processed with the operator $K$. The operator
exicites neighborhood pixels where element $k_{l m}$ is positive while it inhibits neighborhood pixels where element $k_{l m}$ is negative. It should be noted that the template matching and the convolution is the two sides of the same coin, stating the same thing from different perspectives. However, the different viewpoints are sometimes useful as described in the next subsection.

### 2.5.2 Relationship with other image processing methods

There are several image processing methods that have a close relationship with the template matching scheme. These methods include mathematical morphology and discrete time cellular neural network. A brief survey of these two methods is given below to provide a unified and clear viewpoint for understanding template matching scheme.

Mathematical morphology performs image processing operations based on set theory [22]. Morphological operations nonlinearly transform an image by locally modifying the geometric features in an image using the set relationship between the input image and the socalled "structuring element". Before describing the morphological operation, some basic definitions on the set operations are described below. Let $A$ and $B$ be sets in $Z^{2}$ with components $a=\left(a_{1}, a_{2}\right)$ and $b=\left(b_{1}, b_{2}\right)$, respectively ${ }^{2}$.

The translation of $A$ by $x=\left(x_{1}, x_{2}\right)$, denoted $(A)_{x}$, is defined as

$$
\begin{equation*}
(A)_{x}=\{c \mid c=a+x, \quad \text { for } a \in A\} . \tag{2.11}
\end{equation*}
$$

The reflection of $B$, denoted $\widehat{B}$, is defined as

$$
\begin{equation*}
\widehat{B}=\{x \mid x=-b, \quad \text { for } b \in B\} . \tag{2.12}
\end{equation*}
$$

The complement of set $A$, denoted $A^{c}$ is

$$
\begin{equation*}
A^{c}=\{x \mid x \notin A\} . \tag{2.13}
\end{equation*}
$$

The union of two sets $A$ and $B$, denoted $A \cup B$ is defined as

[^1]\[

$$
\begin{equation*}
A \bigcup B=\{x \mid x \in A \quad \text { or } \quad x \in B\} \tag{2.14}
\end{equation*}
$$

\]

The intersection of two sets $A$ and $B$, denoted $A \cap B$, is defined as

$$
\begin{equation*}
A \cap B=\{x \mid x \in A \quad \text { and } \quad x \in B\} . \tag{2.15}
\end{equation*}
$$

Finally, the difference of two sets A and B , denoted $A-B$, is defined as

$$
\begin{equation*}
A-B=\{x \mid x \in A, x \notin B\}=A \cap B^{c} . \tag{2.16}
\end{equation*}
$$

Using the above notation, dilation of set $A$ by the structuring element $B$, denoted $A \oplus B$, is defined as

$$
\begin{equation*}
A \oplus B=\left\{x \mid(\hat{B})_{x} \cap A \neq \varnothing\right\} . \tag{2.17}
\end{equation*}
$$

where $\varnothing$ is the empty set. The dilation of $A$ is the set of all $x$ displacements such that $\widehat{B}$ and $A$ overlap by at least one nonzero element. Another definition of dilation can be written as

$$
\begin{equation*}
A \oplus B=\bigcup_{b \in B}(A)_{b}=\bigcup_{a \in A}(B)_{a}, \tag{2.18}
\end{equation*}
$$

which is exactly the same as the Minkowski addition of two sets $A$ and $B$. The dilated image is obtained as the union of variations of set $A$ translated with different quantities, which are defined by the structuring element $B$, or as the union of variations of set $B$ translated with different quantities, which are defined by $A$.

Erosion can be defined almost in the same manner as dilation. Erosion of $A$ by $B$, denoted $A \ominus B$, is defined as

$$
\begin{equation*}
A \ominus B=\left\{x \mid(B)_{x} \subseteq A\right\} \tag{2.19}
\end{equation*}
$$

This formulation shows that a given pixel in the original image is kept in the eroded image if the structuring element translated to that pixel is fully contained within the set A. This representation is equivalent to the following formula ${ }^{3}$ :

[^2]

Figure 2.12 Example of dilation and erosion. The original set $A$ is processed with the structuring element $B$ to produce the dilated and eroded set. The origin of the structuring element is defined at the center of the $3 \times 3$ region. The combination of the template and the threshold to carry out these two operations are also listed in the figure.

$$
\begin{equation*}
A \Theta B=\bigcap_{b \in B}(A)_{-b} . \tag{2.20}
\end{equation*}
$$

This formula defines the eroded image as the intersection of variations of set $A$ translated with different quantities, which are defined by the structuring element $B$.

Examples of these two operations are shown in Figure 2.12. Let us consider the relationship between these two morphological operations and template matching. The definition of dilation expressed in eqn. (2.17) is naturally mapped into the formula based on template matching expressed in eqn (2.4) with an appropriate template $R$ and a threshold $I$. Note that the dilated image can be obtained by scanning the structuring element $\hat{B}$ in the entire image plane and selecting the pixel that produces the correlation result equal to or greater than 1. This condition is met by setting the threshold value to 0.5 , which corresponds to the lowest stringency. Then the dilation of set $A$ by the structuring element $B$ is represented in terms of template matching operation as follows.

$$
\begin{equation*}
X_{A \oplus B}=T_{X_{\dot{B}}, 0.5}\left(X_{A}\right) \tag{2.21}
\end{equation*}
$$

where $X_{A}$ represents the binary image in which the pixel value is 1 only inside set $A$, i.e.,

$$
X_{A}(i, j)=\left\{\begin{array}{cl}
1 & (i, j) \in A  \tag{2.22}\\
0 & (i, j) \in A^{c},
\end{array}\right.
$$

and $X_{\hat{B}}$ and $X_{A \oplus B}$ are defined almost in the same way. For simlicity reasons, throughout the formulation in the thesis, set $A$ and image $X_{A}$ are used interchangeably. By this definition, expressions like "image $A$ " or $X_{A} \oplus X_{B}$ should be interpreted as "image $X_{A}$ " or $A \oplus B$, respectively.

It is also suggestive to think about the meaning of the second definition of dilation represented in eqn. (2.18). This definition is better interpreted from the viewpoint of convolution operation rather than from the point of correlation. This viewpoint considers $B$ an operator or a kernel rather than a template. Convolution of the original image with the kernel $B$ produces the superimposed version of all possible displacements of image $A$, in which displacements are defined by $B$. Then by digitizing the convolution result with the threshold 0.5 , the union operation in eqn. (2.18) is performed to produce the dilated image $A \oplus B$. Note that the template matching using the reference template $\hat{B}$ is equivalent to the convolution using the kernel $B$ as shown in eqns. (2.9) and (2.10).

The erosion operation can be also related to the template matching operation as well as to the convolution operation. The first definition in eqn. (2.19) indicates that the eroded image can be obtained by scanning the structuring element $B$ in the entire image $A$ and selecting the pixel whose $3 \times 3$ neighbors completely contain $B$. This condition is met by imposing the maximum stringency, i.e., by setting the threshold to 2.5 in the example shown in Figure 2.12. Then the erosion of image $A$ by the structuring element $B$ is rewritten by the following template matching operation:

$$
\begin{equation*}
A \ominus B=T_{B, 2.5}(A) . \tag{2.23}
\end{equation*}
$$

Like in the case of dilation, the interpretation by convolution is more closely associated with the definition based on the modified Minkowski subtraction represented in eqn. (2.20). Convolution of the original image with the kernel $\hat{B}$, equivalent to the template matching with template $B$, produces the superimposed version of all possible displacements of image $A$, in which displacements are defined by $\hat{B}$. Then by thresholding the convolution result with the value 2.5 , the intersection operation in eqn. (2.20) is performed to produce the resultant image $A \Theta B$.

The dilation and erosion operations form the basis of the morphological operations. These two operations do not need -1 in the template, indicating that the template matching is not performed with the maximum stringency. The combined use of these two operations leads to opening and closing operations, which are very useful for image filtering applications. The operation corresponding to the template matching in the strict sense using all the neighborhood pixels is called Hit-or-Miss transform. It is interesting that the computation based on the set theory can be mapped into the template matching scheme. The bridge between these two different methods lies in the thresholding operation, which introduces nonlinearlity.

The other category of image processing method that has some similarity to template matching is the discrete time cellular neural network (DTCNN), which is a discrete time version of the cellular neural network (CNN). The CNN is a class of information processing paradigms which is capable of high speed processing in a parallel fashion [23][24]. It is made of a massive aggregate of regularly spaced analog components, called cells, which communicate with each other in a specified region. When the region of cell interaction is limited to $3 \times 3$ neighbors, the dynamics of the CNN is represented as follows:

$$
\begin{equation*}
\tau \frac{d x_{i j}}{d t}=-x_{i j}+\sum_{l=-1}^{1} \sum_{m=-1}^{1} A_{l m} y_{i+l j+m}+\sum_{l=-1}^{1} \sum_{m=-1}^{1} B_{l m} u_{i+l j+m}+I \tag{2.24}
\end{equation*}
$$

and

$$
\begin{equation*}
y_{i j}=g\left(x_{i j}\right)=\frac{1}{2}\left(\left|x_{i j}+1\right|-\left|x_{i j}-1\right|\right) \tag{2.25}
\end{equation*}
$$

where $x_{i j}$ is the internal state of the cell, $y_{i j}$ is the output of the cell, $u_{i j}$ is the external input to the cell, $\tau$ is the time constant, $I$ is the threshold, $A$ is the feedback template, $B$ is the control template, $g$ is the function which relates $x_{i j}$ to $y_{i j}$. Note that $g$ is linear in the region $-1<x<1$ and either 1 or -1 outside this region. Several interesting parameter sets are proposed to perform different types of image processing operations including noise removal, edge and corner detection [24], shadow detection [25], connected component detection [26], hole filling [27], and thinning [28].

Let us consider the special case where $B$ is 0 . In this case, eqn. (2.24) reduces to

$$
\begin{equation*}
\tau \frac{d x_{i j}}{d t}=-x_{i j}+\sum_{l=-1}^{1} \sum_{m=-1}^{1} A_{l m} y_{i+l}+m . \tag{2.26}
\end{equation*}
$$

The above continuous-time domain representation is transformed into the discrete-time version by replacing the time derivative $d x_{i j} / d t$ with $\left(x_{i j}(t+\tau)-x_{i j}(t)\right) / \tau$. Then eqn. (2.26) reduces to

$$
\begin{equation*}
x_{i j}(t+\tau)=\sum_{l=-1}^{1} \sum_{m=-1}^{1} A_{l m} y_{i+l j+m}(t)+I . \tag{2.27}
\end{equation*}
$$

If the function $g$ is further simplified as the hard limit function as shown below

$$
f(x ; 0)=\left\{\begin{array}{rl}
1 & x \geq 0  \tag{2.28}\\
0 & x<0,
\end{array}\right.
$$

The update rule for the discrete-time version of CNN (DTCNN) is represented as follows

$$
\begin{equation*}
y_{i j}(n+1)=f\left(\sum_{l=-1}^{1} \sum_{m=-1}^{1} A_{l m} y_{i+l}{ }_{j+m}(n) ;-I\right) . \tag{2.29}
\end{equation*}
$$

Eqn. (2.29) is the same as the basic formula for template matching shown in eqn. (2.1), with the following correspondence: $A_{l m}$ to $r_{l m} ; x_{i j}$ to $y_{i j} ; I$ to $-I$. The identity of the two equations indicates that the template matching can be considered the special case of the DTCNN.

Through the discussion above, it is found that the two major classes of image processing paradigm, mathematical morphology and the special form of the DTCNN, can be treated within the common framework of template matching. Although the close relationship between mathematical morphology and the DTCNN has been pointed out elsewhere [29], the discussion above is more comprehensive and serves as a unified framework bridging the two paradigms.

### 2.5.3 Implementation considerations

There are several conditions for an algorithm to be implemented onto VLSI hardware. First, the number of neighbors used for interaction should be kept as small as possible. Second, the computation has to be simple enough so that hardware can easily implement it. Third, processing has to be done in an iterative fashion: each processing result is used for the next processing as is explicitly represented in eqn. (2.1) in the form of image updating. From these points of view, the feasibility of mapping the template matching scheme onto VLSI hardware is briefly described below.

As mentioned earlier, the template matching is nothing but the convolution operation in which the kernel is the flipped version of the template in both horizontal and vertical directions. The convolution kernel, which represents the impulse response of a system, can be easily realized in hardware by a current mirror circuit. A set of current outputs, each of which is proportional to the element specified in the convolution kernel, is generated from the pixel whose value is 1 . Figure 2.13 explains the current distribution scheme. These distributed


Figure 2.13 Implementation of a convolution kernel. The convolution kernel is realized by the current distribution from the pixel whose value is 1 . Each current amount takes one of the three values, $I_{\text {ref }}, 0$, $-I_{r e f}$ where $I_{r e f}$ is the amount of the reference current. Only at the center pixel is allowed to have the value of $2 I_{\text {ref }}$ in addition to the above three values.
currents are summed at each pixel to produce the convolution result, which are then digitized by a threshold current to generate a binary output voltage.

The first requirement for hardware mapping is satisfied if currents are distributed only to its $3 \times 3$ neighbors. The second requirement concerning the simplicity of computation is satisfied by the use of only three current levels, which are $I_{\text {ref }}$, $-I_{\text {ref }}$, and 0 except the center where the value of $2 I_{r e f}$ is also allowed. These three values can be generated by appropriately turning on or turning off current sources. Iterative operation, which is the third requirement, is easily realized in hardware by implementing a memory for each pixel. From these observations, it is concluded that the employment of template matching scheme is suitable for the realization of the feature detection algorithm from the implementation point of view. The detail of implementation is described in Chapter 3.


Figure 2.14 Overall processing flow of the feature detection algorithm. Features are detected by going through several processing steps. These steps include: OD (orientation decomposition), LC (line completion), EIP (elimination of isolated points), LI (line inhibition), LE (line elongation), LT (line thickening), and LSD (linestop detection). The detected features are: JCT-X (X-type junction), JCT-T (Ttype junction), JCT-Y (Y-type junction), trueC (corners), and trueLS (linestops).

### 2.6 Outline of the processing flow

The discussion in the previous section demonstrates the processing flexibility of template matching by the control of stringency and its applicability to the feature detection algorithm. The algorithm is fully explained in this section based on the framework of template matching.

### 2.6.1 Overall processing flow

Figure 2.14 shows the overall processing flow of the proposed feature detection

no change


Figure 2.15 Algorithm for thinning. A pixel is deleted if its $3 \times 3$ neighbors match either $A_{1}^{1}$ or $A_{1}^{2}$ and match neither $A_{1}^{3}$ nor $A_{1}^{4}$.
algorithm, which basically follows the processing shown in Figure 2.6 with additional steps of processing. Each image in the figure is marked with $X_{i}$ for later reference ${ }^{4}$. The original image is sequentially processed in an iterative fashion to finally generate the five features shown in the shaded boxes. These processing steps include: orientation decomposition (OD), line completion (LC), elimination of isolated points (EIP), line inhibition (LI), line elongation (LE), line thickening (LT), and linestop detection (LSD, LSD1). Each of the processing steps is described below in detail.

### 2.6.2 Thinning

The first processing step is thinning. As mentioned in Figure 2.6, thinning is performed to normalize the line segment in the input pattern to a single pixel width so that the width of the line does not affect the feature detection processing. Without this processing, it is impossible to apply the feature detection algorithm for an input pattern containing thick lines since the extent of interaction is limited to $3 \times 3$ neighbors.

[^3]

Figure 2.16 Example of thinning (one cycle). The leftmost letter is thinned in the order of west, south, east, and north (indicated by the arrow) to produced the rightmost letter.

The basic principle of thinning is to erode the line segment from both sides as long as the deletion of pixels does not break the line segment. First of all, the possibility of being able to delete a pixel is examined for all the pixels. Figure 2.15 shows the examination procedure of the possibility of being able to erode a line segment by deleting a pixel from the west side. If the $3 \times 3$ neighbors of the present pixel match either template $A_{1}^{1}$ or $A_{1}^{2}$, indicating the possibility that the present pixel is just an extra or redundant element to the vertical line, it becomes a candidate for deletion. It is deleted unless the deletion breaks a horizontal connection to generate a gap: if the $3 \times 3$ neighbors match either $A_{1}^{3}$ or $A_{1}^{4}$, the deletion of the pixel causes the break between the pixel on the left column and the pixel on the right column. This procedure of conditional deletion of a pixel is repeated from the south side, from the east side, and from the north side, by successively rotating the template to complete one cycle of erosion. This erosion cycle is repeated as many times as required for the thinning result to reach a stable state.

The thinning operation from the west side is represented in the framework of set theory as follows:

$$
\begin{equation*}
X_{1}=X_{0}-\left(\left(T_{A_{1}^{1}, l_{1}^{1}}\left(X_{0}\right) \cup T_{A_{1}^{2}, I_{1}^{2}}\left(X_{0}\right)\right)-\left(T_{A_{1}^{3}, l_{1}^{3}}\left(X_{0}\right) \cup T_{A_{1}^{4}, I_{1}^{4}}\left(X_{0}\right)\right)\right) \tag{2.30}
\end{equation*}
$$



Figure 2.17 Patterns categorized as $0^{\circ}$ with its corresponding template and threshold.
or equivalently represented in the framework of template matching as

$$
\begin{equation*}
X_{1}=T_{E, 0.5}\left(X_{0}-T_{E, 0.5}\left(T_{E, 0.5}\left(T_{A_{1}^{1}, l_{1}^{1}}\left(X_{0}\right)+T_{A_{1}^{2}, I_{1}^{2}}\left(X_{0}\right)\right)-T_{E, 0.5}\left(T_{A_{1}^{3}, l_{1}^{3}}\left(X_{0}\right)+T_{A_{1}^{4}, l_{1}^{4}}\left(X_{0}\right)\right)\right)\right) \tag{2.31}
\end{equation*}
$$

where $X_{0}$ is the input image plane containing patterns and

$$
\begin{gather*}
A_{1}^{1}=\left(\begin{array}{ccc}
0 & 0 & 1 \\
-1 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), A_{1}^{2}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
-1 & 1 & 1 \\
0 & 0 & 1
\end{array}\right), A_{1}^{3}=\left(\begin{array}{ccc}
1 & -1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), \quad A_{1}^{4}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & -1 & 0
\end{array}\right),  \tag{2.32}\\
I_{1}^{1}=I_{1}^{2}=I_{1}^{3}=I_{1}^{4}=0.5 . \tag{2.33}
\end{gather*}
$$

The thinning operation from other sides is represented in the same way by using $90^{\circ}, 180^{\circ}$, and $270^{\circ}$ rotations of the above templates. Figure 2.16 shows the process of thinning for letter "A".

### 2.6.3 Orientation decomposition

After a line drawing is thinned to a single pixel width, it is decomposed into line segments of four orientations. This is performed by locally determining the orientation for each pixel in the image plane based on its $3 \times 3$ neighborhood. The local orientation of a pixel is computed as $0^{\circ}$ if its $3 \times 3$ neighbors match one of the four template patterns listed in Figure 2.17. The first two templates, $A_{2 a}^{1}$ and $A_{2 a}^{2}$, are designed to detect $0^{\circ}$ orientation even for lines which consists of a horizontal line segment and additional pixels on either the top side or the


$$
\begin{array}{ll}
A_{2 b}^{1}=\left(\begin{array}{ccc}
0 & 0 & 1 \\
0 & 1 & -1 \\
1 & -1 & -1
\end{array}\right) & A_{2 b}^{2}=\left(\begin{array}{ccc}
-1 & -1 & 1 \\
-1 & 1 & 0 \\
1 & 0 & 0
\end{array}\right) \\
I_{2 b}^{1}=1.5 & I_{2 b}^{2}=1.5
\end{array}
$$

Figure 2.18 Patterns categorized as $45^{\circ}$ with its corresponding template and threshold.
bottom side. The other two templates, $A_{2 a}^{3}$ and $A_{2 a}^{4}$, are designed to deal with special cases in which the pixel is adjacent to the T-type junction. The pixel is not classified as $0^{\circ}$ using the template $A_{2 a}^{1}$ and $A_{2 a}^{2}$ due to the presence of a vertical line segment on either left or right side. The formula for updating the image is represented as
or

$$
\begin{equation*}
X_{2 a}=T_{E, 0.5}\left(T_{A_{2 a}^{1}, I_{2 a}^{1}}\left(X_{1}\right)+T_{A_{2 a}^{2}, I_{2 a}^{2}}\left(X_{1}\right)+T_{A_{2 a}^{3}, I_{2 a}^{3}}\left(X_{1}\right)+T_{A_{2 a}^{4}, I_{2 a}^{4}}\left(X_{1}\right)\right) \tag{2.35}
\end{equation*}
$$

where

$$
\begin{gather*}
A_{2 a}^{1}=\left(\begin{array}{ccc}
-1 & -1 & -1 \\
1 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), A_{2 a}^{2}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
1 & 1 & 1 \\
-1 & -1 & -1
\end{array}\right), A_{2 a}^{3}=\left(\begin{array}{ccc}
1 & -1 & -1 \\
1 & 1 & 1 \\
1 & -1 & -1
\end{array}\right), A_{2 a}^{4}=\left(\begin{array}{ccc}
-1 & -1 & 1 \\
1 & 1 & 1 \\
-1 & -1 & 1
\end{array}\right),  \tag{2.36}\\
I_{2 a}^{1}=I_{2 a}^{2}=I_{2 a}^{3}=I_{2 a}^{4}=2.5 . \tag{2.37}
\end{gather*}
$$

The $90^{\circ}$ orientation component can be obtained exactly the same way by using the $90^{\circ}$ rotations of the templates $A_{2 a}^{1}, A_{2 a}^{2}, A_{2 a}^{3}$, and $A_{2 a}^{4}$.

The orientation detection for $45^{\circ}$ and $135^{\circ}$ is slightly different from the detection for $0^{\circ}$ and $90^{\circ}$ in terms of stringency. For a pixel to be classified as $45^{\circ}$ orientation, its $3 \times 3$
neighbors have to match one of the two templates shown in Figure 2.18 with some tolerance allowed. The formula using these two templates is represented as

$$
\begin{equation*}
X_{2 b}=X_{1} \cap\left(T_{A_{2 b}^{1}, I_{2 b}^{\prime}}\left(X_{1}\right) \cup T_{A_{2 b}^{2}, I_{2 b}^{2}}\left(X_{1}\right)\right) \tag{2.38}
\end{equation*}
$$

or

$$
\begin{equation*}
X_{2 b}=T_{E, 1.5}\left(X_{1}+T_{E, 0.5}\left(T_{A_{2 b}^{\prime}, I_{2 b}^{\prime}}\left(X_{1}\right)+T_{A_{2 b}^{2}, I_{2 b}^{2}}\left(X_{1}\right)\right)\right) \tag{2.39}
\end{equation*}
$$

where

$$
\begin{gather*}
A_{2 b}^{1}=\left(\begin{array}{ccc}
0 & 0 & 1 \\
0 & 1 & -1 \\
1 & -1 & -1
\end{array}\right), A_{2 b}^{2}=\left(\begin{array}{ccc}
-1 & -1 & 1 \\
-1 & 1 & 0 \\
1 & 0 & 0
\end{array}\right),  \tag{2.40}\\
I_{2 b}^{1}=I_{2 b}^{2}=1.5 . \tag{2.41}
\end{gather*}
$$

The threshold value is lowered to increase orientation tolerance; the threshold of 2.5 is too severe. The lowered threshold value, however, classifies a pixel whose value is 0 as $45^{\circ}$ if its two neighbors in the upper right position and the lower left position have a value of 1 . This categorization is undesirable and can be avoided by the intersection operation in eqn. (2.38), which impose restriction that the selected pixel should be included in the original image. The $135^{\circ}$ orientation detection is possible by using the $90^{\circ}$ rotations of the templates $A_{2 b}^{1}$ and $A_{2 b}^{2}$.

### 2.6.4 Line completion

The result of orientation decomposition is not complete. As shown in Figure 2.14, two pixels along the left diagonal leg of letter " $A$ " are categorized as $90^{\circ}$ while the rest of the pixels are categorized as $45^{\circ}$. Although the classification for these two pixels is locally correct, this generates the two gaps in the $45^{\circ}$ line segment. These two gaps have to be filled so that the line as a whole is classified as $45^{\circ}$. For this purpose, line completion (LC) operation is performed. The detail of the operation is described below. For an input image shown on the


Figure 2.19 Procedure of line completion operation in $0^{\circ}$ orientation.
leftmost side in Figure 2.19, which is a typical result of thinning for a slanted cross, the $0^{\circ}$ orientation plane is computed as $X_{2 a}$. A gap is generated in the middle of the line, resulting in two separate line segments from originally one line segment. The purpose of line completion is to generate $X_{3 a}$, in which the gap is filled to recover the horizontal line. For that purpose, two steps of intermediate processing are required as shown in the figure. First, each of the broken line segments is extended to the horizontal direction by using the following operation

$$
\begin{equation*}
U=T_{L E a}\left(X_{2 a}\right) \tag{2.42}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{L E a}()=T_{A_{3 a}^{\prime}, I_{3 a}^{\prime}}() \tag{2.43}
\end{equation*}
$$

with

$$
A_{3 a}^{1}=\left(\begin{array}{lll}
0 & 0 & 0  \tag{2.44}\\
1 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), \quad I_{3 a}^{1}=0.5
$$

This elongation operation fills the gap. However the line becomes longer than that in $X_{2 a}$ and the extended portion has to be removed. This is done by the detection of linestops and its subsequent deletion. The linestops are detected by the following operation which is already described before in Figure 2.10 as

$$
\begin{equation*}
V=T_{L S D 8}(U) \tag{2.45}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{L S D 8}()=T_{A_{L S D 8}, I_{L S D 8}}() \tag{2.46}
\end{equation*}
$$

with

$$
A_{L S D 8}=\left(\begin{array}{ccc}
-1 & -1 & -1  \tag{2.47}\\
-1 & 2 & -1 \\
-1 & -1 & -1
\end{array}\right), \quad I_{L S D 8}=0.5
$$

The image of a completed or recovered line is then computed as a set of pixels that belong to $U$ and do not belong to $V$. The above procedure is represented as

$$
\begin{equation*}
X_{3 a}=T_{E, 0.5}(U-V)=T_{E, 0.5}\left(T_{L E a}\left(X_{2 a}\right)-T_{L S D 8}\left(T_{L E a}\left(X_{2 a}\right)\right)\right) . \tag{2.48}
\end{equation*}
$$

The line completion procedure described above is very similar to closing operation frequently used in mathematical morphology. The closing of set $A$ by structuring element $B$, denoted $A \bullet B$, is defined as

$$
\begin{equation*}
A \bullet B=(A \oplus B) \ominus B \tag{2.49}
\end{equation*}
$$

which is simply the dilation of $A$ by $B$, followed by the erosion of the result by $B$. Closing is used to fill gaps in the contour as well as to fuse narrow breaks and long thin gulfs. This morphological operation can be implemented in the framework of template matching since both dilation and erosion can be represented in terms of template matching (eqns. (2.21) and (2.23)). However, in the present application, closing with structuring element $A_{3 a}^{1}$ cannot fill the gap: the erosion operation breaks the gap that is filled by the first dilation operation. This is because the gap width is two while the template or the structuring element is limited to $3 \times 3$. For this reason, the erosion operation is replaced with the detection of linestops and subsequent deletion of these linestops from the line segment.

The line completion procedure for $45^{\circ}$ orientation is shown in Figure 2.20, where $X_{2 b}$ is the result of orientation decomposition previously shown in Figure 2.14 and has two gaps in the middle of the diagonal line. A gap filling operation is performed only for detected linestops as


Figure 2.20 Procedure of line completion operation in $45^{\circ}$ orientation. Four templates used to obtain image $V$ from $U$ are shown above and below the arrow.
follows:

$$
\begin{gather*}
U=T_{L S D 8}\left(X_{2 b}\right),  \tag{2.50}\\
V=T_{E, 0.5}\left(T_{A_{3 b}^{1}, I_{3 b}^{1}}(U)+T_{A_{3 b}^{2}, I_{3 b}^{2}}(U)+T_{A_{3 b}^{3}, I_{3 b}^{3}}(U)+T_{A_{3 b}^{4}, I_{3 b}^{4}}(U)\right) \tag{2.51}
\end{gather*}
$$

where

$$
\begin{gather*}
A_{3 b}^{1}=\left(\begin{array}{ccc}
0 & 1 & 1 \\
0 & 2 & 1 \\
1 & 0 & 0
\end{array}\right), \quad A_{3 b}^{2}=\left(\begin{array}{ccc}
0 & 0 & 1 \\
1 & 2 & 0 \\
1 & 1 & 0
\end{array}\right), \quad A_{3 b}^{3}=\left(\begin{array}{lll}
0 & 0 & 0 \\
1 & 2 & 1 \\
0 & 0 & 0
\end{array}\right), \quad A_{3 b}^{4}=\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 2 & 0 \\
0 & 1 & 0
\end{array}\right),  \tag{2.52}\\
I_{3 b}^{1}=I_{3 b}^{2}=I_{3 b}^{3}=I_{3 b}^{4}=1.5 . \tag{2.53}
\end{gather*}
$$

The above four templates describe the possible configuration of the gap. The purpose of the assignment of value 2 for the center element is not to delete isolated pixels. Although the templates, $A_{3 b}^{3}$ and $A_{3 b}^{4}$, may look strange at first sight, this types of gap can happen for a line deviated from $45^{\circ}$, and are hence taken care of by the above templates. The gap-filled image is obtained by simply taking OR of two images $X_{2 b}$ and $V$, i.e.,

$$
\begin{equation*}
X_{3 b}=T_{E, 0.5}\left(X_{2 b}+V\right) \tag{2.54}
\end{equation*}
$$

The reason why the gap filling operation is performed for linestops is to reduce the
number of templates while allowing some tolerance. If the template matching is performed directly for the original image, additional points will be detected which results in an unnecessarily thick line. The line completion for $90^{\circ}$ and $135^{\circ}$ orientations is possible with the $90^{\circ}$ rotations of the templates for $0^{\circ}$ and $45^{\circ}$, respectively, as is the case for orientation detection. In the following subsections, the processing procedure is explained only for $0^{\circ}$ and $45^{\circ}$; the procedure for $90^{\circ}$ and $135^{\circ}$ is obtained by using their $90^{\circ}$ rotations unless otherwise noted.

### 2.6.5 Elimination of isolated points

The small gaps between line segments generated by orientation decomposition have been filled by the line completion operation as shown in the orientation plane $X_{3 b}$ in Figure 2.14. Then pixels that are incorrectly classified as other orientation planes (for example $X_{3 c}$ ) have to be removed. This is done by elimination of isolated points (EIP) operation as described below. This operation deletes a single isolated pixel as well as two adjacent pixels since the result of erroneous orientation detection results in pixel(s) whose length is one or two. A line segment that consists of three pixels or more survives this operation. In other words, in the proposed algorithm, the line segment has to have a length of at least three pixels. Note that the EIP operation should follow the LC operation: otherwise, a set of broken line segments, each of which can be one or two pixels long, generated immediately after orientation decomposition, would be eliminated by the EIP operation.

The concept of the EIP operation is opposite to that of the LC operation. Figure 2.21 shows the procedure of the EIP operation in $0^{\circ}$ orientation and $45^{\circ}$ orientation. First, linestops are detected in image $U$ and are removed from the original image to generate image $V$. This process is represented in the following way.


Figure 2.21 Procedure of elimination of isolated points operation in $0^{\circ}$ orientation (top) and in $45^{\circ}$ orientation (bottom).

$$
\begin{gather*}
U=T_{L S D 8}\left(X_{3 a}\right),  \tag{2.55}\\
V=T_{E, 0.5}\left(X_{3 a}-U\right) . \tag{2.56}
\end{gather*}
$$

Then the image is elongated to $0^{\circ}$ with $T_{L E a}$ operation and the intersection between this elongated image $W$ and the original image $X_{3 a}$ is computed to generate an updated image $X_{4 a}$. This procedure is written as:

$$
\begin{gather*}
W=T_{L E a}(V),  \tag{2.57}\\
X_{4 a}=T_{E, 1.5}\left(X_{3 a}+W\right) . \tag{2.58}
\end{gather*}
$$

The above procedure is identical to the opening operation using structuring element $A_{L E a}$. The opening of set $A$ by structuring element $B$, denoted $A \circ B$, is defined as

$$
\begin{equation*}
A \circ B=(A \ominus B) \oplus B \tag{2.59}
\end{equation*}
$$

which is simply the erosion of $A$ by $B$, followed by dilation of the result by $B$. Opening is frequently used in image processing applications to break narrow isthmuses and eliminates thin protrusions.


Figure 2.22 Procedure of line inhibition operation.

The procedure for the EIP operation is the same for $45^{\circ}$ orientation except the template used to generate image $W$ from image $V$. Instead of the $45^{\circ}$ rotations of template $A_{4 a}^{1}$, the following template is used:

$$
A_{4 b}^{1}=\left(\begin{array}{lll}
0 & 1 & 1  \tag{2.60}\\
1 & 1 & 1 \\
1 & 1 & 0
\end{array}\right) .
$$

This is because pixels in a line segment of $45^{\circ}$ orientation can be mutually connected not only in an exactly $45^{\circ}$ orientation but also in horizontal and vertical directions as the example in Figure 2.21 shows.

### 2.6.6 Line inhibition

After EIP operation is performed, errors generated by orientation calculation should have been fixed. However, there is no guarantee at this point that there is not any pixel of value 1 that is present in multiple orientation planes. Figure 2.22 demonstrates this situation. The line segment which has an orientation between $45^{\circ}$ and $90^{\circ}$ is decomposed into two orientation planes $X_{2 b}$ and $X_{2 c}$. These two images are updated by LC and EIP operations to $X_{4 b}$ and $X_{4 c}$,


Figure 2.23 Schematic explaining the orientation tolerance of the proposed algorithm. (a) Result of the orientation decomposition followed by the three steps of subsequent processing. (b) Orientation range to be classified as one of the four orientations.
respectively. At this stage, several pixels have both $45^{\circ}$ and $90^{\circ}$ orientations. To allow only one orientation for each pixel, line inhibition (LI) operation simply keeps the content of $90^{\circ}$ orientation $\left(0^{\circ}\right.$ orientation as well) and suppresses pixels in $45^{\circ}$ and $135^{\circ}$ orientation planes to produce $X_{5 b}$ and $X_{5 c}$. Suppression of the diagonal element by $0^{\circ}$ and $90^{\circ}$ element, not the other way around, is reasonable considering that the stringency of template matching for diagonal directions is lower than that for $0^{\circ}$ and $90^{\circ}$ orientations, and hence generates redundant outputs. The procedure of the LI operation is expressed as

$$
\begin{align*}
& X_{5 b}=T_{E, 0.5}\left(X_{4 b}-X_{4 a}-X_{4 c}\right)  \tag{2.61}\\
& X_{5 d}=T_{E, 0.5}\left(X_{4 d}-X_{4 a}-X_{4 c}\right) \tag{2.62}
\end{align*}
$$

The LI operation sometimes generates gaps in line segments of $45^{\circ}$ and $135^{\circ}$ orientations. If the fragmented segment consists of a single pixel or two pixels, it does not satisfy the requirement of a line segment. These pixels are removed by the subsequent EIP operation and produce the final result of orientation decomposition in which originally
generated detection errors should have been corrected. Now it is possible to investigate the range of orientation that is classified into each of the four orientations. Figure 2.23 (a) shows the simulation result of the orientation decomposition followed by the above-explained three steps of processing for three lines of different orientations. The line shown at the top, which has an orientation of $18.4^{\circ}=\tan ^{-1}(1 / 3)$, connecting the origin and the pixel $(3,1)$, is still present in the $0^{\circ}$ orientation plane but is removed from the $45^{\circ}$ orientation plane. This indicates that the original line is categorized as $0^{\circ}$, which is correct since $18.4^{\circ}$ is closer to $0^{\circ}$ than to $45^{\circ}$. Note the importance of the LI operation: without this processing, the line is categorized as $0^{\circ}$ as well as $45^{\circ}$ orientation. This is the maximum orientation deviation from $0^{\circ}$ for a line to be classified as $0^{\circ}$.

On the other hand, the line shown at the bottom, which has an orientation of $26.6^{\circ}=\tan ^{-1}(1 / 2)$, connecting the origin and the pixel $(2,1)$, is classified as $45^{\circ}$. This is the maximally deviated line to be classified as $45^{\circ}$. In other words, the orientation tolerance is again $18.4^{\circ}=(45-26.6)^{\circ}$, which is the same as that for $0^{\circ}$ orientation. The line shown in the middle, whose orientation is $23.3^{\circ}=\tan ^{-1}(3 / 7)$, connecting the origin and the pixel $(7,3)$, is categorized neither as $0^{\circ}$ nor as $45^{\circ}$. Figure 2.23 (b) shows the range of orientation that is categorized as one of the four orientations. The coverage of the orientation is $82 \%$, which is reasonably high considering the simplicity of the algorithm for orientation detection.

### 2.6.7 Line elongation and line thickening

As explained above, the first several steps of processing decomposes an input image into different orientations. Based on this result, higher level features are detected. Note at this
point, no intersection exists between the four orientation planes: no tatalJCT is detected. To ensure the presence of intersection, line elongation (LE) and line thickening (LT) operations are performed. For $0^{\circ}$ and $90^{\circ}$ orientations, only single LE operation is necessary. This operation has been explained already as part of the LC operation (see eqns. (2.42) and (2.44)), which simply elongates or extends the line segment using the directional template as shown in Figure 2.24. This operation recovers the original length of the line, which is observed in Figure 2.24 when compared to Figure 2.19. With this operation, some pixels are intentionally allowed to have multiple orientations to be detected as totalJCT by the intersection operation in the proposed algorithm. LE operation has another purpose of establishing 4-connectivity ${ }^{5}$. This simplifies the process of linestop detection as explained later on. Note that up to this point the line segment has been treated as a set of pixels that are 8 -connected ${ }^{6}$.

For $45^{\circ}$ orientation, the simple line elongation using the following template:

$$
A_{7 b}^{2}=\left(\begin{array}{lll}
0 & 0 & 1  \tag{2.63}\\
0 & 1 & 0 \\
1 & 0 & 0
\end{array}\right), \quad I_{7 b}^{2}=0.5
$$

does not automatically generate a line in which pixels are 4-connected. Another possible problem is that the simple intersection operation may not produce any pixel of value 1 for letter " X ", depending on how it is decomposed into line segments of $45^{\circ}$ and $135^{\circ}$ orientations. Therefore, the LT operation needs to be performed for an image extended in $45^{\circ}$ orientation. The overall operation is defined as

$$
\begin{equation*}
X_{7 b}=T_{A_{7_{7}^{2}}, I_{l b}^{2}}\left(T_{A_{l b}^{\dagger}, I_{l b}^{1}}\left(X_{6 b}\right)\right) \tag{2.64}
\end{equation*}
$$

where

[^4]

Figure 2.24 Line elongation operation and line thickening operation explained for $0^{\circ}$ and $45^{\circ}$ orientations.

$$
A_{7 b}^{1}=\left(\begin{array}{ccc}
-1 & 1 & 0  \tag{2.65}\\
1 & 2 & 0 \\
0 & 0 & 0
\end{array}\right), \quad I_{7 b}^{1}=1.5 .
$$

The above template finds a pixel that has a pixel of value 1 at its upper position and its left position and a pixel of value 0 at its upper left position, and set the pixel value to 1 . This operation thickens the line segment. The purpose of the assignment of value 2 for the center element is to avoid deletion of pixels whose $3 \times 3$ neighbors do not match the template pattern.

### 2.6.8 Linestop detection

For the line segment for which 4-connectivity is established by the LE and the LT operation, the linestop detection (LSD) operation with a simplified template is performed. The LSD operation is expressed as

$$
\begin{equation*}
X_{8 p}=T_{L S D 4}\left(X_{7 p}\right) \quad(p=a, b, c, d) \tag{2.66}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{L S D 4}()=T_{A_{L S D 4}, I_{S S D 4} 4}( \tag{2.67}
\end{equation*}
$$

with

$$
A_{L S D 4}=\left(\begin{array}{ccc}
0 & -1 & 0  \tag{2.68}\\
-1 & 2 & -1 \\
0 & -1 & 0
\end{array}\right), \quad I_{L S D 4}=0.5 .
$$

With this operation, pixels that are not linestops are removed since they have two neighbors in horizontal or vertical directions. Note that this template can be commonly used for the four orientations.

### 2.6.9 Detection of higher level features

Using the thickened oriented line segment and linestops, detection of higher level features is possible. The first step for junction detection is to find the pixel where at least two orientations are designated, which is represented as

$$
\begin{equation*}
\mathrm{X}_{10}=T_{E, 1.5}\left(X_{7 a}+X_{7 b}+X_{7 c}+X_{7 d}\right) . \tag{2.69}
\end{equation*}
$$

At this stage, however, one junction may consist of a set of mutually connected pixels. Then it is desirable to shrink each junction to a single pixel so that the number of pixels in the image plane represents the number of junctions. A normal way of shrinking is to prepare several templates representing possible connections between pixels for removing pixels that are located at "peripheries". Iterative execution of this process should finally shrink a set of mutually connected pixels into a single pixel. Instead of the above method, a simplified approach with smaller number of templates is employed in the proposed algorithm. The method first expands a set of mutually connected pixels to a circumscribing rectangle and then shrinks the rectangle to a single pixel. Generation of the circumscribing rectangle is performed by the following operation:

$$
\begin{equation*}
X_{11}=T_{\text {CirRect }}\left(T_{\text {CirRect }}\left(X_{10}\right)\right) \tag{2.70}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{\text {CirRect }}(O)=T_{A_{11}^{4}, I_{11}^{4}}\left(T_{A_{11}^{3}, I_{11}^{3}}\left(T_{A_{11}^{2}, I_{11}^{2}}\left(T_{A_{11}^{1}, I_{11}^{1}}()\right)\right)\right) \tag{2.71}
\end{equation*}
$$

with

$$
\begin{gather*}
A_{11}^{1}=\left(\begin{array}{lll}
0 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & 0
\end{array}\right), A_{11}^{2}=\left(\begin{array}{lll}
0 & 0 & 0 \\
1 & 2 & 0 \\
0 & 1 & 0
\end{array}\right), \quad A_{11}^{3}=\left(\begin{array}{lll}
0 & 0 & 0 \\
0 & 2 & 1 \\
0 & 1 & 0
\end{array}\right), \quad A_{11}^{4}=\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 2 & 1 \\
0 & 0 & 0
\end{array}\right),  \tag{2.72}\\
I_{11}^{1}=I_{11}^{2}=I_{11}^{3}=I_{11}^{4}=1.5 . \tag{2.73}
\end{gather*}
$$

In other words, $X_{l l}$ is obtained by applying two cycles of rectangular expansion operation, which consists of a series of template matching using $A_{11}^{1}, A_{11}^{2}, A_{11}^{3}$, and $A_{11}^{4}$. The maximum number of the rectangualr expansion operation required to obtain a circumscribing rectangle is the number of pixels forming a longer edge of the rectangle minus one. After the circumscribing rectangle is formed, further application of the rectangular expansion operation does not modify the shape any more. The process of rectangular expansion is shown at the top row in Figure 2.25. Since the size of the circumscribing rectangle is $3 \times 3$, execution of the rectangular expansion operation twice generates a circumscribing rectangle.

The shrinking operation is performed for the circumscribing rectangle obtained. The procedure is represented as

$$
\begin{equation*}
X_{\text {totalJCT }}=X_{12}=T_{\text {EdgeRem }}\left(X_{11}\right) \tag{2.74}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{E d g e R e m}()=T_{A_{12}^{4}, I_{12}^{4}}\left(T_{A_{12}^{3}, I_{12}^{3}}\left(T_{A_{12}^{2}, I_{12}^{2}}\left(T_{A_{12}^{1}, I_{12}^{1}}()\right)\right)\right) \tag{2.75}
\end{equation*}
$$

with

$$
\begin{gather*}
A_{12}^{1}=\left(\begin{array}{ccc}
0 & 1 & 0 \\
0 & 2 & 0 \\
0 & -1 & 0
\end{array}\right), A_{12}^{2}=\left(\begin{array}{ccc}
0 & -1 & 0 \\
0 & 2 & 0 \\
0 & 1 & 0
\end{array}\right), \quad A_{12}^{3}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
1 & 2 & -1 \\
0 & 0 & 0
\end{array}\right), \quad A_{12}^{4}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
-1 & 2 & 1 \\
0 & 0 & 0
\end{array}\right),  \tag{2.76}\\
I_{12}^{1}=I_{12}^{2}=I_{12}^{3}=I_{12}^{4}=1.5 . \tag{2.77}
\end{gather*}
$$



Figure 2.25 Procedure of shrinking. A set of mutually connected pixels are expanded first to form a circumscribing rectangle. Then the formed rectangle is shrunken to a single pixel.

The first template and the second template are used for the removal of the top edge and the bottom edge, respectively. Likewise, the third and the fourth templates are used for the removal of the left edge and the right edge, respectively. The process of successive shrinking using these four templates is shown at the bottom part of Figure 2.25. For a rectangle whose size is $3 \times 3$, application of the procedure only once shrinks a rectangular image to a single pixel. The procedure of shrinking is summarized as

$$
\begin{equation*}
X_{\text {totalJCT }}=X_{12}=T_{\text {Shrink }}\left(X_{10}\right) \tag{2.78}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{\text {Shrink }}()=T_{\text {EdgeRem }}\left(T_{\text {CirRect }}()\right) \text {. } \tag{2.79}
\end{equation*}
$$

Note that the variety of the original pattern $X_{10}$ is absorbed in the common form of the circumscribing rectangle, which simplifies the procedure of shrinking with only four templates.

Like the junction, the corner is defined as the point where at least two linestops are present in different orientation planes. However, the detected linestops may not exactly coincide with each other in different orientation planes. In Figure 2.14, for example, the top
corner of letter "A" cannot be detected since the linestop at the upper end of the $45^{\circ}$ line segment and the linestop at the upper end of the $135^{\circ}$ line segment are not located at the same pixel position. Therefore, the position of the linestop should be adjusted. For this purpose, the following processing is performed first:

$$
\begin{equation*}
\mathrm{X}_{13}=T_{E, 0.5}\left(X_{8 a}+X_{8 b}+X_{8 c}+X_{8 d}+X_{10}\right) \tag{2.80}
\end{equation*}
$$

In contrast to eqn. (2.69), which corresponds to AND2 operation for four images, eqn. (2.80) is obtained by OR operation for the same set of images plus an additional image $X_{l 0}$. For the obtained image, the shrinking procedure consisting of rectangular expansion and the subsequent edge removal is carried out. The procedure is slightly more involved than the procedure described before to obtain $X_{12}$. The shrinking operation is represented as

$$
\begin{equation*}
X_{15}=T_{\text {Shrink }}\left(T_{\text {Connect }}\left(T_{\text {Shrink }}\left(X_{13}\right)\right)\right) \tag{2.81}
\end{equation*}
$$

which is equivalent to

$$
\begin{gather*}
X_{14}=T_{\text {CirRect }}\left(T_{\text {Connect }}\left(T_{\text {Shrink }}\left(X_{13}\right)\right)\right),  \tag{2.82}\\
X_{15}=T_{\text {EdgeRem }}\left(X_{14}\right) \tag{2.83}
\end{gather*}
$$

where

$$
\begin{equation*}
T_{\text {Connect }}()=T_{A_{14}^{8}, l_{14}^{8}}\left(\cdots\left(T_{A_{14}^{2}, l_{14}^{2}}\left(T_{A_{14}^{1}, l_{14}^{1}}()\right)\right) \cdots\right) \tag{2.84}
\end{equation*}
$$

with

$$
\begin{align*}
& A_{14}^{1}=\left(\begin{array}{lll}
0 & 0 & 0 \\
1 & 2 & 1 \\
0 & 0 & 0
\end{array}\right), A_{14}^{2}=\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 2 & 0 \\
0 & 1 & 0
\end{array}\right), \\
& A_{14}^{3}=\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 2 & 1 \\
1 & 0 & 0
\end{array}\right), A_{14}^{4}=\left(\begin{array}{lll}
0 & 0 & 1 \\
1 & 2 & 0 \\
0 & 1 & 0
\end{array}\right), A_{14}^{5}=\left(\begin{array}{lll}
0 & 0 & 1 \\
0 & 2 & 0 \\
1 & 0 & 0
\end{array}\right),  \tag{2.85}\\
& A_{14}^{6}=\left(\begin{array}{lll}
1 & 0 & 0 \\
0 & 2 & 1 \\
0 & 1 & 0
\end{array}\right), A_{14}^{7}=\left(\begin{array}{lll}
0 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & 1
\end{array}\right), A_{14}^{8}=\left(\begin{array}{lll}
1 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{array}\right),
\end{align*}
$$

$$
\begin{equation*}
I_{14}^{1}=I_{14}^{2}=I_{14}^{3}=I_{14}^{4}=I_{14}^{5}=I_{14}^{6}=I_{14}^{7}=I_{14}^{8}=1.5 . \tag{2.86}
\end{equation*}
$$

Except for the first shrinking operation, the meaning of which may not be immediately clear and hence explained later, the basic idea of $T_{\text {Connect }}$ followed by $T_{\text {CirRect }}$ is to form an circumscribing rectangle for a set of pixels, for which a gap of one pixel is allowed between pixels. To be more specific, $T_{\text {Connect }}$ connects two pixels that match the template shown above, and then forms a circumscribing rectangle. This process is shown in the top row in Figure 2.26. The rational of this operation is that two pixels detected as linestops, each of which is separated from the other by one pixel gap, is derived from one pixel. In this example, $T_{\text {Shrink }}$ operation prior to $T_{\text {Connect }}$ operation does not have any effect. Without this operation, however, the four pixels shown in the bottom row in Figure 2.26, two of which are mutually connected, are merged to produce a big rectangle in $X_{14}$, which leads to a single pixel in $X_{15}$ after the final edge removal operation. With $T_{\text {Shrink }}$ operation, generation of a big rectangle containing initial four pixels is avoided, which results in two isolated pixels in $X_{15}$.

The above-explained operation defines the new position of the linestop for a set of line segments in different orientation planes. The position of each linestop in different orientation planes is adjusted to the newly defined position:

$$
\begin{equation*}
X_{9 p}=T_{E, 1.5}\left(X_{15}+T_{E x p 8}\left(X_{8 p}\right)\right) \quad(p=a, b, c, d) \tag{2.87}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{E x p 8}()=T_{A_{E x p}, I_{E x p 8}}() \tag{2.88}
\end{equation*}
$$

with

$$
A_{E x p 8}=\left(\begin{array}{lll}
1 & 1 & 1  \tag{2.89}\\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right), I_{E x p 8}=0.5
$$

Using linestops with their position adjusted, the following three images are computed:


Figure 2.26 Merging of linestops which originally belong to different orientation planes.

$$
\begin{gather*}
X_{\text {totalLS }}=X_{16}=T_{E, .5}\left(X_{9 a}+X_{9 b}+X_{9 c}+X_{9 d}\right),  \tag{2.90}\\
X_{\text {totalC }}=X_{17}=T_{E, 1.5}\left(X_{9 a}+X_{9 b}+X_{9 c}+X_{9 d}\right),  \tag{2.91}\\
X_{J C T-Y}=X_{18}=T_{E, 2.5}\left(X_{9 a}+X_{9 b}+X_{9 c}+X_{9 d}\right) . \tag{2.92}
\end{gather*}
$$

Note that these three operations correspond to OR, AND2, AND3 operations shown in Figure 2.6. $X_{\text {JCT-Y }}$ is one of the finally produced five features (Y-type junction).

Based on those features detected so far, higher level features are computed. First, junctions including both X-type and T-type are detected as an intermediate feature:

$$
\begin{equation*}
X_{19}=T_{E, 0.5}\left(X_{12}-T_{E x p 8}\left(X_{17}\right)\right) . \tag{2.93}
\end{equation*}
$$

Then the final features are detected as follows:
X-type junction (JCT-X):

$$
\begin{equation*}
X_{20}=T_{E, 0.5}\left(X_{12}-T_{E x p 8}\left(X_{16}\right)\right) ; \tag{2.94}
\end{equation*}
$$

T-type junction (JCT-T):

$$
\begin{equation*}
X_{21}=T_{E, 0.5}\left(X_{19}-X_{20}\right) ; \tag{2.95}
\end{equation*}
$$

true linestops (trueLS):

$$
\begin{equation*}
\left.X_{22}=T_{E, 0.5}\left(X_{16}-T_{E x p 8}\left(X_{12}\right)-X_{17}\right)\right) ; \tag{2.96}
\end{equation*}
$$

true corners (trueC):

$$
\begin{equation*}
X_{23}=T_{E, 0.5}\left(X_{17}-X_{18}\right) . \tag{2.97}
\end{equation*}
$$

These relationships can be represented in a more understandable way as:

$$
\begin{gather*}
X_{J C T-X T}=T_{E, 0.5}\left(X_{\text {totaJCT }}-T_{\text {Exp } 8}\left(X_{\text {totalC }}\right)\right),  \tag{2.98}\\
X_{J X T-X}=T_{E, 0.5}\left(X_{\text {totalJCT }}-T_{E x p 8}\left(X_{\text {totalLS }}\right),\right.  \tag{2.99}\\
X_{J C T-T}=T_{E, 0.5}\left(X_{J C T-X T}-X_{J C T-X}\right),  \tag{2.100}\\
X_{\text {trueLS }}=T_{E, 0.5}\left(X_{\text {totalLS }}-T_{E x p 8}\left(X_{\text {totalJCT }}\right)-X_{\text {totalC }}\right),  \tag{2.101}\\
X_{\text {trueC }}=T_{E, 0.5}\left(X_{\text {totalC }}-X_{J C T-Y}\right) . \tag{2.102}
\end{gather*}
$$

These formulas directly describe the relationship between several features shown in Figure 2.6 and Figure 2.7. The use of $T_{\text {Exp } 8}$ is to compensate possible mis-alignment of the linestop position between four orientation planes. The only expression which is not represented in Figure 2.6 is the inhibition of $X_{\text {totalls }}$ by $X_{\text {totalc }}$ to derive $X_{\text {truels. }}$. (eqn. (2.101)). This should be unnecessary since $X_{\text {totalC }}$ is a subset of $X_{\text {totalJCT }}$. There are cases, however, in which the corner is not detected in $X_{\text {totalJCT }}$ as the intersection of two line segments in different orientations but is detected in $X_{\text {totalC }}$ as the intersection of two linestops in different orientation planes as a result of the alignment of linestop position.

### 2.6.10 Key points of the algorithm

The processing flow for the detection of important image features has been described in detail in this section. The processing flow consists of thinning, orientation decomposition, linestop detection, and further processing based on interaction between images of line segments and linestops in different orientation planes. The two key points of the processing flow are summarized below.

In the error correction stage after orientation decomposition, the minimum length of the line segment is defined as three pixels: line segments that are one or two pixels long are removed (EIP operation). Likewise, the gap between two line segments has to be one pixel or two pixels long so that they are considered a candidate to be filled to recover the originally one line segment (LC and LE operation). Another important operation in the error correction stage is the line inhibition operation, which assigns only one orientation for one pixel and consequently produces the same orientation range $\left( \pm 18.4^{\circ}\right)$ for all the four orientations. These additional steps of processing virtually increase the interaction area to its original $3 \times 3$ region to a larger area, which leads to the increased robustness of the proposed algorithm.

In the stage for the detection of higher level features, the adjustment of the linestop position is important to absorb the difference in the position of linestops in different orientation planes. Since prior to this stage, the processing is carried out in each orientation plane independently of other orientation planes with the only exception of line inhibition operation, the position of the linestop corresponding to the same corner could be different. Several linestops in different orientation planes whose mutual distance is one or two pixels, i.e., a maximum of one pixel gap is allowed, are considered to represent the same corner and hence merged together to produce a single linestop. Note that this rule does not contradict with the above-mentioned rule of the minimum line length of three. The LE operation ensures that the minimum length of the line segment is five, avoiding the merging of two linestops belonging to the same line. Note also that almost similar idea of compensating the difference in the position of features in the four orientation planes is realized by the use of $T_{\text {Exp } 8}$ operation.

### 2.7 Evaluation of the algorithm

The feature detection algorithm explained in detail in the previous section is applied to


Figure 2.27 Features detected by the proposed algorithm for alphabetical characters. The center of a closed rectangle indicates the position of a feature. The shade of the rectangle represents the type of the feature.
printed and handwritten characters as well as simple patterns to evaluate its performance.

### 2.7.1 Printed characters

Figure 2.27 shows the simulation result for printed characters with the proposed algorithm. For twenty-six alphabetical characters, the result of thinning is shown below the corresponding original character; the detected features are superimposed on the thinned character. The font type is Arial with a $32 \times 32$ resolution. The simulation result demonstrates that the algorithm is able to detect almost all important features in a discriminative fashion. These features include T-type junctions in letters "E", "F", "H" "K", "P", "R", "T", and X-type


Figure 2.28 Simulation result for letter "V". Each of the two diagonal line segments is represented as an alternating sequence of $45^{\circ}$ and $90^{\circ}$ line segments, resulting in unexpected corners (shown in the rectangle of a thick boundary).
junctions in letters "Q" and " X ", and Y -type junctions in letters " B " and " Y ". Corners are detected correctly for most letters including "D", "E", "F".

Corners are also detected along the circular stroke in letters "C", "D", "G", "O", "Q", "S". The circular strokes are nicely represented as a sequence of line segment with its orientation gradually changing. For example, letter "O", which has a closed circular stroke, is represented as eight line segments whose orientation changes to complete two cycles of $0^{\circ}$, $45^{\circ}, 90^{\circ}$, and $135^{\circ}$. In other words, the closed circular stroke is represented as an octagon. It should be also mentioned that the point of corner does not indicate the point of strong curvature. This is in contrast to conventional corner detection algorithms, which respond to local curvature. No corner would be generated by these algorithms for a large letter "O" while the


Figure 2.29 Simulation result for letter "W". The line segments in $45^{\circ}$ and $135^{\circ}$ orientations constituting the three corners (shown in the circle) are removed by the EIP operation since the line length is two. This result in the failure of corner detection; linestops are detected instead (shown in the rectangle of a thick boundary).
entire periphery would be detected as corners for a small "O". For recognition purposes, the representation in the proposed algorithm, which generates eight corners regardless of the size of the letter, reflects the topological information more faithfully than the conventional algorithms.

However some of the results contradict with human perception. For example, for letters "V", four corners are detected along the left and right stroke. This is because these two strokes are not represented as a line segment of a single orientation as shown in Figure 2.28: for example, the right stroke is represented as an alternating sequence of $45^{\circ}$ and $90^{\circ}$ line segments. This alternating sequence generates unexpected corners along these strokes. This
type of unexpected corners are also observed in letters "A" and "M". Although generation of these unexpected corners for lines in a certain orientation range is not erroneous, it may be problematic in some cases.

Another problem, which is even more serious, is the failure of corner detection. It is observed for letter "M" and "W". The acute angle at the bottom corner in "M" and the three corners in "W" is not well represented as a sequence of line segments that are allowed to have only four orientations. As is shown in Figure 2.29, the three corners, one at the top and the two at the bottom, are decomposed as line segments of $45^{\circ}$ and $135^{\circ}$ orientations immediately after orientation decomposition. However, these line segments are removed by the EIP operation since the length of these line segments is two. As this examples shows, the deletion of line segments of length two, especially when it happens near corners and junctions, sometimes results in the failure of detection of these features and results in the detection of unexpected linestops instead.

Figure 2.30 shows another simulation result performed for the same alphabetical characters with italic modifications. Overall, most of the features successfully detected in Figure 2.27 are also detected. This is because with this degree of slanting, line segments that are originally aligned in either horizontal or vertical direction are still categorized as the same orientation. Slanting caused favorable effect for letters "M" and "W", which were previously difficult to handle as shown in Figure 2.27. Diagonal lines whose original orientation is either $45^{\circ}$ or $135^{\circ}$ are categorized as either $0^{\circ}$ or $90^{\circ}$ in the slanted letter: the slanted "W" is represented as line segments of $45^{\circ}$ and $90^{\circ}$. As a result of this, all the three corners were correctly detected.


Figure 2.30 Features detected by the proposed algorithm for italic alphabetical characters.

However, several patterns causing problems are found by this simulation. T-type junctions are not correctly detected for letters " $K$ " and " $R$ " since the line segment corresponding to the stem ${ }^{7}$ of the T -junction is detected in a shorter form than it should be. This situation is shown in Figure 2.31 (a) for letter "R". The upper end point of the leg extending toward the bottom right direction is categorized as $90^{\circ}$ while the rest of the line segment is categorized as $135^{\circ}$ orientation. Therefore, the $135^{\circ}$ line segment cannot reach the horizontal line segment (bottom part of the loop) even with the help of the LE operation. For letter "K", no orientation is assigned to the point indicated in the figure, which makes the $90^{\circ}$ line segment shorter to fail

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Figure 2.31 Pixel point where the stem is separated from the bar of the T-type junction for two letters " $R$ " and " $K$ ". It is superimposed on the original letter displayed with half-tone color. (a) The local orientation of this point is computed as $90^{\circ}$. (b) No orientation is assigned to this point.
to reach the $45^{\circ}$ line.

There was another problem found for letters " $X$ " and " $Y$ ". For letter " $X$ ", instead of the X-type junction at the center of the letter, the T-type junction is detected. For letter "Y", instead of the Y-type junction, the T-type junction and the corner are detected. These somewhat confusing results are associated with the result of thinning as shown in Figure 2.32 for letter " X ". The thinning operation generated a short vertical line segment between two L-shaped corners, one is facing upward and the other facing downward. The presence of this vertical line segment eventually leads to the breaking of the $135^{\circ}$ line segment. As a result of this, two Ttype junction are detected as the point of intersection between the $45^{\circ}$ line segment and two $135^{\circ}$ line segments. These two T-type junctions are merged to finally produce single T-type junction. What happens to letter "Y" is somewhat similar. A closer look of the thinning result indicates the presence of the T-type junction formed between the $45^{\circ}$ and the $135^{\circ}$ line segments, and the corner formed by the $45^{\circ}$ and the $90^{\circ}$ line segments instead of the single Y-


Figure 2.32 Simulation results for slanted letter " X ". The vertical line segment (shown in the circle) generated by thinning leads to the detection of the T-type junction instead of the X-type junction (shown in the rectangle with a thick boundary).
type junction where the above three line segments of different orientations are supposed to meet.

Figure 2.33 shows the effect of the order of thinning on the detected features. The result shown in (b) and (d) is obtained by performing the thinning in the order of south, east, north, and west, while the result in (a) and (c) is obtained in the order of west, south, east, and north. In (b) and (d), the thinning result maintains the structural shape of the original letter without generating any redundant line segments as observed in (a) and (c), which resulted in the correct detection of the features.


Figure 2.33 Effect of thinning order on the feature detection. Thinning was performed in the order of west, south, east, and north for (a) and (c), while it was performed in the order of south, east, north, and west for (b) and (d).

### 2.7.2 Handwritten characters

The feature detection algorithm was applied to handwritten characters to further investigate its performance. The characters were obtained from the database produced by the Center of Excellence for Document Analysis and Recognition (CEDAR) at the State University of New York at Buffalo (http://www.cedar.buffalo.edu/Databases/CDROM1/chars.html) [30]. The result is shown in Figure 2.34. Even for these deformed character sets, the algorithm was successfully able to detect the important features. Some results that are inconsistent with a human perception are due to the problems discussed earlier.

For example, the unexpected break in the middle of the left leg in letter "A" and the absence of X-type junction in letter "Q" is due to the deletion of some important line segments whose length is two. The Y-type junction in letters " M " and " N " are generated by the thinning process. It is difficult to thin a line drawing that has corners of an acute angle: it tends to generate a Y-type junction instead of a V-type corner. Despite these problems, the algorithm proved to be applicable to characters whose shape is not as nicely defined as printed characters


Figure 2.34 Features detected by the proposed algorithm for handwritten characters. These data were obtained from the database at http://www.cedar.buffalo.edu/Databases/CDROM1/chars.html [30].
due to some deformation.

### 2.7.3 Patterns

The algorithm was also applied to several patterns of binary values to test its performance for more general cases. The simulation was performed without thinning process before orientation decomposition. The simulation result shown in Figure 2.35 demonstrates that all the features are successfully detected. For the edge image of a three-dimensional view of the cube, Y-type junctions and corners are correctly detected. Note that the Y-type junction at the center bottom would be detected as the T-type junction when the $3 \times 3$ template matching is performed without considering the neighborhood information. The more global information obtained by


Figure 2.35 Features detected for various patterns. From left to right, they are indicated as wire frame of the rectangle, open circle, closed circle, closed triangles, checkerboard pattern.
the additional steps of processing (line completion, elimination of isolated points operations) indicated the Y-type junction instead of the T-type junction as the point where line segments of three different orientations meet.

Note also that the open circle and the closed circle produce exactly the same result. This is because the procedure of orientation detection simultaneously extracts edges of the closed circle. Due to this edge extraction capability, X-type junctions are correctly detected from the checkerboard pattern.

### 2.7.4 Discussion

The simulation results presented in this section proved that the feature detection algorithm works well for binary images. The algorithm is not only useful for line drawings but also can be applied for more general patterns as long as they are represented in binary images. The algorithm was robust enough to deal with handwritten characters that have a large degree of deformation thanks to additional steps of processing for error correction. Corners and junctions are detected in a discriminative fashion as well as linestops. It is also possible to further describe the type of each of these features in terms of orientation if necessary. For example, an X-type junction which consists of the horizontal line segment and the vertical line segment can
be discriminated from an X-type junction which consists of two diagonal line segments.
Several problems concerning orientation decomposition were found. The first problem is the representation of a slanted line. A straight line whose orientation is almost in the middle of two adjacent orientations may be represented as a sequence of line segments whose orientation fluctuates between the two orientations, which results in unexpected corners. This representation contradicts with a human perception, which would like to assign only one orientation for a single straight line. Therefore, it may be more appropriate to merge these broken line segments to create a longer line, which is defined as a loosely diagonal line. This is possible to realize, for example, by newly defining the $45^{\circ}$ orientation by including both $0^{\circ}$ and $90^{\circ}$ line segments in addition to the originally present $45^{\circ}$ line segment.

The other problem concerning the orientation decomposition is that there are some pixels for which no orientation is assigned. Although no orientation is probably the right judge, this results in the failure of corner detection and failure of junction detection. One of the possible solutions for this problem is the assignment of the four orientations to those pixels instead of no orientation assignment. The combined use of this new orientation assignment with the above mentioned loose representation of diagonal lines is expected to further improve the performance of the algorithm although detailed investigation needs to be carried out.

It was found that thinning could be problematic especially for line drawings having a sharp corner: letter "V" could become "Y" after thinning. Another problem is the instability of the thinning algorithm: the topological information could be different depending on the order of thinning. Actually there have been a large number of thinning algorithms proposed (see [31] for review, for example). Each algorithm has its advantages and disadvantages in different aspects. These aspects include the degree of excessive erosion, generation of spurious branches, and so on. The most important requirement for the thinning algorithm to be used as preprocessing for
the feature detection is being able to preserve the structural shape. Therefore, further refinement of the present thinning algorithm as well as investigation for other existing algorithms would be required so that the overall feature detection algorithm works for line drawing images without being affected by the line width of the pattern.

### 2.8 Summary

The feature detection algorithm, which is able to detect corners, three types of junctions (T-type, X-type, Y-type) and linestops of a binary image, is presented in this chapter. The advantages of the present algorithm over other existing ones is the detection capability of three types of junctions in a discriminative fashion even though its input is limited to a binary image.

The algorithm is inspired by the biological visual processing, which performs detection and hierarchical integration of features. The proposed algorithm first decomposes an input image into four orientations and detects linestops for line segments in each orientation plane. Corners and junctions are detected as a result of interactions between line segments and linestops in different orientation planes.

The algorithm is formulated on the basis of template matching which iteratively performs a $3 \times 3$ neighborhood processing. The degree of matching is determined by comparing the result of correlation between $3 \times 3$ neighbors and the template to a specified threshold value. The stringency of template matching can be lowered by decreasing the threshold value, which results in a smaller number of templates for certain types of processing. The template matching is suitable for hardware implementation because of its nearest neighborhood interaction. It is also found that the template matching is closely related to other image processing paradigms, mathematical morphology and the special form of DTCNN.

To supplement the limitation of a small $3 \times 3$ interaction area, additional steps of processing are implemented, which includes line completion, elimination of isolated points, and
so on. These additional steps of processing virtually increase the area of interaction, which results in the increased robustness of the algorithm to the deformation of an object. Due to this mechanism, the features are successfully detected even for handwritten characters with a large degree of deformation.

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## Chapter 3 Architectural design of the sensor

In the previous chapter, the algorithm for feature detection has been described with possible hardware implementation in current-mode configuration in mind. In this chapter, architectural and design issues for the realization of the proposed algorithm is described. First, related research concerning the implementation of several functions on-chip is reviewed focusing on the phototransduction and the processing circuitry. Based on these observations, a conceptual schematic of the processing circuit is derived, which employs a phototransistor as an optical input and performs processing in an analog/digital mixed-mode fashion.

### 3.1 Overview of related research

There are numerous types of image sensors that enable some preliminary processing onchip. In the following, they are referred to as vision sensors. The general architecture of these vision sensors is shown in Figure 3.1. The sensor consists of an array of pixels and the peripheral circuit. Each pixel further consists of the photosensor and the processing circuit. The pixel is usually connected to its four neighbors or eight neighbors to perform some processing on-chip. The operation is controlled by several signal lines, which usually runs both horizontally and vertically to cover an entire pixel array. The peripheral circuit should at least include a scanning circuit to read the processing result from each pixel to the output node. The most important aspect of the vision sensor is that some processing takes place on chip in a parallel fashion due to the presence of processing circuitry at each pixel.

The functionality of the sensor is determined by the processing circuitry, which can be implemented in either the analog or digital domain. Depending on the type of implementation,


Figure 3.1 General architecture of vision sensors. A rectangular array of pixels, each of which consists of the photosensor and the processing circuit, and the peripheral circuit constitute the vision sensor. The sensor receives a set of control signal and produces an output.
vision sensors are roughly divided into two categories: analog vision sensors and digital vision sensors. The analog vision sensor usually realizes some functionality using the nature of an analog circuit; neither control clock nor memory is usually required for the operation. They are usually dedicated for specific purposes. The digital vision sensor, on the other hand, incorporates memory and a Boolean operation unit. A set of operations is executed in a sequential fashion at each control clock. They are oriented for general-purpose use. There is another class of implementation methodology based on the cellular neural network (CNN) scheme, which can be considered a hybrid of the above two implementations.

The configuration of the photosensor is important as well as the processing circuit. The phototransduction method must be carefully chosen to facilitate the subsequent processing. In general, analog vision sensors prefer steady-state phototransduction while digital vision sensors
prefer phototransduction based on charge integration.
In the following section, various types of phototransduction methods are reviewed first. Then the type of processing circuitry, including analog, digital, or CNN schemes, is explained. Based on these observations, a new mixed-mode processing scheme will be proposed.

### 3.1.1 Phototransduction

The first task of vision sensors is phototransduction, which converts the intensity of the incoming light into an electrical signal. The most popular device for phototransduction is the CCD (Charge Coupled Device), which is basically an analog shift register. The advantages of a CCD are its high sensitivity, low noise, high fill-factor, and good matching [1][2]. However, there are several disadvantages as well. First, CCD consumes a considerable amount of power: it has to drive a large capacitance; it is inherently difficult to lower the voltage due to the need of creating a deep depletion region under the gate. Second, a clocking scheme is complicated: multiple voltages have to be applied with high speed to enable nearly perfect charge transfer. Third, CCD process is not compatible with CMOS process, which is widely accessible and well-understood technology suitable for the implementation of on-chip functions.

Although phototransduction can be carried out in several ways even with CMOS processes, a higher noise level than that of CCD has limited its use in imager applications despite several advantages such as random access capability, simple clocking scheme, integration with on-chip processing functions. The most dominant noise source is a fixed pattern noise, which is generated due to mismatch in transistors used at each pixel. However, ever increasing demands for low power consumption, together with new technological improvements such as correlated double sampling [3][4] to decrease the noise level, have attracted lots of attention toward CMOS as an alternative technology for imager applications. Most vision sensors have been implemented using CMOS processes mainly due to its capability


Figure 3.2 Schematic explaining the process of phototransduction. The node voltage $\mathrm{V}_{\mathrm{d}}$, which is first precharged to $\mathrm{V}_{\mathrm{DD}}$, starts decreasing after Reset is set low due to the discharging by the photogenerated current.
for implementing both phototransduction and processing functions on the same chip.

Even with CMOS processes, there are several methods for phototransduction. These methods can be roughly divided into two categories: integration type and steady-state type. The integration type phototransduction is frequently used in standard imager applications. Figure 3.2 shows the schematic of integration type phototransduction using the photodiode [5][6]. When Reset is logic High, the n-terminal of the photodiode, which has a parasitic capacitance C, is charged up to $\mathrm{V}_{\mathrm{DD}}$. After Reset is set to logic Low, the photogenerated current starts discharging the capacitance until Reset is set to logic High again. The slope of discharge depends on the photogenerated current and the capacitance. The amount of the photogenerated current is estimated as

$$
\begin{align*}
& I_{p h}=q \eta \frac{E A}{h v}  \tag{3.1}\\
& I_{p h}=q \eta \frac{E A}{h v} \tag{3.1}
\end{align*}
$$

where $q$ is the charge per electron $\left(1.6 \times 10^{-19}\right.$ [C]), $\eta$ is the quantum efficiency, $E$ is the irradiance, $A$ is the area of the photodiode, $h$ is the Plank constant, $v$ is the frequency. The voltage drop is represented as a function of the integration time $T$ :

$$
\begin{equation*}
\Delta V=\frac{I_{p h} T}{C}=\frac{I_{p h} T}{C_{d} A}=q \eta \frac{E A}{h v} T \frac{1}{C_{d} A}=\frac{q \eta}{h v} \frac{E T}{C_{d}} \tag{3.2}
\end{equation*}
$$

where $C$ is the total parasitic capacitance, $C_{d}$ is the capacitance of the depletion region per unit area. To get a rough idea about the amount of $I_{p h}$ and $\Delta V$, the following figures are substituted for eqns. (3.1) and (3.2): $A=5 \times 5=25 \mu \mathrm{~m}^{2} ; C_{d}=1\left[\mathrm{fF} / \mu \mathrm{m}^{2}\right] ; E=0.1\left[\mathrm{~W} / \mathrm{m}^{2}\right]^{8} ; \eta=0.4 ; T=33$ [msec]. The photogenerated current and the resultant voltage are 0.48 pA and 0.63 V , respectively. Note that the amount of photogenerated voltage is independent of the area of the photodiode. This is desirable for further shrinking of the photosensor area to meet the demand for higher pixel densities on chip.

Another phototransduction method belonging to the integration type is reported [3]. This method uses photogate, which is actually made of polysilicon, and has some similarity to CCD. Electrons generated by light illumination are accumulated under the photogate while the gate voltage is maintained at $\mathrm{V}_{\mathrm{DD}}$. After some integration time, the gate voltage is brought down to GND to transfer the accumulated charge to the floating diffusion node (FD), which is initially set to $\mathrm{V}_{\mathrm{DD}}$. The transfer of electrons lowers the voltage of FD, which is detected by a source follower. Correlated double sampling is possible by reading the signal twice. First reading is carried out when the floating node is connected to $\mathrm{V}_{\mathrm{DD}}$, and the second reading is carried out after the accumulated electrons are transferred. The result of each reading is stored on two capacitors to produce a differential output, which is free from noise.

The second category of phototransduction method, which is steady-state type, simply

[^6]

Figure 3.3 Structure of the phototransistor. The phototransistor is formed vertically by defining p+ emitter region in the n-well. The n-well serves as the base while the p -substrate works as the collector. The emitter region is defined in a stripe manner for allowing the light to easily reach the base region.
uses a current output of a photodiode or a phototransistor. This type of phototransduction scheme is mainly used in analog vision sensors for the following reasons. First, no external clocking is necessary to obtain the signal: the output becomes available immediately after light falls on the photodiode. This is suitable for analog implementation, which usually operates in a continuous mode without any control clocks.

The phototransistor has been more preferably used than the photodiode simply because of its higher current sensitivity. Figure 3.3 shows the structure of the phototransistor [7]. Note that the base is left floating. It has been implemented using the parasitic aspects of CMOS processes. When the light falls on the phototransistor region, electron-holes pairs are created underneath. Electrons generated in the base region are swept to the emitter region while holes generated in the emitter region are swept into the base region due to the builtin potential. Carriers within a diffusion length contribute to this action. The same kind of action takes place at the base-collector boundary. As a result of this distribution process of carriers, the base region becomes more negatively charged, lowering the emitter-base potential barrier. This further increases the flow of holes from the emitter to the base. The gain of this process, current


Figure 3.4 Logarithmic compression circuit. (a) Mahowald and Mead's logarithmic compression circuit [9]. (b) Etienne's compression circuit with an additive transistor to produce an amplified current output [7].
gain $\beta$, is determined by the number of holes that can cross the base before one hole recombines with an electron in the base [8].

The current gain $\beta$ measured by treating the phototransistor as a normal three-terminal device was larger than 100 [7]. Therefore, the current output of the phototransistor is estimated more than 100 times larger than that obtained by the photodiode, leading to an estimate of 48 pA for geometry of $5 \mu \mathrm{~m} \times 5 \mu \mathrm{~m}$. Figure 3.4 (a) shows the schematic of the logarithmic compression circuit, which is developed by Mahowald and Mead and has been frequently used in analog vision sensors [9]. The small current enforces the transistor to operate in the subthreshold region and hence produces an output voltage that varies as a function of the logarithm of the photogenerated current. Due to this range compression capability, which is also found in biological visual system, the phototransistor can cover several orders of light intensities without signal saturation. The circuit shown in Figure 3.4 (b), which is a slight
modification of the circuit in (a) [7], is able to produce an amplified current output in the $\mu \mathrm{A}$ order, while still maintaining the capability of covering a broad range of light intensities.

To conclude this subsection, there are mainly two types of phototransduction methods. The photosensor in the first category performs phototransduction by charge integration and has the following characteristics: (1) the sensitivity can be controlled by the integration time;. (2) the shrinking of the pixel size does not decrease the output voltage; (3) clocks are necessary for the operation. The photosensor in the second category produces a steady-state output current or voltage, which can be just a current output of a photodiode/phototransistor or its converted form by some processing. This type of sensor has the following characteristics: (1) the output is immediately available after the light falls on the sensor; (2) logarithmic range compression is possible by operating the transistor in the subthreshold region; (3) no external clock is required.

Starting from the next subsection, various types of implementation method for the processing circuit is discussed with some reference to the phototransduction method.

### 3.1.2 Processing circuit - analog implementation

The first analog vision sensor was developed by a group led by Carver Mead at Caltech [8]. The fabricated sensor is called silicon retina, which has been cited frequently by researchers involved in this field. Their motivation was to model and implement the first stage of retinal processing onto a silicon chip. The processing that takes place in the outer plexiform layer of the retina is explained in the previous chapter. Briefly, the photoreceptor output, which is proportional to the logarithm of the light intensity, is spatially and temporally averaged by the horizontal cells. The output of the bipolar cells is proportional to the difference between the photoreceptor signal and the horizontal cell signal. This results in the concentric ON-center type receptive field: the light which falls onto the center area excites the bipolar cell while the light which falls onto the periphery inhibits the bipolar cell. This receptive field responds well


Figure 3.5 Resistive grid (left) and its impulse response (right). The impulse response is defined as the voltage profile for a current injected at one node. The voltage exponentially decreases as a function of distance from the node of current injection. The spread is larger for larger values of $\mathrm{R}_{\mathrm{V}} / \mathrm{R}_{\mathrm{H}}$.
to edges of an object where intensity changes very sharply. It is reasonable to perform such a processing at the entry level of visual processing since the edge is considered a important feature of an object [10].

Since their objective is to mimic biology, the logarithmic compression circuit explained in Figure 3.4 (a) was a perfect choice. For the implementation of the spatial spreading function of the horizontal cell, the resistive grid scheme was employed. Figure 3.5 shows the resistive grid, which consists of a series of horizontal resistors $\left(\mathrm{R}_{\mathrm{H}}\right)$ and vertical resistors $\left(\mathrm{R}_{\mathrm{V}}\right)$, and its impulse response. The impulse response is defined as the voltage profile for a current injected at one node. The voltage decreases in an exponential way as a function of the distance from the node of current injection. This is the result of gradually decreasing current leakage from each node of the resistive grid to the ground as the current spreads from the injection node to the periphery. Note that the extent of spreading is controlled by $R_{V} / R_{H}$ : larger values of $R_{V} / R_{H}$ produces a broader spreading. Any input image is convolved with this impulse response to produce a smoothed output. The photoreceptor output is converted into a current through transconductance amplifier to be injected into this resistive grid. The resultant smoothed output
is then subtracted from the photoreceptor output to produce the difference signal, which corresponds to the bipolar cell output in biology. The fabricated sensor showed a good resemblance to the biological retina in terms of spatial and time response.

Several researchers have tried to enhance the quality of smoothing by the resistive grid. The first attempt is characterized by the use of the so-called resistive fuse [11][12]. The resistive fuse acts as a normal resistor to produce a smoothed image when the voltage difference between neighboring pixels is not very large. When the voltage difference becomes larger than a specified threshold, indicating that this point is likely to correspond to the boundary of an object, the resistive fuse shuts off the connection to separate these two regions. As a result of the above action, the smoothing takes place in each segmented region while the edge is kept sharp since blurring does not take place. The second attempt was to use the Gaussian function instead of a conventional exponentially decaying function as a convolution kernel [13]. The cusp shape of the exponentially decaying function, which has a discontinuity in derivative at the center, would produce undesirable results when this function is applied to a noisy image and then followed by edge detection. The Gaussian kernel was realized by connecting a pixel to the next-nearest pixels by the negative resistance in addition to the commonly used connection between nearest neighbors.

Boahen and Andreou reported another type of implementation of the smoothing and subsequent edge detection function [14]. They developed a silicon retina, which is even simpler than Mead's retina but more closely parallels biological counterpart. In addition to the resistive grid that mimics signal spreading due to the bipolar cell, they also implemented the signal spreading through a photoreceptor layer, which takes an input current directly from the phototransistor. They also realized mutual interaction between these two resistive grids, excitatory action from the photoreceptor layer to the horizontal cell layer, and the inhibitory action from the horizontal cell layer to the photoreceptor layer. These bilateral interactions are
implemented in a smart way using two transistors facing each other. This circuit is considered the special case of the winner-take-all circuit proposed by Lazzaro [15] where strength of the horizontal interaction is weakened due to the resistance which lies between pixels.

It should be mentioned that one of the reasons why research activity on vision sensors based on the resistive grid scheme was so high is the close relationship between the resistive grid and the regularization theory [16]. Several problems in early vision can be mapped into the resistive grid scheme. The solution is obtained by simply getting the network to reach the steady state, which corresponds to the state of the minimum energy dissipation.

Most of the above mentioned studies employ transistors operating in the subthreshold region. The main advantage of using the subthreshold region is its low power consumption. In addition, the exponential relationship between the gate voltage and the drain current can be useful for compact implementation for complex computations [8][17]. However, those devices suffer from severe transistor mismatch effect and the operational speed is slow due to the small current. Based on these observations, Etienne-Cummings has designed an edge detection circuit, which serves as a front-end for his motion detection sensor, in the strong inversion region [7]. Edge detection function is realized by a resistive grid and an Opamp, which is used to compute the difference between the input image and the smoothed image.

Another important image feature that has been detected by analog vision sensors is orientation of lines and edges. The importance of orientation detection can be found in biology as described in the previous chapter. An array of orientation sensors has been reported by several researchers. Camp has fabricated an orientation sensor which consists of $8 \times 8$ pixels [18]. Each orientation sensor has six outputs, corresponding to six orientations ( 0 to $150^{\circ}$ by $30^{\circ}$ interval). The largest output indicates the orientation of the image that falls onto the orientation sensor. Each of these six outputs is produced by summation of currents from each
pixel with appropriate weights. These weights are chosen so that the resultant receptive field forms a first-order Gabor function. The Gabor function has some directionality and hence responds most to an image aligned along the preferential direction. This is in contrast to the concentric receptive field realized for smoothing purposes. These orientation sensors are combined in the second layer to detect a specific curvature. Although this study showed a good example of feature detection based on decomposed oriented segments, each orientation sensor is still too large and complicated to enable larger array configuration.

A simplified orientation sensor with lower orientation resolution was reported by Nishimura [19]. Instead of using a sophisticated receptive field, the illumination pattern defined by which photosensor located at the four corners of a pixel is under illumination, was used to encode orientation. A sensor chip which consists of a $6 \times 6$ array of these orientation sensors was fabricated. The fabricated sensor was able to successfully decompose several images projected on the sensor into four orientation planes. Almost the similar concept, but slightly different approach, was demonstrated by Lin [20]. They have used hydrogenated amorphous silicon technology to implement four pin diodes of trapezoidal shape placed at the edge of the pixel. The orientation was calculated by using the current output from these photodiodes. At each sensor, which is part of the fabricated $8 \times 8$ array, the orientation was locally calculated for the image projected on the sensor. In contrast to refs. [18][19], the sensor output is represented as an analog continuous value, which may be difficult to process at later stages for the detection of higher features. The similar idea of calculating the orientation from the four position sensitive devices is found in ref. [21].

The directional receptive field can be realized by giving different voltages to the gate of the transistors for horizontal connection in different orientations. However, the rapidly decreasing nature of the exponential function does not produce a large receptive field. Venier et al. have replaced vertical resistors forming a resistive grid with current sources so that the
voltage decaying profile becomes linear instead of an exponential profile [22]. The voltage is compared to a given threshold to produce a binary-valued receptive field. The experimental result showed that by properly controlling the gate voltages of horizontal resistors in three directions and choosing the proper injection current level, the orientation and the size of the receptive field could be well controlled.

Instead of preparing a set of receptive fields for different orientations, it is possible to estimate the orientation based on the two responses, one obtained from the receptive field for $0^{\circ}$ and the other for $90^{\circ}$. Etienne-Cummings et al. have demonstrated this idea in ref. [23]. The receptive field for $0^{\circ}$ is represented as a $3 \times 3$ template matrix, i.e., a set of weights multiplied for $3 \times 3$ neighbors for current summation. The template consists of three horizontal 2 's in the center row and three -1 's both in the top row and in the bottom row. The receptive field for $90^{\circ}$ is obtained by rotating the $0^{\circ}$ receptive field by $90^{\circ}$. It was shown that the response from the $0^{\circ}$ receptive field is largest for $0^{\circ}$ line and linearly decreases as the orientation increase up to $45^{\circ}$. Above $45^{\circ}$, the response levels off at almost zero. On the other hand, the response of the $90^{\circ}$ receptive field is almost zero up to $45^{\circ}$ but starts linearly increasing to reach the maximum at $90^{\circ}$ stimulus. At each pixel, the local orientation was reconstructed from these two outputs. The size and the configuration of the receptive field are programmable by a smart circuit configuration. The circuit for generating the receptive field is placed outside the pixel array and the computation is performed at the time of read-out. The proposed scheme thus converts the computation performed in the space domain in most vision sensors to the time-domain, enabling the increase of the pixel density and providing exactly the same kernel for every pixel without suffering from transistor mismatch. The idea of programmable kernel was also demonstrated in a different fashion by Kyuma et al. [24]. Their
sensor consists of a two-dimensional variable sensitivity photosensors. The convolution kernel is limited to one-dimension and is given as the voltage pattern along the column, which controls the sensitivity of the photosensor. The convolution is computed by the current summation; the computation result is scanned along all column lines simultaneously.

The orientation detection explained so far is a local computation: each pixel detects the orientation using the information projected on itself and neighbors. The extent of neighbors being used for calculation determines the size of the receptive field. In other words, this type of processing is pixel-based. In contrast to these local computations, a principal orientation of an object projected on the entire chip is sometimes useful. Standley et al. have fabricated a sensor that detects the centroid and orientation of an object [25]. The centroid and the orientation can be defined using the first order and the second order moment. The fabricated sensor produces a set of current, which is the result of some computation using the two-dimensional resistive grid for the given intensity distribution. The centroid and the orientation can be reconstructed from these outputs. This type of processing can be referred to as object-based, since the detected quantities, centroid and orientation, are the attributes of each object.

Several papers have been reported on these types of object-based processing. Liu and Harris reported the concept of the dynamic wire which can be used for different calculations [26]. The concept of dynamic wire is: configuring the switches in a two-dimensional network to connect a group of pixels that share a common property, and then utilizing the resultant dynamic connections for computation. By closing the switches that correspond to the edge of an object, pixels on the contour are connected with each other. From a set of these mutually connected pixels, each having a current source, a leader pixel is arbitrary chosen to establish a current path to the ground. The total current, which flows into the ground, is proportional to the number of pixels belonging to the contour.

Luo et al. have presented an interesting sensor which performs figure-ground segregation
by labeling all points inside a given contour with one voltage and all remaining points outside this contour with another voltage [27]. The resistive network is configured with communication switches. The input of the sensor is again the contour, or the binary edge map. A pixel corresponding to the contour is isolated from the surrounding pixels. This is the opposite action that takes place on the above-mentioned sensor for contour length computation. The pixels inside an object are connected with each other and isolated from the surrounding region. The current injected at the center, which is based on the assumption that the object is always centered, spreads through a resistor path until it hits the contour where the switch is shut off. If the contour is broken at some points, the current spreads outside to seal off the incomplete contour, isolating the object from the background.

Nishimura et al. have presented a sensor that counts the number of objects [28]. The sensor produces a current that is proportional to the number of objects. The object is defined as a set of mutually connected pixels in either horizontal or vertical direction. Two adjacent pixels in either horizontal or vertical direction are connected with each other when both of these two pixels are under illumination. Thus the region under illumination is defined as one object and isolated from the surroundings. The winner-take-operation is performed within each object to select one winner. A current that is proportional to the number of winners and hence proportional to the number of objects is produced. The sensor not only counts the number of objects but also measures the size of each object. This is possible because the current at the winning pixel collects all the current from other pixels inside the object and thus represents the object size.

Several types of analog vision sensors have been reviewed above. It was observed that most of these vision sensors favor phototransistors as a photosensor due to its operation in the subthreshold region and the steady-state output. In terms of functionality, several functions such as smoothing, edge detection, orientation detection, and detection of object-based features
have been implemented. The beauty of analog processing lies in its simplicity. By properly mapping the target problem into the voltage and current domain, several functions are realized with much smaller number of circuit elements than the digital counterpart. For example, the resistive grid scheme allows the interaction between pixels that are several pixels apart although the wiring is limited to nearest neighbors. This type of interaction is not easy to realize in digital implementation, which usually necessitates the direct wiring between pixels that are several pixels apart, thus drastically complicating the layout. Another example that shows the advantages of analog implementation is the orientation computation performed in Standley's sensor [25]. This computation would be difficult to implement in digital fashion since it is a fairly complex computation. A small number of circuit elements at each pixel naturally leads to less area and lower power consumption. The number of control lines and clock lines is significantly smaller than the digital implementation, which also contributes to lower power consumption.

However, the analog vision sensors are not very suitable for performing different types of processing in an iterative manner, which is required for the proposed feature detection algorithm, due to lack of a memory. However, the nature of analog processing which naturally maps the interaction between pixels into the form of the receptive field is attractive and will be considered again later in this work.

### 3.1.3 Processing circuit - digital implementation

In contrast to the analog vision sensors, most of which are dedicated for specific purposes, the digital vision sensors are intended for more general purposes. This is possible by the use of the SIMD (Single Instruction Multiple Datastream) architecture, which assigns a processor per each pixel. Researches on digital vision sensors have been conducted mainly by three different groups.

Bernard et al. have fabricated a digital vision sensor, which is probably the simplest of this kind in terms of hardware [29]. The binary image is obtained by thresholding the output voltage of a p-n junction photodiode operated in the integration mode. Each pixel has only 28 transistors for processing purposes. A set of basic Boolean operations is defined for three memories. Despite the use of a slightly outdated $2 \mu \mathrm{~m}$ CMOS technology, the fabricated sensor contains a relatively large number of pixels, $65 \times 76$ pixels, each measuring $100 \mu \mathrm{~m} \times 80 \mu \mathrm{~m}$. The maximum operational clock frequency was 20 MHz . Several operations such as erosion, edge detection were implemented as a sequence of basic Boolean operations. The execution time was in the order of $\mu \mathrm{sec}$. More complex operations such as thinning resulted in the execution time in the order of msec. The relatively large execution time is required because a large number of operations, each of which is very simple but functionally limited, has to be performed. This example clearly shows the trade-off between the simplicity of the operation at each pixel and the execution time. Simplicity leads to increased pixel density and higher speed per operation but needs a lot of operations to perform a specific task. On the other hand, generality leads to reduced number of operation cycles but requires more pixel area, resulting in lower density.

Another type of digital vision sensors, which is based on the NSIP (Near Sensor Image Processing) concept, has been designed by Eklund et al. [30]. The sensor contains $32 \times 32$ pixels in an area of $25 \mathrm{~mm}^{2}$, each pixel measuring $118 \mu \mathrm{~m} \times 118 \mu \mathrm{~m}$ and containing 97 transistors. No explicit ALU (Arithmetical and Logical Unit) is implemented at each pixel. Instead, Boolean operations can be performed by properly combining NAND operation, which is implemented as a precharged logic. For phototransduction, the integration type photosensor was employed again but in a slightly different manner. Gray level information is obtained by counting the number of readout until the output voltage of the photosensor, which keeps
decreasing, reaches a certain threshold value, instead of measuring the voltage after a certain integration time. Six memories are used as a counter to store this gray level information. Furthermore, the global operation, such as counting the number of pixels meeting a specific requirement, can be executed. The sensor realizes a high spatial density while achieving basic neighborhood processing as well as global processing.

The third type of digital vision sensors was reported by Ishikawa's group at the University of Tokyo [31]-[33]. The latest version of the vision sensor was realized in an array of $16 \times 16$ pixels in a $2.4 \mathrm{~mm} \times 2.4 \mathrm{~mm}$ chip area using CMOS $0.35 \mu \mathrm{~m}$ technology. Each pixel contains 510 transistors, which is significantly larger than that implemented in the sensors mentioned above. As this number suggests, this sensor has the most general processing capability. There are 24 memories included in each pixel. The sensor performs A/D conversion by using the same procedure as is employed in the NSIP chip. An 8 bit arithmetic operation is possible by bit serial processing. The communication between 4 -neighbors is realized by memory mapped I/O. One operation cycle is divided into four subcycles to save the number of control lines, although this time-division quadruples the processing time for one instruction. The fabricated sensor was able to perform fairly complex operations such as line thinning in the order of microseconds. It was also used for the purpose of high speed target tracking.

As the above three examples indicate, digital implementation favors phototransduction in the integration mode. It is partly because there have not been so much motivation to mimic biological range compression which is obtained by operating transistors in the subthreshold region, and partly because that inherent necessity of clocking in the digital implementation facilitates the phototransduction by integration. The only problem is that phototransduction takes time, in the order of msec in the normal lighting conditions. This seems to contradict with the demand for high-speed operation. The digital processing time turns out to be a mere fraction of the entire processing time that includes phototransduction in addition to the
processing time.
The other key point of the digital vision sensors is how to find the optimal balance between processing generality and high spatial density. Although it depends on the target application, the trend seems to support having more generality at each pixel without too much degrading the spatial resolution, which is boosted by the continuing drive for smaller feature size. For examples, a sensor which has practically useful number of pixels, say, $64 \times 64$, would be possible to implement on chip based on Ishikawa's architecture.

However, the present configuration of the vision sensors is not very suitable for the implementation of the feature detection algorithm for the following reasons. First, the interaction between pixels is limited to 4 -neighbors, meaning that the information on diagonal pixels has to be retrieved in two steps. For the purpose of feature detection, which needs diagonal information as well as horizontal and vertical information, communication between diagonal pixels is crucial. Second, even within the interaction between 4-neighbors, the vision sensors cannot perform the logical operation using all these values at once. For example, Ishikawa's vision sensor can take only two inputs for one Boolean operation, meaning that at least four operations are required for the operation based on its 4 -neighbors. This is because the digital vision sensors are built on the framework of standard Boolean operations, which is purely arithmetic and does not have any concept of describing the interaction between pixels.

### 3.1.4 Processing circuit - CNN implementation

The cellular neural network (CNN) is a powerful framework for systematic formulation of low-level image processing functions [34] as explained in the previous chapter. Several sensors have been fabricated based on this paradigm [35]-[37]. The CNN sensor designed by Domínguez-Castro [37] seems to be the most sophisticated one and hence explained below as a representative example of this type of implementation. The CNN computation is realized in an
array form of nonlinear dynamic analog processing units, arranged on regular grids where direct interactions among cells are usually limited to $3 \times 3$ neighbors. In addition to the internal analog operations in continuous time, an iterative operation based on the external clock is possible. This implementation is based on the CNN universal machine (CNN-UM) [38], which is an extended concept of the standard CNN [34], and can be considered a hybrid of the analog vision sensor and the digital vision sensor.

The sensor is intended for processing a binary image. Phototransduction was carried out using a phototransistor arranged in Darlington configuration: the output of the phototransistor is connected to the base of the vertical pnp transistor for further amplification. The output current was thresholded to produce a binary image. The threshold was automatically adjusted to the average of the photocurrent on the entire array of pixels to avoid an image of completely black or completely white pixels.

The processing function is determined by 19 analog coefficients whose values are programmable with 7-bit accuracy. These 19 analog coefficients completely determine the evolution pattern of the pixel: nine coefficients are necessary for the feedback template; the other nine for the control template; and the remaining one for the offset term (see eqn. (2.24)). The sensor can store a sequence of operations in the memory. The sensor also has some digital flavor by incorporating four memories per pixel and allowing Boolean operations between these memories. This sensor is very powerful since operations based on templates known for different image processing applications can be executed on-chip. However, the processing speed is not very fast: each operation takes about $2 \mu \mathrm{~s}$. This amount of time is necessary for the network to settle into the steady state.

### 3.1.5 The selected architecture

The three types of implementation schemes for on-chip processing, which are closely
related to the method of phototransduction, have been reviewed. The analog vision sensor is able to define cell interaction by either resistive grid or programmable kernels, but is not very suitable for iterative processing. The digital vision sensor, on the other hand, is suitable for iterative processing. However, the computation using 4-neighbors or 8 -neighbors in one step is not easy to implement due to its gate-level based Boolean logic. Compared to these two schemes, the sensor based on the CNN-UM framework works in a hybrid way: it specifies the pixel interaction by the programmable kernel; the operation can be executed in an iterative fashion for multiple memories. Therefore, it is an appropriate architecture for the implementation of the feature detection algorithm. It is a natural consequence since the proposed feature detection algorithm, which is based on the template matching scheme, is considered the special case of the cellular neural network.

The implementation, however, can be drastically simplified by getting rid of unnecessary elements, which includes analog multipliers (synapse), circuit element associated with control templates, and capacitors. The resultant processing circuit turns out to be a programmable current distribution circuit and thresholding circuit with some memories for storing images. Due to this simplification, the circuit dynamics is completely changed from the operation governed by $R C$ time constants to the operation governed by constant current sources. Therefore, much faster operation is expected.

As for the selection of the phototransduction method, the phototransistor operated in the steady-state mode (without integration) seems to be the best choice. For the present objective of capturing an image in a binary level, the simple thresholding scheme is just enough. Range compression capability found in most analog vision sensors is not necessary. The choice of phototransistor over photodiode is due to its larger current. The increased current level should work for faster charging and discharging of the thresholding node.

In summary, the considerations based on the related work on vision sensors led us to the


Figure 3.6 Conceptual architecture of the pixel circuit. In the phototransduction stage, the phototransistor output is compared to a threshold current $\mathrm{I}_{\mathrm{th} 1}$ to produce a binary output $\mathrm{V}_{\mathrm{ph}}$, which is transferred to the memory when signal photo is set to logic High. In the processing stage, the reference current $I_{\text {ref }}$ is distributed to neighbors according to a specified kernel pattern (not shown above). The currents from neighbor pixels, in turn, are summed to generate $\mathrm{I}_{\text {sum }}$, which is then compared with $\mathrm{I}_{\mathrm{th} 2}$ to generate a binary output $\mathrm{V}_{\mathrm{o}}$ This resultant voltage is transferred and written into the memory by a twophase clock ( $\phi_{1}$ and $\phi_{2}$ ).
following architecture for the implementation of the feature detection algorithm. A phototransistor operating in the steady-state mode with a thresholding circuit is used as an input to the processing circuit, which iteratively performs neighborhood processing based on programmable kernels. In the next section, the selected pixel architecture is described in detail.

### 3.2 Pixel architecture

The objective of this section is to describe the circuit schematic of the pixel. Before going into the detail, the target specification of the feature detection sensor is described below. Based on the survey of the related researches, it would be possible for the prototype sensor to
have an array of $16 \times 16$ pixels, each measuring $150 \mu \mathrm{~m} \times 150 \mu \mathrm{~m}$, in a chip area of $3.2 \mathrm{~mm} \times$ 3.2 mm using CMOS $0.5 \mu \mathrm{~m}$ technology. The margin between $2.4 \mathrm{~mm}(150 \mu \mathrm{~m} \times 16)$ and 3.2 mm is for peripheral circuits and for pads. If the design is successfully tested, it should be easily scaled up to an array of $64 \times 64$ pixels, each measuring $100 \mu \mathrm{~m} \times 100 \mu \mathrm{~m}$, in a chip area of $7.2 \mathrm{~mm} \times 7.2 \mathrm{~mm}$ with CMOS $0.35 \mu \mathrm{~m}$ technology. This is technically feasible and the array size of $64 \times 64$ would be practically useful for some applications. Therefore, the prototype sensor is considered a small-size model for the verification of the circuit design stated below.

### 3.2.1 Overall architecture

The conceptual architecture of the pixel circuit is shown in Figure 3.6. It consists of a phototransduction circuit (shown on the right-hand side) and a processing circuit (shown on the left-hand side). As explained before, the phototransistor is selected as a transducer element. The output current of the phototransistor $\mathrm{I}_{\mathrm{ph}}$ is compared to a threshold current $\mathrm{I}_{\mathrm{th} 1}$ to generate a binary output $\mathrm{V}_{\mathrm{ph}}$, which is always available as a steady-state output.

The processing circuit is constructed in a mixed-mode fashion in the sense that the internal operation is performed in the analog domain while the control of the operation is performed in the digital domain. The circuit consists of the memory and the convolution unit with programmable kernels. The basic function is to distribute a current to neighborhood pixels if the memory content is logic High. The distribution pattern, which corresponds to the convolution kernel, is specified by the setting of switches along the connection path to neighbors, which are not shown in the figure. The currents from neighboring pixels, in turn, are summed to generate $\mathrm{I}_{\text {sum }}$ and compared with $\mathrm{I}_{\text {th2 }}$ to generate a binary output $\mathrm{V}_{\mathrm{o}}$. This binary voltage is then transferred through a parasitic capacitor to the memory by the use of a nonoverlapping two-phase clock ( $\phi_{1}$ and $\phi_{2}$ ).


Figure 3.7 Schematic of image digitization circuit with an adjustable threshold current. The transistor sizes $(W / L)$ are as follows: $\mathrm{M}_{1}, \mathrm{M}_{2 \mathrm{a}}, \mathrm{M}_{2 \mathrm{~b}}, \mathrm{M}_{2 \mathrm{c}}, \mathrm{M}_{2 \mathrm{~d}}, \mathrm{M}_{6}, \mathrm{M}_{7}: 1.5 / 2.7 ; \mathrm{M}_{3 \mathrm{a}}: 9 / 0.9 ; \mathrm{M}_{3 \mathrm{~b}}: 30 / 0.9 ; \mathrm{M}_{3 \mathrm{c}}$ : 90/0.9; $\mathrm{M}_{3 \mathrm{~d}}: 300 / 0.9 ; \mathrm{M}_{4}, \mathrm{M}_{5}: 1.5 / 4.2$.

A standard operation goes as follows. Signal photo is set to logic High to store the result of phototransduction $\mathrm{V}_{\mathrm{ph}}$ into the memory. Then signal photo is set to logic Low to start a sequence of operation. Different kernels and different threshold values are specified at each operation to perform various types of processing. In the following, the phototransduction circuit and the processing circuit are explained in detail.

### 3.2.2 Phototransistor and thresholding circuit

The only circuit that is necessary for phototransduction is the threshold current generator. For the design of the circuit, it is necessary to estimate the level of a photogenerated current. The level of the phototransistor current is estimated before in the order of 48 pA under the following conditions: $A=5 \mu \mathrm{~m} \times 5 \mu \mathrm{~m} ; C_{d}=1 \mathrm{fF} / \mu \mathrm{m}^{2} ; E=0.1 \mathrm{~W} / \mathrm{m}^{2} ; \eta=0.4$. Suppose that the area of the phototransistor is $50 \mu \mathrm{~m} \times 50 \mu \mathrm{~m}$, which results in a fill factor of 0.11 for the assumed pixel area of $150 \mu \mathrm{~m} \times 150 \mu \mathrm{~m}$, the photogenerated current increases 100 times to 4.8 nA. The threshold current level should be adjustable below this value to deal with different lighting conditions.


Figure 3.8 Relationship between the input current and the output current for different switch settings for the circuit shown in Figure 3.7. The value shown at the right end of the line indicates an output current for the input current of $2 \mu \mathrm{~A}$.

Figure 3.7 shows the schematic of the circuit. The circuit converts an input current in the order of $\mu \mathrm{A}$ to an output current in the order of pA to nA , to be used as a current for image thresholding. The attenuation factor can be controlled depending on which of the four switches $\left(\mathrm{SW}_{\mathrm{a}}, \mathrm{SW}_{\mathrm{b}}, \mathrm{SW}_{\mathrm{c}}, \mathrm{SW}_{\mathrm{d}}\right)$ is selected. When $\mathrm{SW}_{\mathrm{d}}$ is selected, the attenuation factor is largest since a NMOS current mirror is formed between $\mathrm{M}_{3 \mathrm{~d}}$ and $\mathrm{M}_{4}$, producing a largest $W / L$ ratio of $(300 / 0.9)$ to (1.5/4.2). The simulation result shown in Figure 3.8 demonstrates the relationship between the input current and the output current for different switch settings. The current in the order of pA to nA is obtained from an input current of $\mu \mathrm{A}$ order, which is easy to generate on chip.


Figure 3.9 Schematic of the processing circuit.

### 3.2.3 Processing circuit

The complete schematic of the processing circuit is shown in Figure 3.9, which is a detailed representation of the conceptual architecture shown in Figure 3.1. Note that there are six memories. Four of them $\left(\mathrm{M}_{\mathrm{a}}, \mathrm{M}_{\mathrm{b}}, \mathrm{M}_{\mathrm{c}}, \mathrm{M}_{\mathrm{d}}\right)$ are used to store the image in four orientation planes while the other two memories are used as working memories. These memories are specified by a set of signals (a1, b1, c1, d1, x1, y1). Corresponding to these memories, there are six reference current sources, whose amount is determined by the bias voltage $\mathrm{V}_{\text {ref }}$. If at least one of these signals is logic High, and the corresponding memory content is logic High, (suppose RE is also logic High), the reference current $I_{\text {ref }}$ brings down the voltage of Nc , and initiates current spreading to neighbor pixels.


Figure 3.10 Timing diagram for chip operation. The function associated with each phase is described in the box. The control signals consists of those shown in Figure 3.9 (a1, b1, ..., NW1, NW2). The twophase clock ( $\phi_{1}$ and $\phi_{2}$ ) are generated from CLK.

The current distribution pattern is determined by the convolution kernel, which is controlled by a set of switches (C, N1, N2, NE1, NE1, ..., NW1, NW2). Note that there is always current flow to itself and switch C controls the presence/absence of an additional current, indicating that the center value of the kernel is either one or two. A set of eight switches (N1, NE1, ... , NW1), each corresponding to eight neighbors, determines the presence/absence of the outward current flow from PMOS transistors: current flows out to the pixel whose corresponding switch is ON. A set of the other eight switches (N2, NE2, ... , NW2) determines the presence/absence of the inward current flow into NMOS transistors: current flows in from the pixel whose corresponding switch is ON. Only one of the two switches is allowed to be ON for each direction, i.e., both N1 and N2 cannot be logic High at the same time. Thus, the convolution kernel except the center pixel takes a value of 1,0, or -1 .

While the current is distributed to neighboring pixels, the currents generated at
neighboring pixels are accumulated on the thresholding node. The accumulated current $\mathrm{I}_{\text {sum }}$ is compared to the threshold current $\mathrm{I}_{\mathrm{th}}$, specified by signals $\mathrm{V}_{\mathrm{th} 1}$ and $\mathrm{V}_{\mathrm{th} 2}$. With these switches set to logic Low, the threshold current is $0.5 I_{r e f}$. It can be increased to $1.5 I_{\text {ref }}$ and $2.5 I_{r e f}$ by setting one or two of these switches. The result of comparison is transferred to the parasitic capacitor at the output of the inverter when clock $\phi_{1}$ is logic High for temporal information storage. The stored charge is then transferred to the memory by setting $\phi_{2}$ to logic High. Then by bringing down $\phi_{2}$ and raising $\phi_{1}$, the information is secured in the memory. This processing initiates a new operation cycle since control signals are already set to logic High or Low before $\phi_{1}$ goes High. Figure 3.10 shows the above-mentioned operational cycle. The operation can be repeated as many times as specified.

There is a difference in the configuration among six memories. For memories $M_{a}, M_{b}$, $M_{c}, M_{d}$, one control signal (a1, b1, c1, d1) is used for specifying both the source (from which memory to read) and the destination (to which memory to write), resulting in the automatic updating of the content when some memory is selected for reading. If RE is set to logic Low, the memory content is updated with the operation result (voltage $V_{o}$ in Figure 3.9) even though the present content is not used for computation. Memories $\mathrm{M}_{\mathrm{x}}$ and $\mathrm{M}_{\mathrm{y}}$ do not have the restriction of automatic updating of the memory content by having separate lines for specifying source and destination (x1, y2 for source and $\mathrm{x} 2, \mathrm{y} 2$ for destination). Memory $\mathrm{M}_{\mathrm{y}}$ has another useful feature. If this memory is specified as the destination of storing the operation result ( y 2 logic High), the automatic updating of the memory content is disabled for $\mathrm{M}_{\mathrm{a}}, \mathrm{M}_{\mathrm{b}}, \mathrm{M}_{\mathrm{c}}$, and $\mathrm{M}_{\mathrm{d}}$, making it possible to use the content of these memories for computation without affecting their content. Another unique feature of memory $\mathrm{M}_{\mathrm{y}}$ is that it can be used in an inhibitive way by asserting signal yln, which enables the sourcing of current flow into node Nc , resulting in the decrease in the current that is distributed to neighboring pixels.

| CLK |  | a1 | b1 | c1 | d1 | RE | x1 | x2 | y1 | y1n | y2 | C | N1 | N2 |  |  | E1 | E2 |  | SE2 | S1 |  |  |  |  |  |  |  |  | V t |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |  | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |  | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 |  | 0 | 0 | 0 |  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Figure 3.11 Example of an operational sequence. The sequence, which is intended for line completion in $0^{\circ}$, consists of the following operations.
(1) Line elongation in $0^{\circ}$
(2) Linestop detection
(3) Deletion of linestops

The kernels and thresholds for each of these operations are specified below.

Figure 3.11 shows an example of the operational sequence for line completion operation in $0^{\circ}$, which is explained in Section 2.6.4 (see eqns. (2.42)-(2.48)). A total of 31 bits are required for completely specifying the sensor operation. The first bit CLK refers to the control clock for generating a two-phase clock. The second bit $\mathrm{V}_{\mathrm{res}}$, which corresponds to signal photo in Figure 3.6, is set to logic High at the onset of processing for image capture and digitization, and set to logic Low during the rest of the operation. The other bits are used for memory specification (a1, b1, ..., y2), convolution kernel specification (C, N1, ... , NW2), and threshold current specification $\left(\mathrm{V}_{\text {th1 }}, \mathrm{V}_{\text {th2 }}\right)$.

To see the correspondence between the template and the switch setting, the formulae for line completion operation are listed below with slight modification:

$$
\begin{equation*}
\text { Line elongation: } M_{a}=T_{A_{3 a}^{1}, I_{3 a}^{1}}\left(M_{a}\right) \text {, } \tag{3.3}
\end{equation*}
$$

Linestop detection: $M_{y}=T_{A_{L S D 8}, I_{\text {LSD8 }}}\left(M_{a}\right)$,
Deletion of linestops: $M_{a}=T_{E, 0.5}\left(M_{a}-M_{y}\right)$
with

$$
\begin{gather*}
A_{3 a}^{1}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), \quad I_{3 a}^{1}=0.5,  \tag{3.6}\\
A_{L S D 8}=\left(\begin{array}{ccc}
-1 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & -1
\end{array}\right), \quad I_{L S D 8}=0.5 . \tag{3.7}
\end{gather*}
$$

Note that the operation is defined for memories in the above expression. It is clear from this example that the convolution kernel required for the feature detection algorithm can be implemented with these control signals and associated switch settings.

### 3.3 Summary

In this chapter, the architectural design of the feature detection sensor is described. Based on the considerations of related work on vision sensors, an analog/digital mixed-mode configuration is proposed to incorporate the advantages of both implementations

The feature detection sensor digitizes an input image first by comparing the steady-state output of a phototransistor with a threshold current. Then the neighborhood processing is carried out in a current-mode analog fashion while the external interface is made in a digital fashion. The way of interaction between adjacent pixels, which corresponds to the convolution kernel, is represented by the current distribution pattern. The result of the convolution computation is digitized with respect to the threshold current and stored into specified memories. The stored result can be used for the next processing. The above process can be performed as many times as required.

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## Chapter 4 Circuit design based on transistor mismatch analysis

The topic presented in this chapter is the design procedure of analog circuits based on transistor mismatch. First, the importance of such a design procedure, especially for currentmode processing circuits, is explained. Then transistor mismatch is analyzed in detail to represent the current variation as a function of the transistor dimension and the reference current. Then the operational speed is also represented using these design parameters. Consideration on the operating region of the transistor is also presented. Taking these relationships into account, the design parameters are determined to satisfy both the accuracy and the speed requirement while keeping the transistor in the saturation region.

### 4.1 Problem definition

One of the key aspects in the design of analog circuits is how to determine the transistor dimension in order to satisfy accuracy and speed requirements. Let us think about the case of the pixel circuit proposed in the previous chapter for the feature detection sensor. The circuit extensively employs the reference current source and the current mirror circuit. The reference current, which should be ideally the same for all the pixels, varies from pixel to pixel due to transistor mismatch. In addition, the mirrored output of the current mirror circuit cannot be the same as the input current due to the mismatch in transistor characteristics. Therefore, extensive employment of the current mirror circuit further increases the original variation of the current and may lead to the classification error at the thresholding node.

Considering a large number of pixels in future practical applications, the error rate has to be kept reasonably low. Suppose that the sensor has a pixel array of $64 \times 64(=4096$ pixels $)$,
even an error rate of $0.1 \%$ generates four pixels whose output is erroneous. It can be fatal for some applications. Since the accuracy of the current mirror circuit is higher for larger transistor dimensions, the common practice is to choose larger transistor dimensions, which sometimes turn out to be unnecessarily large. Large size transistors lead to slower operational speed. Therefore, a systematic design procedure for determining transistor dimension is required to keep the necessary accuracy without too much degrading the operational speed. Let us specify the requirement on the accuracy and the speed as follows:

- Accuracy: error rate in the order of ten thousandth ( $0.01 \%$ )
- Speed: operating frequency of 10 MHz , which is comparable to the digital vision sensors described in Section 3.1.3.

The next section analyzes the transistor mismatch behavior and derives important formulas for characterizing current variation. Based on this result, a systematic procedure for determining the design parameters is described in the subsequent section.

### 4.2 Transistor mismatch

In analog processing circuits, mismatch in transistor characteristics has a lot of impact on the circuit performance. A considerable amount of research has been carried out to characterize and model the mismatch behavior. In the following, a brief overview on the modeling of the transistor mismatch is given. Next, several important formulas will be derived which serves as a framework to construct the systematic design procedure for current-mode processing circuits.

### 4.2.1 Background

Mismatch is the process that causes time-independent random variations in physical quantities of identically designed devices. For MOS transistors, the mismatch is studied mainly for the threshold voltage and the current factor by several researchers (refer to refs. [1]-[5], for
example). The threshold voltage of the transistor is given by the following formula [1]:

$$
\begin{equation*}
V_{T}=\phi_{M S}+2 \phi_{F}+\frac{Q_{B}}{C_{o x}}-\frac{Q_{f}}{C_{o x}} \tag{4.1}
\end{equation*}
$$

where $\phi_{M S}$ is the gate-semiconductor work function difference, $\phi_{F}$ is the Fermi potential in the bulk, $Q_{B}$ is the bulk depletion charge density, $Q_{f}$ is the fixed oxide charge density, and $C_{o x}$ is the gate oxide capacitance per unit area. It is widely believed that the mismatch of the threshold voltage is mainly attributed to the mismatch of the bulk depletion charges in two devices (the third term in eqn. (4.1)). Due to the random process of the ion implantation and the drive-in diffusion process, the doping ions are distributed randomly and the depletion charge fluctuates randomly. The assumption that the depletion charge follows a Poisson distribution leads to the model where the variance of the threshold voltage difference between two transistors $\sigma_{\Delta V_{T}}$ is inversely proportional to the gate area:

$$
\begin{equation*}
\sigma_{\Delta V_{T}}^{2}=\frac{A_{\Delta V_{T}}^{2}}{W L} \tag{4.2}
\end{equation*}
$$

where $A_{\Delta V_{T}}$ is a process-dependent constant, $W$ is the transistor channel width, $L$ is the channel length. Note that this proportionality constant $A_{\Delta V_{T}}$ is usually designated as $A_{V_{T}}$ in most studies. The notation is intentionally changed to clearly differentiate the two proportionality constants, $A_{V_{T}}$ and $A_{\Delta V_{T}}$. $A_{V_{T}}$ relates the gate area to the variance of the random variable $V_{T}$, while $A_{\Delta V_{T}}$ relates the gate area to the variance of the new random variable $\Delta V_{T}$ that is defined as the difference between two values arbitrarily chosen from random variable $V_{T}$.

The mismatch of the current factor $\beta=\mu C_{o x}$, where $\mu$ is the mobility, is generated by the mismatch of the mobility and can be also modeled in the same manner:

$$
\begin{equation*}
\sigma_{\Delta \beta}^{2}=\frac{A_{\Delta \beta}^{2}}{W L} \tag{4.3}
\end{equation*}
$$

where $A_{\Delta \beta}$ is another process-dependent constant. In ref. [2], experiments were carried out using pairs of transistors having various sizes and various mutual distances. For each transistor pair that consists of two transistors of the identical dimension, $V_{T}$ and $\beta$ are extracted form the current measured in the linear region first, and then $\Delta V_{T}$ and $\Delta \beta$ are calculated as the difference of the two parameters. Based on these measurement results, the above process-dependent constants were determined.

In contrast to the above method where the current measurement was done in the linear region, Bastos et al. have devised a new parameter extraction method which enables the current measurement in the saturation region [3]. This method is advantageous especially for the characterization in sub-micron processes where the resistance of the transistor in the linear region becomes smaller, making the first method more prone to error due to the parasitic resistance in the drain path. These experimental results are summarized in Table 4.1, which is adapted from ref. [4] with the following modifications. The original notation $A_{V_{T}}$ and $A_{\beta}$ are replaced with $A_{\Delta V_{T}}$ and $A_{\Delta \beta}$, respectively; $A_{V_{T}}$ and $A_{\beta}$ are newly calculated based on the formula derived below and included in the table.

In these experiments, the influence of distance between two transistors on mismatch was also investigated: the constants that relate the distance to the variation in the threshold voltage and current factor were calculated. It was found that the influence of the distance is much smaller than the influence of the gate area. Therefore, the influence of distance is not treated in the following analysis. Another interesting finding which is commonly observed in these experiments is that almost no correlation was found between $\Delta V_{T}$ and $\Delta \beta$. This finding suggests that the gate oxide thickness is a well-controlled parameter with little variation since these two variables ( $\Delta V_{T}$ and $\Delta \beta$ ) have the term $C_{o x}$ in common.

Table 4.1 Matching proportionality constants for difference processes. (adapted from ref. [4] and modified)

| Technology | Type | $A_{\Delta V_{T}}$ <br> $[\mathrm{mV} \mu \mathrm{m}]$ | $A_{\Delta \beta}$ <br> $[\% \mu \mathrm{~m}]$ | $\left(V_{G S}-V_{T}\right)_{m}$ <br> $[\mathrm{~V}]$ | $A_{V_{T}}$ <br> $[\mathrm{mV} \mu \mathrm{m}]$ | $A_{\beta}$ <br> $[\% \mu \mathrm{~m}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2.5 \mu \mathrm{~m}[2]$ | NMOS | 30 | 2.3 | 2.6 | 21.3 | 1.6 |
|  | PMOS | 35 | 3.2 | 2.2 | 24.8 | 2.3 |
| $1.2 \mu \mathrm{~m}[3]$ | NMOS | 21 | 1.8 | 2.3 | 14.9 | 1.3 |
|  | PMOS | 25 | 4.2 | 1.2 | 17.7 | 3.0 |
| $0.7 \mu \mathrm{~m}[]$ | NMOS | 13 | 1.9 | 1.4 | 9.2 | 1.3 |
|  | PMOS | 22 | 2.8 | 1.6 | 15.6 | 2.0 |

Note that from Table 4.1 the two matching proportionality constants ( $A_{V_{T}}$ and $A_{\beta}$ ) for PMOS transistors are larger than those for NMOS transistors. This phenomenon is also found in ref. [1], which is probably due to the effect of a compensating threshold adjust implant. Note also that as the feature size decreases, the proportionality constant $A_{V_{T}}$ also decreases. Technologies with smaller feature sizes employ a thinner gate oxide layer, which increases the gate capacitance $C_{o x}$, and hence reduces the effect of the depletion charge fluctuations (refer to eqn. (4.1)). This type of dependence on the feature size is not clearly observed for $A_{\beta}$.

It should be mentioned that the effective channel width and length should be used for $W$ and $L$ in eqns. (4.2) and (4.3) instead of a drawn width and length. The effective width and length are obtained by

$$
\begin{gather*}
W_{\text {eff }}=W_{\text {drawn }}-D W,  \tag{4.4}\\
L_{e f f}=L_{\text {drawn }}-D L \tag{4.5}
\end{gather*}
$$

where $D W$ and $D L$ are the channel width and length reduction parameters. $D W$ reflects the effect of encroachment of the field oxide into the channel, while $D L$ reflects the effect of lateral diffusion of the source and drain regions. Although this shortening effect is not very significant


Figure 4.1 Current output from a set of transistors that has the common gate voltage. The transistor parameters ( $V_{T}$ and $\beta$ ) are different from transistor to transistor, leading to the variation in the output current. $\beta(i), V_{T}(i), V_{G S}(i)$ indicate variables associated with $i$-th transistor $\mathrm{M}_{\mathrm{i}}$.
when the feature size is large, the effect becomes more prominent as the feature size becomes smaller [5].

### 4.2.2 Formulation of the current variation

In current-mode processing circuits, once the circuit configuration is fixed, what needs to be determined is the following three variables: the channel width, the channel length, the amount of the reference current. The reference current, which corresponds to $I_{\text {ref }}$ in Figure 3.9 forms the basis of current processing circuits in the sense that current sources in the circuit usually produces some multiples of the reference current. In the following, the variation of the current is formulated as a function of these design variables based on the simplified models (eqns. (4.2), (4.3)). The current variation indicates the variation found in current outputs from a set of transistors whose gate voltage is common as shown in Figure 4.1. The analysis starts from the following basic relationship:

$$
\begin{equation*}
I=\frac{\beta}{2}\left(V_{G S}-V_{T}\right)^{2} \tag{4.6}
\end{equation*}
$$

where $\beta$ is the current factor and $V_{T}$ is the threshold voltage. There is variation in these variables from transistor to transistor, leading to the variation in the current output. A small
difference in $\beta$ and $V_{T}$ between two transistors generates a small difference in $I$. Let us consider a pair of transistors $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ as a representative set and define the following variables:

$$
\begin{gather*}
\Delta I=I(1)-I(2),  \tag{4.7}\\
\Delta \beta=\beta(1)-\beta(2),  \tag{4.8}\\
\Delta V_{T}=V_{T}(1)-V_{T}(2), \tag{4.9}
\end{gather*}
$$

and

$$
\begin{gather*}
\bar{I}=\frac{I(1)+I(2)}{2},  \tag{4.10}\\
\bar{\beta}=\frac{\beta(1)+\beta(2)}{2},  \tag{4.11}\\
\overline{V_{T}}=\frac{V_{T}(1)+V_{T}(2)}{2} . \tag{4.12}
\end{gather*}
$$

The small change in the current $\Delta I$ is represented as

$$
\begin{equation*}
\Delta I=\frac{\partial I}{\partial \beta} \Delta \beta+\frac{\partial I}{\partial V_{T}} \Delta V_{T} . \tag{4.13}
\end{equation*}
$$

By calculating each derivative, we get

$$
\begin{equation*}
\Delta I=\frac{1}{2}\left(V_{G S}-V_{T}\right)^{2} \Delta \beta-\beta\left(V_{G S}-V_{T}\right) \Delta V_{T} . \tag{4.14}
\end{equation*}
$$

The following equation, which is a modified version of eqn. (4.6), is valid if higher order terms are neglected:

$$
\begin{equation*}
\bar{I}=\frac{\bar{\beta}}{2}\left(V_{G S}-\overline{V_{T}}\right)^{2} . \tag{4.15}
\end{equation*}
$$

By dividing eqn. (4.14) by (4.15), the representation for the relative change is expressed as follows:

$$
\begin{equation*}
\frac{\Delta I}{\bar{I}}=\frac{\Delta \beta}{\bar{\beta}}-\frac{2 \Delta V_{T}}{V_{G S}-\overline{V_{T}}} . \tag{4.16}
\end{equation*}
$$

Eqn. (4.16) is valid only for a pair of transistors and needs to be extended to explain the overall statistics on the current variation for a large number of transistors. This is possible by considering each variable in eqn. (4.16) as a random variable. Since it is experimentally confirmed that there is almost no correlation between $\beta$ and $V_{T}$ [2][3], hence no correlation between $\Delta \beta$ and $\Delta V_{T}$, the relative variation of $\Delta I$ is represented as the summation of the relative variation of $\Delta \beta$ and $\Delta V_{T}$ :

$$
\begin{equation*}
\left(\frac{\sigma_{\Delta I}}{\bar{I}}\right)^{2}=\left(\frac{\sigma_{\Delta \beta}}{\bar{\beta}}\right)^{2}+\left(\frac{2 \sigma_{\Delta V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2} \tag{4.17}
\end{equation*}
$$

where $\bar{I}, \bar{\beta}, \overline{V_{T}}$ represents the mean of each variable for an entire set of transistors. In general, for any random variable $P$, the variance of the difference of any two, which is defined as $\Delta \square P=P_{1}-P_{2}$, is represented as

$$
\begin{equation*}
\sigma_{\Delta P}^{2}=\sigma_{P}^{2}+\sigma_{P}^{2}=2 \sigma_{P}^{2} . \tag{4.18}
\end{equation*}
$$

Applying this identity for variables which appear in eqn. (4.17) yields

$$
\begin{equation*}
\left(\frac{\sigma_{I}}{\bar{I}}\right)^{2}=\left(\frac{\sigma_{\beta}}{\bar{\beta}}\right)^{2}+\left(\frac{2 \sigma_{V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2} \tag{4.19}
\end{equation*}
$$

This expression statistically relates the variation of $\beta$ and $V_{T}$ to that of $I$ for a large number of transistors. Eqn. (4.18) can be applied to eqn. (4.2) to yield

$$
\begin{equation*}
\sigma_{V_{T}}^{2}=\frac{A_{V_{T}}^{2}}{W L} \tag{4.20}
\end{equation*}
$$

where

$$
\begin{equation*}
A_{V_{T}}=A_{\Delta V_{T}} / \sqrt{2} \tag{4.21}
\end{equation*}
$$

is the mismatch proportionality constant which relates the variance of the threshold voltage to the gate area. Likewise,

$$
\begin{equation*}
\sigma_{\beta}^{2}=\frac{A_{\beta}^{2}}{W L} \tag{4.22}
\end{equation*}
$$

where

$$
\begin{equation*}
A_{\beta}=A_{\Delta \beta} / \sqrt{2} . \tag{4.23}
\end{equation*}
$$

Substituting eqns. (4.20) and (4.22) into (4.19) yields

$$
\begin{equation*}
\left(\frac{\sigma_{I}}{\bar{I}}\right)^{2}=\frac{1}{W L}\left(A_{\beta}^{2}+\left(\frac{2 A_{V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2}\right) . \tag{4.24}
\end{equation*}
$$

Let us compare the first term with the second term to find out the relative importance of each term. These two terms become equal when the transistor is biased such that:

$$
\begin{equation*}
V_{G S}-\overline{V_{T}}=\frac{2 A_{V_{T}}}{A_{\beta}} . \tag{4.25}
\end{equation*}
$$

This voltage, which is termed as corner gate-drive voltage in ref. [4], is shown in the fifth column in Table 4.1 as $\left(V_{G S}-V_{T}\right)_{m}$. If transistors are biased below this value, the contribution of the second term in eqn. (4.24) is larger than the first term. The ratio of the second term to the first term is given as

$$
\begin{equation*}
\left(\frac{2 A_{V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2} / A_{\beta}^{2}=\left(\frac{\left(V_{G S}-\overline{V_{T}}\right)_{m}}{V_{G S}-\overline{V_{T}}}\right)^{2} . \tag{4.26}
\end{equation*}
$$

As one of the typical examples, this ratio becomes 9 for $\left(V_{G S}-V_{T}\right)_{m}$ equal to $1.5[\mathrm{~V}]$ and $\left(V_{G S}-V_{T}\right)$ equal to $0.5[\mathrm{~V}]$, which is large enough to neglect the effect of the first term in eqn. (4.24). Therefore, eqn. (4.24) can be simplified as

$$
\begin{equation*}
\left(\frac{\sigma_{I}}{\bar{I}}\right)^{2}=\frac{1}{W L}\left(\frac{2 A_{V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2} . \tag{4.27}
\end{equation*}
$$

or equivalently

$$
\begin{equation*}
\left(\frac{\sigma_{I}}{\bar{I}}\right)^{2}=\left(\frac{2 \sigma_{V_{T}}}{V_{G S}-\overline{V_{T}}}\right)^{2} . \tag{4.28}
\end{equation*}
$$

Let $s$ be the relatvie variaion of the currents obtained from a set of transistors with the gate dimension of $W / L$ to produce a mean current $\bar{I}=I_{\text {ref }}$, i.e.,

$$
\begin{equation*}
s^{2}\left(I_{r e f}, W, L\right)=\left(\frac{\sigma_{I}}{I_{r e f}}\right)^{2}=\left(\frac{2 \sigma_{V_{T}}}{V_{G S}\left(I_{r e f}\right)-\overline{V_{T}}}\right)^{2}=\frac{1}{W L}\left(\frac{2 A_{V_{T}}}{V_{G S}\left(I_{r e f}\right)-\overline{V_{T}}}\right)^{2} \tag{4.29}
\end{equation*}
$$

where $V_{G S}\left(I_{\text {ref }}\right)$ is the gate voltage to produce a mean current $\bar{I}=I_{\text {ref }}$.
Since from eqn. (4.15)

$$
\begin{equation*}
\frac{2 I_{r e f}}{\bar{\beta}}=\left(V_{G S}\left(I_{r e f}\right)-\overline{V_{T}}\right)^{2} \tag{4.30}
\end{equation*}
$$

the following relationship is obtained by substituting eqn. (4.30) into eqn. (4.29):

$$
\begin{equation*}
s^{2}\left(I_{r e f}, W, L\right)=\frac{4 A_{V_{T}}^{2}}{W L} \frac{\bar{\beta}}{2 I_{r e f}}=\frac{4 A_{V_{T}}^{2}}{W L} \frac{\mu C_{o x}(W / L)}{2 I_{r e f}}=\frac{2 A_{V_{T}}^{2} \mu C_{o x}}{L^{2} I_{r e f}} \tag{4.31}
\end{equation*}
$$

where the relationship $\beta=\mu C_{o x}(W / L)$ is used. The above equation represents the relative current variation as a function of the process parameters $\left(A_{V_{T}}, \mu C_{o x}\right)$ and design parameters ( $L, I_{\text {ref }}$ ). Since circuit designers do not have any control over the process parameters, one can only adjust the design parameters to meet some specifications.

It is interesting to note that the relative current variation is inversely proportional to $L^{2}$ and is independent of $W$. This is intuitively explained as follows. Suppose that $W$ becomes larger for a fixed value of $L$. As a result, the amount of $V_{G S}-\overline{V_{T}}$ to produce a current $I_{r e f}$ becomes smaller, resulting in a larger current variation. The other consequence is the increase in the gate area, which results in a smaller current variation. These two effects cancel each other. On the other hand, suppose that $L$ becomes smaller for a fixed value of $W$. As a result,
the amount of $V_{G S}-\overline{V_{T}}$ to accommodate a current $I_{\text {ref }}$ becomes smaller and the gate area also becomes smaller, both of which work to increase the current variation. This is why the current variation is inversely proportional to the square of the channel length. To explicitly express the above dependence, $s$ is expressed as $s\left(I_{\text {ref }}, L\right)$ instead of $s\left(I_{\text {ref }}, W, L\right)$ hereafter. The current variation $\sigma_{I}$ in eqn. (4.29) can be also explicitly expressed as a function of $I_{\text {ref }}$ and $L$ :

$$
\begin{equation*}
\sigma_{I}^{2}\left(I_{r e f}, L\right)=s^{2}\left(I_{r e f}, L\right) I_{r e f}^{2} . \tag{4.32}
\end{equation*}
$$

Using eqn. (4.31), let us consider the relative current variation for the case where the current level is $m I_{\text {ref }}$, and the channel length is $b L$. The relative current variation decreases by a factor of $m b^{2}$ since

$$
\begin{equation*}
s^{2}\left(m I_{r e f}, b L\right)=\frac{2 A_{V_{T}}^{2} \mu C_{o x}}{(b L)^{2} m I_{\text {ref }}}=\frac{1}{m b^{2}} \frac{2 A_{V_{T}}^{2} \mu C_{o x}}{L^{2} I_{r e f}}=\frac{1}{m b^{2}} s^{2}\left(I_{r e f}, L\right) . \tag{4.33}
\end{equation*}
$$

The variance of the current increases by a factor of $\mathrm{m} / \mathrm{b}^{2}$ since

$$
\begin{align*}
\sigma_{I}^{2}\left(m I_{\text {ref }}, b L\right) & =s^{2}\left(m I_{\text {ref }}, b L\right)\left(m I_{\text {ref }}\right)^{2}=\frac{1}{m b^{2}} s^{2}\left(I_{\text {ref }}, L\right)\left(m I_{\text {ref }}\right)^{2}  \tag{4.34}\\
& =\frac{m}{b^{2}} s^{2}\left(I_{\text {ref }}, L\right) I_{\text {ref }}^{2}=\frac{m}{b^{2}} \sigma_{I}^{2}\left(I_{\text {ref }}, L\right) .
\end{align*}
$$

Hereafter, $s^{2}\left(I_{\text {ref }}, L\right)$ will be simply designated as $s^{2}$ for simplicity since this term is frequently used in the derivations described below.

### 4.2.3 Error introduced by current mirror

A formula for the current variation as a function of the design parameters is derived in the previous subsection. The formula is extended to incorporate the effect of $V_{G S}$ variation to represent the error, i.e., the increase of the variance of the current, by the current mirror operation. Figure 4.2 shows the effect of $V_{G S}$ variation on the current variation. The difference between Figure 4.2 and Figure 4.1 lies in the treatment of $V_{G S}$, which is not a fixed parameter


Figure 4.2 Variation of the output currents from a set of transistors with different gate voltages and process parameters. Note that Figure 4.1 is a special case where the gate voltage is common for all transistors.
any more. Eqn. (4.13) is now expressed as follows:

$$
\begin{equation*}
\Delta I=\frac{\partial I}{\partial \beta} \Delta \beta+\frac{\partial I}{\partial V_{T}} \Delta V_{T}+\frac{\partial I}{\partial V_{G S}} \Delta V_{G S} . \tag{4.35}
\end{equation*}
$$

Therefore, we get

$$
\begin{equation*}
\frac{\Delta I}{\bar{I}}=\frac{\Delta \beta}{\bar{\beta}}-\frac{2 \Delta V_{T}}{\overline{V_{G S}}-\overline{V_{T}}}+\frac{2 \Delta V_{G S}}{\overline{V_{G S}}-\overline{V_{T}}} . \tag{4.36}
\end{equation*}
$$

where $\overline{V_{G S}}$ is the mean of $V_{G S}$ for the entire set of transistors. Again by discarding the effect of the current factor $\beta$, the relative current variation is simplified as

$$
\begin{equation*}
\frac{\Delta I}{\bar{I}}=-\frac{2 \Delta V_{T}}{\overline{V_{G S}}-\overline{V_{T}}}+\frac{2 \Delta V_{G S}}{\overline{V_{G S}}-\overline{V_{T}}} . \tag{4.37}
\end{equation*}
$$

The above equation can be statistically expressed as

$$
\begin{equation*}
\left(\frac{\sigma_{I}}{\bar{I}}\right)^{2}=\left(\frac{2 \sigma_{V_{T}}}{\overline{V_{G S}}-\overline{V_{T}}}\right)^{2}+\left(\frac{2 \sigma_{V_{G S}}}{\overline{V_{G S}}-\overline{V_{T}}}\right)^{2} . \tag{4.38}
\end{equation*}
$$

The variation in $V_{T}$ and $V_{G S}$ equally contributes to the variation in $I$ as schematically shown at


Figure 4.3 Schematic showing the increase of the current variation by current mirror operation.
the bottom of Figure 4.2. Now let us formulate the error introduced by the current mirror circuit based on the schematic shown in Figure 4.3. Eqn. (4.38) is modified to represent the relative variation of the current $I_{2}$ as

$$
\begin{equation*}
\left(\frac{\sigma_{I_{2}}}{\overline{I_{2}}}\right)^{2}=\left(\frac{2 \sigma_{V_{T 2}}}{\overline{V_{G S}}-\overline{V_{T 2}}}\right)^{2}+\left(\frac{2 \sigma_{V_{G S}}}{\overline{V_{G S}}-\overline{V_{T 2}}}\right)^{2} . \tag{4.39}
\end{equation*}
$$

In order to express the second term in the right-hand side as a function of the variation in the input current $I_{l}$ and the variation in the threshold voltage $V_{T I}$, eqn. (4.37) is expressed in a slightly different way:

$$
\begin{equation*}
\frac{2 \Delta V_{G S}}{\overline{V_{G S}}-\overline{V_{T 1}}}=\frac{\Delta I}{\bar{I}}+\frac{2 \Delta V_{T 1}}{\overline{V_{G S}}-\overline{V_{T 1}}} . \tag{4.40}
\end{equation*}
$$

Eqn. (4.40) clearly explains that the variation in $V_{G S}$ is generated by the variation in $I$ and $V_{T}$, which can be again statistically represented as

$$
\begin{equation*}
\left(\frac{2 \sigma_{V_{G S}}}{\overline{V_{G S}}-\overline{V_{T 1}}}\right)=\left(\frac{\sigma_{I_{1}}}{\overline{I_{1}}}\right)^{2}+\left(\frac{2 \sigma_{V_{T 1}}}{\overline{V_{G S}}-\overline{V_{T 1}}}\right)^{2} . \tag{4.41}
\end{equation*}
$$

By substituting eqn. (4.41) to (4.39), we get

$$
\begin{equation*}
\left(\frac{\sigma_{I_{2}}}{\overline{I_{2}}}\right)^{2}=\left(\frac{\sigma_{I_{1}}}{\overline{I_{1}}}\right)^{2}+\left(\frac{2 \sigma_{V_{T 1}}}{\overline{V_{G S}}-\overline{V_{T 1}}}\right)^{2}+\left(\frac{2 \sigma_{V_{T 2}}}{\overline{V_{G S}}-\overline{V_{T 2}}}\right)^{2} . \tag{4.42}
\end{equation*}
$$

Suppose that the mean input current for transistor $\mathrm{M}_{1}$ and the output current from transistor $\mathrm{M}_{2}$ are

$$
\begin{equation*}
\overline{I_{1}}=\frac{a_{1}}{b_{1}} I_{r e f}=m_{1} I_{r e f} \tag{4.43}
\end{equation*}
$$

and

$$
\begin{equation*}
\overline{I_{2}}=\frac{a_{2}}{b_{2}} I_{r e f}=m_{2} I_{r e f}, \tag{4.44}
\end{equation*}
$$

respectively, with the corresponding transistor dimension of $a_{1} W / b_{1} L$ for transistor $\mathrm{M}_{1}$ and $a_{2} W / b_{2} L$ for transistor $\mathrm{M}_{2}$. In the above expressions, $m_{1}=a_{1} / b_{1}$ and $m_{2}=a_{2} / b_{2}$ represent the current amplification coefficient. Substituting eqns. (4.43) and (4.44) into (4.42), we get

$$
\begin{equation*}
\left(\frac{\sigma_{I_{2}}}{m_{2} I_{r e f}}\right)^{2}=\left(\frac{\sigma_{I_{1}}}{m_{1} I_{\text {ref }}}\right)^{2}+\frac{1}{m_{1} b_{1}^{2}} s^{2}+\frac{1}{m_{2} b_{2}^{2}} s^{2} \tag{4.45}
\end{equation*}
$$

where eqns. (4.29) and (4.33) are used to obtain the second and the third term in the right-hand side. It can be further modified as

$$
\begin{align*}
\sigma_{I_{2}}^{2} & =\left(\frac{m_{2}}{m_{1}} \sigma_{I_{1}}\right)^{2}+\frac{m_{2}^{2}}{m_{1} b_{1}^{2}} s^{2} I_{\text {ref }}^{2}+\frac{m_{2}^{2}}{m_{2} b_{2}^{2}} s^{2} I_{\text {ref }}^{2}  \tag{4.46}\\
& =g_{2}^{2} \sigma_{I_{1}}^{2}+g_{2}^{2} \frac{m_{1}}{b_{1}^{2}} s^{2} I_{\text {ref }}^{2}+\frac{m_{2}}{b_{2}^{2}} s^{2} I_{\text {ref }}^{2}
\end{align*}
$$

where $g_{2}$ represents the current gain defined as $m_{2} / m_{1}=I_{2} / I_{1}$. The first term and the second term represent the contribution of the variance of the input current and the variance of the threshold
voltage of transistor $\mathrm{M}_{1}$, respectively. Note that these contributions are magnified by the square of the current gain. The third term represents the contribution of the variance of the threshold voltage of transistor $\mathrm{M}_{2}$. Note that in the special case where transistor $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ have the same gate dimension, i.e., $a_{1}=a_{2}=a, b_{1}=b_{2}=b$, and subsequently $m_{1}=m_{2}=m=a / b, g_{2}=1$, eqn. (4.46) is simplified as

$$
\begin{equation*}
\sigma_{I_{2}}^{2}=\sigma_{I_{1}}^{2}+2 \frac{m}{b^{2}} s^{2} I_{r e f}^{2} . \tag{4.47}
\end{equation*}
$$

### 4.2.4 Error introduced by current summation and subtraction

In addition to the current mirroring operation, current summation and subtraction are extensively employed in current-mode processing circuits. Therefore, the effect of these operations on the variation should be formulated. In general, for a random variable $X$, which is defined as the summation or subtraction of $X_{1}, X_{2}, \ldots, X_{N}$, i.e.,

$$
\begin{equation*}
X=p_{1} X_{1}+p_{2} X_{2}+\ldots+p_{N} X_{N} \tag{4.48}
\end{equation*}
$$

where $p_{i}$ is either +1 or -1 , the variance of $X$ is expressed as

$$
\begin{equation*}
\sigma_{X}^{2}=\sum_{i=1}^{N} \sigma_{X_{i}}^{2}+2 \sum_{i=1}^{N} \sum_{j \neq i}^{N} p_{i} p_{j} \sigma_{X_{i} X_{j}}^{2} \tag{4.49}
\end{equation*}
$$

where $\sigma_{X_{i} X_{j}}^{2}$ is the covariance between two random variables $X_{i}$ and $X_{j}$. Based on this formula, two cases, a case where no correlation exists between currents and the other case where correlation exists between currents, are analyzed below.

Figure 4.4 shows the example of the first case where no correlation exists between any pair of current. The output current $I_{3}$ is obtained as the summation of the three currents, $I_{3 a}, I_{3 b}$, and $I_{3 c}$. These three current outputs are generated from three sets of current mirror circuit. Each of the input currents for these current mirror circuits, $I_{1 \mathrm{a}}, I_{1 \mathrm{~b}}$, and $I_{1 \mathrm{c}}$, has the variance of $s^{2} I_{r e f}^{2}$. The variance of $I_{l a}, I_{l b}$, and $I_{l c}$ is calculated as


Figure 4.4 Current summation circuit. Note that there is no correlation between three currents $I_{3 a}, I_{3 b}$, and $I_{3 c}$. Expression in a parenthesis represents the variance of the current shown above the parenthesis.

$$
\begin{equation*}
\sigma_{I_{3 a}}^{2}=\sigma_{I_{3 b}}^{2}=\sigma_{I_{3 c}}^{2}=s^{2} I_{r e f}^{2}+2 s^{2} I_{r e f}^{2}=3 s^{2} I_{r e f}^{2} \tag{4.50}
\end{equation*}
$$

using eqn. (4.47) with $m=b=1$. Since there is no correlation between these three current outputs, the variance of the output current $I_{3}$ is obtained by simply calculating the sum of the variance of $I_{3 a}, I_{3 b}$, and $I_{3 c}$ :

$$
\begin{equation*}
\sigma_{I_{3}}^{2}=\sigma_{I_{3 a}}^{2}+\sigma_{I_{3 b}}^{2}+\sigma_{I_{3 c}}^{2}=3 \times 3 s^{2} I_{r e f}^{2}=9 s^{2} I_{r e f}^{2} . \tag{4.51}
\end{equation*}
$$

It should be noted that the relative variance decreases by a factor of three after current summation since the quantity

$$
\begin{equation*}
\left(\frac{\sigma_{I_{3}}}{3 I_{r e f}}\right)^{2}=\frac{9 s^{2} I_{r e f}^{2}}{9 I_{r e f}^{2}}=s^{2} \tag{4.52}
\end{equation*}
$$

is one third the amount represented in eqn. (4.50).
Figure 4.5 demonstrates the example of the second case where correlation exists between currents. Since three transistors $\mathrm{M}_{3 \mathrm{a}}, \mathrm{M}_{3 \mathrm{~b}}$, and $\mathrm{M}_{3 \mathrm{c}}$ have a common gate voltage, which has some variation due to the input current variation, correlation exists between these three current outputs. To calculate the total variance, we first take a brute-force approach where the


Figure 4.5 The other type of current summation circuit. Note that correlation exists between three current outputs $I_{3 a}, I_{3 b}$, and $I_{3 c}$.
contribution is decomposed into a set of variables, which are independent with each other. From eqn. (4.37), we get

$$
\begin{align*}
& \frac{\Delta I_{3 a}}{I_{r e f}}=\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 a}}{\overline{V_{G S 3}}-\overline{V_{T 3}}},  \tag{4.53}\\
& \frac{\Delta I_{3 b}}{I_{r e f}}=\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 b}}{\overline{V_{G S 3}}-\overline{V_{T 3}}},  \tag{4.54}\\
& \frac{\Delta I_{3 c}}{I_{r e f}}=\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 c}}{\overline{V_{G S 3}}-\overline{V_{T 3}}} . \tag{4.55}
\end{align*}
$$

Therefore,

$$
\begin{align*}
\frac{\Delta I_{3}}{I_{r e f}} & =\frac{\Delta I_{3 a}}{I_{\text {ref }}}+\frac{\Delta I_{3 b}}{I_{r e f}}+\frac{\Delta I_{3 c}}{I_{\text {ref }}} \\
& =\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 a}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}+\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{\overline{V_{T 2}}}}-\frac{2 \Delta V_{T 3 b}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}+\frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 c}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}}  \tag{4.56}\\
& =3 \frac{2 \Delta V_{G S 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}-\frac{2 \Delta V_{T 3 a}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}-\frac{2 \Delta V_{T 3 b}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}-\frac{2 \Delta V_{T 3 c}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}} .
\end{align*}
$$

Substituting eqn. (4.37) into (4.56) yields

$$
\begin{equation*}
\frac{\Delta I_{3}}{I_{r e f}}=3\left(\frac{\Delta I_{1}}{\overline{I_{1}}}+\frac{2 \Delta V_{T 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}\right)-\frac{2 \Delta V_{T 3 a}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}-\frac{2 \Delta V_{T 3 b}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}-\frac{2 \Delta V_{T 3 c}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}}, \tag{4.57}
\end{equation*}
$$

which further leads to

$$
\begin{equation*}
\frac{\Delta I_{3}}{I_{r e f}}=3\left(-\frac{2 \Delta V_{T 1}}{\overline{V_{G S 1}}-\overline{V_{T 1}}}+\frac{2 \Delta V_{T 2}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}\right)-\frac{2 \Delta V_{T 3 a}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}-\frac{2 \Delta V_{T 3 b}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}-\frac{2 \Delta V_{T 3 c}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}} \tag{4.58}
\end{equation*}
$$

by using eqn. (4.37) with $\Delta V_{G S}=0$ since the gate voltage of transistor $\mathrm{M}_{1}$ is fixed. Now the above equation represents the current variation as a total contribution of the variation of each threshold voltage with different weigths, i.e., heigher contribution from transistors $M_{1}$ and $M_{2}$, and lower contribution from transistors $M_{3 a}, M_{3 b}$, and $M_{3 c}$. Since there is no correlation between these random variables, the variation in $\Delta I_{3} / I_{\text {ref }}$ is represented as the summation of the variance of each variable:

$$
\begin{align*}
\left(\frac{\sigma_{\Delta I_{3}}}{I_{r e f}}\right)^{2}= & 9\left(\frac{2 \sigma_{\Delta V_{T 1}}}{\overline{V_{G S 1}}-\overline{V_{T 1}}}\right)^{2}+9\left(\frac{2 \sigma_{\Delta V_{T 2}}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}\right)^{2}  \tag{4.59}\\
& +\left(\frac{2 \sigma_{\Delta V_{T 3 a}}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}\right)^{2}+\left(\frac{2 \sigma_{\Delta V_{T 3 b}}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}\right)^{2}+\left(\frac{2 \sigma_{\Delta V_{T 3 c}}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}}\right)^{2} .
\end{align*}
$$

Using eqn. (4.18), we get

$$
\begin{align*}
\left(\frac{\sigma_{I_{3}}}{I_{\text {ref }}}\right)^{2} & =9\left(\frac{2 \sigma_{V_{T 1}}}{\overline{V_{G S 1}}-\overline{V_{T 1}}}\right)^{2}+9\left(\frac{2 \sigma_{V_{T 2}}}{\overline{V_{G S 2}}-\overline{V_{T 2}}}\right)^{2} \\
& +\left(\frac{2 \sigma_{V_{T 3 a}}}{\overline{V_{G S 3}}-\overline{V_{T 3 a}}}\right)^{2}+\left(\frac{2 \sigma_{V_{T 3 b}}}{\overline{V_{G S 3}}-\overline{V_{T 3 b}}}\right)^{2}+\left(\frac{2 \sigma_{V_{T 3 c}}}{\overline{V_{G S 3}}-\overline{V_{T 3 c}}}\right)^{2}  \tag{4.60}\\
& =9 s^{2}+9 s^{2}+s^{2}+s^{2}+s^{2} \\
& =21 s^{2}
\end{align*}
$$

or equivalently

$$
\begin{equation*}
\sigma_{I_{3}}^{2}=21 s^{2} I_{r e f}^{2} . \tag{4.61}
\end{equation*}
$$

The variance is more than twice that described in eqn. (4.51) due to the presence of the


Figure 4.6 Generalized current summation circuit. Note that correlation exists between $I_{21}, I_{22}, \ldots, I_{2 N}$.
correlation. The approach to derive eqn. (4.60) is based on the decomposition of the total variation to the summation of several variables which are independent with each other. This was done by tracing back the circuit to the point where no correlation exists between variables.

The above example can be extended to more general cases using eqn. (4.46). Consider the example shown in Figure 4.6 where transistors $\mathrm{M}_{1}, \mathrm{M}_{2}, \ldots, \mathrm{M}_{\mathrm{N}}$ have the dimension of $\left(a_{21} W / b_{21} L\right),\left(a_{22} W / b_{22} L\right), \ldots,\left(a_{2 N} W / b_{2 N} L\right)$, respectively. Using eqn. (4.46), the variation in the current from each of these transistors $\sigma_{I_{2 i}}^{2}(i=1, \cdots, N)$, is expressed as

$$
\begin{equation*}
\sigma_{I_{2 i}}^{2}=g_{2 i}^{2} \sigma_{I_{1}}^{2}+g_{2 i}^{2} \frac{m_{1}}{b_{1}^{2}} s^{2} I_{r e f}^{2}+\frac{m_{2 i}}{b_{2 i}^{2}} s^{2} I_{\text {ref }}^{2} \tag{4.62}
\end{equation*}
$$

where $m_{2 i}=I_{2 i} / I_{1}$ and $g_{2 i}=m_{2 i} / m_{1}$. Note that the first term and the second term appear for all $\sigma_{I_{2 i}}^{2}(i=1, \cdots, N)$ and hence generate correlation between these variables. Taking this effect into account,

$$
\begin{equation*}
\sigma_{I_{2}}^{2}=\left(\sum_{i=1}^{N} g_{2 i}\right)^{2} \sigma_{I_{1}}^{2}+\left(\sum_{i=1}^{N} g_{2 i}\right)^{2} \frac{m_{1}}{b_{1}^{2}} s^{2} I_{r e f}^{2}+\sum_{i=1}^{N} \frac{m_{2 i}}{b_{2 i}^{2}} s^{2} I_{r e f}^{2} . \tag{4.63}
\end{equation*}
$$

By defining the total current multiplication factor $g_{2}$ as

$$
\begin{equation*}
g_{2}=\sum_{i=1}^{N} g_{2 i}, \tag{4.64}
\end{equation*}
$$

eqn. (4.63) can be simply represented as

$$
\begin{equation*}
\sigma_{I_{2}}^{2}=g_{2}^{2} \sigma_{I_{1}}^{2}+g_{2}^{2} \frac{m_{1}}{b_{1}^{2}} s^{2} I_{r e f}^{2}+\sum_{i=1}^{N} \frac{m_{2 i}}{b_{2 i}^{2}} s^{2} I_{r e f}^{2} . \tag{4.65}
\end{equation*}
$$

A set of transistors $\mathrm{M}_{2 \mathrm{i}}$ act as a lumped one big transistor having a current gain of $g_{2}$ with respect to transistor $\mathrm{M}_{1}$, which magnifies the variance prior to $\mathrm{M}_{2 i}$, i.e., variance in the input current $I_{1}$ and the threshold voltage of transistor $\mathrm{M}_{1}$. On the other hand, the variance of the threshold voltage of each transistor $\mathrm{M}_{2 \mathrm{i}}$ is independent with each other, and hence just adds up without generating the cross term due to the correlation effect. Note that the result shown in (4.60) is immediately obtained by substituting 3 for $g_{2}, s^{2} I_{\text {ref }}^{2}$ for $\sigma_{I_{1}}^{2}$ and setting other variables to 1 .

The approach described so far to calculate the total variance is based on the decomposition of the variance of each current into the sum of the variance of the threshold voltage of each transistor which exists prior to the current summation stage. In this way, the effect of correlation can be clearly observed.

To summarize this section, the three useful formulas have been derived starting from the model that characterize transistor mismatch:
(1) The relative current variation of the transistor is a function of $L$ and $I_{r e f}$. Larger values of $L$ and $I_{r e f}$ give smaller variations (eqn. (4.31)).
(2) Current mirror operation increases variation by $2\left(m / b^{2}\right) s^{2} I_{\text {ref }}^{2}$, equally contributed from two transistors used in a pair, where $s^{2}$ is defined as the current variation due to the threshold mismatch for current $I_{\text {ref }}$ (eqn. (4.47)).
(3) The variance of the current which is represented as summation of multiple input currents is
obtained by simply adding the variance of each current. The variance becomes larger when there is correlation between these currents.

### 4.3 Systematic design procedure for current-mode processing circuits

Based on the formulas derived in the previous section, a method to systematically design a circuit is discussed in this section. To be more specific, the design parameters including the reference current, the channel width and length, are determined to satisfy accuracy and speed requirement while ensuring the operation in the correct operating region.

### 4.3.1 Background

Few papers have been published on a design procedure that takes transistor mismatch into account. In his paper dealing with transistor mismatch [1], Lakshmikumar et al. have presented a method for determining required accuracy for current sources in order to satisfy a specific yield for an 8 -bit current-steering CMOS DAC. The procedure is briefly described below as an introduction of a systematic design procedure.

The output of the current-steering type DAC is generated by the selective summation of the eight current sources whose current level is represented as $2^{i} I_{\text {unit }}(i=1, \cdots, 8)$ where $I_{u n i t}$ is a unit current amount. The selection of the current source is controlled by the input word. The authors have linked the linearity error of the DAC output to the random variation of the current, and based on this link, represented the circuit yield as a function of the matching accuracy. In other words, the required matching accuracy can be determined to satisfy a given yield requirement. The nonlinearity of the output is expressed as the following normalized form:

$$
\begin{equation*}
z=\frac{x}{x+y} \tag{4.66}
\end{equation*}
$$

where $z$ is the normalized output, and $x$ and $y$ are the output of the DAC for a given input word
and the analog complement of the output, respectively. After a series of calculations, the variance of $z$ can be expressed as

$$
\begin{equation*}
\sigma_{z}^{2}=\frac{\bar{z}(1-\bar{z})}{\bar{I}(\bar{x}+\bar{y})} \sigma^{2} \tag{4.67}
\end{equation*}
$$

where $\bar{x}, \bar{y}$, and $\bar{z}$ represent the mean of each random variable, $\bar{I}$ and $\sigma^{2}$ indicate the mean and variance of $I_{u n i t}$, respectively. The circuit yield is defined in their paper as the percentage of functional devices that have integral nonlinearity less than $1 / 2$ LSB. With this definition, a theoretical estimate of the circuit yield is obtained by multiplying the probabilities that each of the 256 outputs have less than $1 / 2$ LSB error. Assuming the normal distribution with variance given above, the yield is calculated as

$$
\begin{equation*}
G=\prod_{i=2}^{255} \frac{1}{\sqrt{2 \pi} \sigma_{z}} \int_{z_{i}-1 / 512}^{\overline{z_{i}+1 / 512}} \exp \left(-\frac{\left(z-\overline{z_{i}}\right)^{2}}{2 \sigma_{z}^{2}}\right) d z \tag{4.68}
\end{equation*}
$$

It was found from the above equation that the yield is a very sharp function of the relative current variation $\sigma / \bar{I}$, giving almost $100 \%$ yield for $\sigma / \bar{I}=0.004$ and decreasing sharply toward almost $0 \%$ for $\sigma / \bar{I}=0.008$. They actually manufactured a DAC by setting $\sigma / \bar{I}=0.045$, which corresponds to the yield of $95 \%$, and confirmed the validity of their design methodology.

What has been proposed in the above procedure is a top-down approach, which converts the specification given in terms of an error rate, or equivalently yield, to the required accuracy for current sources. This conversion procedure is represented as the solid line in Figure 4.7. However, it is not clearly specified how the current amount $I_{\text {unit }}$ and the transistor dimension are chosen to satisfy the required current accuracy. For the final determination of these design parameters, the specification on the operational speed, sometimes power consumption as well, should be taken into account. The information flow corresponding to the procedure described


Figure 4.7 Schematic explaining the design flow of the systematic circuit design.
above is schematically shown as two dashed lines in Figure 4.7. Starting from the next subsection, the design procedure incorporating all these three lines of information flow is described in detail.

### 4.3.2 Relationship between error rate and design parameters

In this subsection, the procedure to determine the maximum allowable current variation to satisfy the specification given in terms of the error rate is described. This procedure corresponds to the flow shown as the solid line in Figure 4.7, and has some similarity to Lakshmikumar's method described before in terms of analyzing the circuit behavior from the point of a random process. Let us analyze the circuit shown in Figure 4.8, which is an extracted part of the pixel circuit designed prior to that shown in Figure 3.9. Each of the three sets of current-sourcing PMOS transistors and each of the three sets of current-sinking NMOS transistors corresponds to one of the neighboring pixels.

Note that the current thresholding scheme is slightly different from the one shown in Figure 3.9. The current summation result $I_{4}$ is mirrored to $I_{5}$ to be compared to the thresholding current designated as $I_{t h}$. The operation analyzed below is the detection of $45^{\circ}$ line segments, which is described in Chapter 2 The template consists of three 1's at the right diagonal position, three -1 's at the upper triangle or the lower triangle, and three 0 's at the other side of triangle.


Figure 4.8 Current thresholding circuit as an example to demonstrate the design procedure. The thresholding scheme is slightly different from the one shown in Figure 3.9.

The threshold is 1.5 . The reason why this operation is chosen is that the error rate is expected to be higher than any other operations required for the feature detection algorithm. This is because the number of neighbors used in the computation is larger than other kernels. The current output of $I_{5}=2 I_{\text {ref }}$, which indicates $45^{\circ}$ point, should be classified as logic Low, and the current output $I_{5}=I_{\text {ref }}$, which does not indicate $45^{\circ}$ point, should be classified as logic High by the choice of $I_{t h}=1.5 I_{r e f}$. Note that the polarity is inverted with respect to that in Figure 3.9. The variation of the output current and the thresholding current are analyzed below to estimate the error rate and later associate it with the design parameters.

As shown in the previous example (Figure 4.4), each of the current from PMOS transistors has a variance of $3 s^{2} I_{\text {ref }}^{2}$. On the other hand, each of the current sunk into NMOS transistors has a variance of $5 s^{2} I_{\text {ref }}^{2}$ due to an additional current mirror operation by two NMOS
transistors. Two cases are analyzed below to calculate the variance of $I_{5}$ and $I_{t h}$. In the first case, the pixel is categorized as $45^{\circ}$ by the presence of three 1 's and one -1 , corresponding to the output current of $2 I_{\text {ref. }}$. The variance of current $I_{4}$ is $14 s^{2}\left(=3 \times 3 s^{2}+5 s^{2}\right)$. This current is next mirrored to produce output current $I_{5}$. This current mirror operation increases the current variance by $4 s^{2} I_{\text {ref }}^{2}\left(=2 \times 2 s^{2} I_{\text {ref }}^{2}\right)$ since the current level is $2 I_{\text {ref }}$ (refer to eqn. (4.47) with $m=2$ and $b=1$ ), which results in the variance of $I_{5}$ equal to $18 s^{2} I_{r e f}^{2}$.

In the second case, the pixel is not categorized as $45^{\circ}$ by the presence of three 1 's and two -1 's, corresponding to the output current of $I_{r e f}$. The variance of the output current $I_{4}$ is calculated as $19 s^{2}\left(=3 \times 3 s^{2}+2 \times 5 s^{2}\right)$. The subsequent current mirror operation increases the current variance by $2 \mathrm{~s}^{2}$, which results in the variance of $I_{5}$ equal to $21 s^{2}$. On the other hand, the variation of the threshold current is $1.5 \mathrm{~s}^{2} I_{\text {ref }}^{2}$.

Although so far no assumption has been made on the shape of the probability distribution, the Gaussian distribution is assumed for generality and analytical simplicity. Hence, hereinafter, the above three distributions are desiganted as

$$
\begin{gather*}
p_{2}(x)=N\left(2 I_{r e f}, 18 s^{2} I_{r e f}^{2}\right),  \tag{4.69}\\
p_{1}(x)=N\left(I_{r e f}, 21 s^{2} I_{r e f}^{2}\right), \tag{4.70}
\end{gather*}
$$

and

$$
\begin{equation*}
p_{1.5}(x)=N\left(1.5 I_{r e f}, 1.5 s^{2} I_{r e f}^{2}\right), \tag{4.71}
\end{equation*}
$$

respectively, where $N\left(m, \sigma^{2}\right)$ is the Gaussian distribution with the mean value $m$ and the variance $\sigma^{2}$. The distributions for different values of $s(s=0.03$ and $s=0.05)$ are shown in Figure 4.9. Note that the $x$-axis is scaled to $I_{\text {ref }}$. For $s=0.03$, which is shown on the left-hand side, the probability distribution is not very broad and almost no overlap exists between $p_{1}(x)$ and $p_{1.5}(x)$,


Figure 4.9 Probability distributin function of the output current and the threshold current for different relative current variations (left: $s=0.03$; right: $s=0.05$ ). Note that the $x$-axis is scaled to $I_{r e f} . p_{1}(x)$ and $p_{2}(x)$ represents the output current distribution for the current whose mean is $I_{r e f}$ and $2 I_{r e f}$, respectively. $P_{1.5}(x)$ represents the distribution of the threshold current whose mean value is $1.5 I_{r e f}$.
and between $p_{2}(x)$ and $p_{1.5}(x)$, On the other hand, for the case $s=0.05$, which is shown on the right-hand side, the distribution is broader, resulting in the overlap between $p_{1}(x)$ and $p_{1.5}(x)$, and the overlap between $p_{2}(x)$ and $p_{1.5}(x)$.

The generation of these overlap regions indicates the classification error. For example, the classification error occurs when the output current is $1.5 I_{r e f}$ instead of $I_{r e f}$ and the threshold current is $1.4 I_{\text {ref }}$ instead of $1.5 I_{\text {ref }}$. This happens relatively frequently for $s=0.05$ since both $p_{1}(1.5)$ and $p_{1.5}(1.4)$ are not very small. The error rate of this type, i.e., the probability of an output current which is originally $I_{r e f}$ is erroneously classified as logic High with respect to the threshold current of $1.5 I_{r e f}$, is computed as

$$
\begin{equation*}
p_{l e f t}=\int_{-\infty}^{\infty} p_{1}(x) d x\left(\int_{-\infty}^{x} p_{1.5}(\eta) d \eta\right) . \tag{4.72}
\end{equation*}
$$

Likewise, the error rate in the right side, i.e., the probability of an output current which is originally $2 I_{\text {ref }}$ is erroneously classified as logic Low with respect to the threshold current of $1.5 I_{r e f}$, is represented as

Table 4.2 Error rate (\%) for $45^{\circ}$ detection as a function of the relative current variation.

| $s$ | 0.03 | 0.04 | 0.05 | 0.06 |
| ---: | ---: | ---: | ---: | ---: |
| $p_{\text {right }}$ | 0.0081 | 0.23 | 1.18 | 2.97 |
| $p_{\text {left }}$ | 0.0238 | 0.42 | 1.74 | 3.91 |

$$
\begin{equation*}
p_{r i g h t}=\int_{-\infty}^{\infty} p_{1.5}(x) d x\left(\int_{-\infty}^{x} p_{2}(\eta) d \eta\right) \tag{4.73}
\end{equation*}
$$

These two integrals cannot be solved analytically and are hence numerically computed by Monte-Carlo simulation. The procedure for the computation is briefly explained for $p_{\text {left }}$. The simulation generates two random numbers based on the two probability distribution functions, $p_{1}(x)$ and $p_{1.5}(x)$. If the number generated from the former distribution is larger than that from the latter distribution, the trial is classified as an error. After each trial is tested, the number of trials categorized as an error is divided by the number of the total trials. The obtained number is an estimate of the error rate and the estimate should become more accurate as the number of trials become larger. In the simulation, one million trials were performed.

Table 4.2 shows the error rate as a function of the relative current variation. The error rate is lower for smaller values of $s$. The error occurrence on the left-hand side is higher than that on the right-hand because the distribution $p_{1}(x)$ has broader distribution than $p_{2}(x)$. If the specification for the circuit is given in terms of accuracy, or equivalently an error rate, it is possible to select a value of $s$ to satisfy that requirement. To keep the error rate in the order of ten thousandth, $s$ should be chosen smaller than 0.03 . The selected value of $s$ can be obtained by various combinations of the reference current $I_{\text {ref }}$ and the channel length $L$ if the process parameters listed in eqn. (4.31), $\mu_{\mathrm{n}} C_{o x}$ and $A_{V_{T}}$, are known. The HP $0.5 \mu \mathrm{~m}$ technology, which is chosen for the fabrication of the feature detection sensor, has the following values for $\mu C_{o x}$ :

$$
\begin{equation*}
\mu_{n} C_{o x}=104 \quad\left[\mu \mathrm{~A} / \mathrm{V}^{2}\right] \tag{4.74}
\end{equation*}
$$

and

$$
\begin{equation*}
\mu_{p} C_{o x}=35\left[\mu \mathrm{~A} / \mathrm{V}^{2}\right] \tag{4.75}
\end{equation*}
$$

where suffixes $n$ and $p$ indicate that the parameter is for NMOS transistors and PMOS transistors, respectively.

The other parameter required for the calculation of the design parameters, the mismatch proportionality constant $A_{V_{T}}$, should be experimentally obtained from a test chip which includes a lot of transistors of different gate areas. For the present design, however, this proportionality constant $A_{V_{T}}$ is estimated as $15[\mathrm{mV} \mu \mathrm{m}]$ for both NMOS and PMOS transistors. This is probably a rather conservative estimate considering experimentally obtained values of $A_{V_{T}}$ listed in Table 4.1. Table 4.3 shows the channel length for different values of $s$ and different values of $I_{\text {ref }}$ obtained by substituting those values listed in eqns. (4.74) and (4.75), and $A_{V_{T}}=15[\mathrm{mV} \mu \mathrm{m}]$ into eqn. (4.31). Numbers shown prior to the parenthesis is the calculated value of $L$; numbers shown in the parenthesis is the drawn dimension. Larger values of $L$ and $I_{r e f}$ are required to obtain smaller values of $s$, or equivalently, higher accuracy. The constraint at this point for $L$ and $I_{\text {ref }}$ is given in the product form of $L^{2} I_{r e f}$. How to choose the optimal combination of $L$ and $I_{r e f}$ is explained in the next section.

Before moving onto that topic, it is worth confirming by simulation that these obtained values of $L$ and $I_{\text {ref }}$ really produce the error as expected. A Monte-Carlo simulation was done using Hspice circuit simulator in the following way. For each transistor in the circuit, the threshold voltage was randomly chosen so that the overall distribution for the entire set of transistors follow the normal distribution with its mean equal to the threshold voltage provided by the vendor (HP) and with its variance equal to $A_{V_{T}}^{2} / W L$. Simulations were performed ten

Table 4.3 Channel lengths to satisfy the specified accuracy $s$ for different values of $I_{r e}$.

|  |  | 0.03 | 0.04 | 0.05 | 0.06 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $I_{r e f}=1[\mu \mathrm{~A}]$ | $L_{n}[\mu \mathrm{~m}]$ | $7.2(7.3)$ | $5.4(5.5)$ | $4.3(4.4)$ | $3.6(3.7)$ |
|  | $L_{p}[\mu \mathrm{~m}]$ | $4.2(4.3)$ | $3.1(3.2)$ | $2.5(2.6)$ | $2.1(2.2)$ |
| $I_{\text {ref }}=4[\mu \mathrm{~A}]$ | $L_{n}[\mu \mathrm{~m}]$ | $3.6(3.7)$ | $2.7(2.8)$ | $2.2(2.3)$ | $1.8(1.9)$ |
|  | $L_{p}[\mu \mathrm{~m}]$ | $2.1(2.2)$ | $1.6(1.7)$ | $1.3(1.4)$ | $1.1(1.2)$ |

thousand times. Figure 4.10 shows the simulation result for different values of the channel length $(L)$ and the channel width $(W)$. The reference current $I_{\text {ref }}$ is set to $1 \mu \mathrm{~A}$. The two graphs at the top display the effect of $L$ on the error rate. The left graph and the right graph correspond to the error rate $p_{\text {left }}$ and $p_{\text {right }}$, respectively. For both cases, the error rate decreases as the channel length increases as is expected from Table 4.2 (shown as a dashed line).

On the other hand, the two graphs at the bottom, where the simulation results are replotted as a function of $W$ for different values of $L$, demonstrate that the influence of $W$ on the error rate is much smaller than the channel length. There is, however, a slight dependence of the error rate on $W$ especially when $L$ is small possibly for the following reason. For larger values of $W / L$, eqn. (4.6) is valid for larger current levels. In other words, for a current level of $1 \mu \mathrm{~A}$, the relationship between $V_{G S}-V_{T}$ and $I$ is not as strong as specified in eqn. (3.6), indicating an exponent smaller than 2 . In this case, the proportionality constant relating $\left(\sigma_{V_{T}} /\left(V_{G S}-V_{T}\right)\right)^{2}$ to $\left(\sigma_{I} / I\right)^{2}$ in eqn. (4.19) becomes smaller than $4\left(=2^{2}\right)$, which leads to a smaller current variation and hence a smaller error rate.

The simulation results shown above demonstrate the validity of the proposed design procedure which converts the initial requirement for the error rate to the requirement for the relative current accuracy and subsequently to the requirement for the design variables. The


Figure 4.10 Hspice Monte-Carlo simulation result. The two graphs at the top represent the error rate as a function of the transistor channel length. The two graphs at the bottom represent the error rate as a function of the channel width. For these two cases, the graph on the left side and on the right side corresponds to $p_{\text {left }}$ and $p_{\text {right }}$, respectively.
minimum value of $L^{2} I_{r e f}$ is obtained to satisfy the required current accuracy. What is established by this procedure is the information flow given by the solid line and the dashed line, designated as 1 and 2a, in Figure 4.7. By this procedure, the minimum value of $L^{2} I_{\text {ref }}$ can be obtained.

There are, however, still questions unanswered for the choice of the design parameters. These questions include (1) how to choose the channel width $W$. What aspect of the circuit performance does this parameter affect? (2) How to choose $L$ and $I_{r e f}$ among several candidate combinations which generates the same value of $L^{2} I_{r e f}$ ? These two questions are addressed below from the point of operational speed. This is nothing but the establishment of the design


Figure 4.11 Schematic to explain the charging and discharging of the capacitance associated with the gate. The rise time $t_{r}$ is defined as the time for the current to reach $95 \%$ of $I_{r e f}$ while the fall time $t_{f}$ is defined as the time for the current to reach $5 \%$ of $I_{r e f}$.
flow shown as a dashed line designated as 2 b in Figure 4.7.

### 4.3.3 Effect of the transistor dimension on the operational speed

The operational speed is an important parameter for specifying the performance of the circuit. It is determined by the charging and discharging of the capacitance associated with the gate, which is schematically shown in Figure 4.11. The capacitance drawn as a dashed line represents the parasitic capacitance connected to that node, which consists of the gate and drain capacitance of $\mathrm{M}_{1}$ and the gate capacitance of $\mathrm{M}_{2}$. A formula that relates the design parameters to the operational speed is derived below. The Kirchhoff current law at the gate node requires that

$$
\begin{equation*}
I_{r e f}=I_{t r}+I_{c a p} \tag{4.76}
\end{equation*}
$$

where $I_{\text {ref }}$ represents the reference current for the current mirror circuit. $I_{t r}$ and $I_{\text {cap }}$ represent the current that flows through the transistor and the capacitor, respectively. Expressing each current in terms of voltages yields

$$
\begin{equation*}
\frac{\beta}{2}\left(V_{\text {ref }}-V_{T}\right)^{2}=\frac{\beta}{2}\left(V-V_{T}\right)^{2}+C \frac{d V}{d t} \tag{4.77}
\end{equation*}
$$

where $V_{\text {ref }}$ represents the gate voltage required to generate the current $I_{r e f}$. The above equation can be further simplified to

$$
\begin{equation*}
\frac{\beta}{2}\left(\left(V_{\text {ref }}-V_{T}\right)^{2}-\left(V-V_{T}\right)^{2}\right)=C \frac{d V}{d t} \tag{4.78}
\end{equation*}
$$

or equivalently

$$
\begin{equation*}
\frac{\beta}{2 C} d t=\frac{d V}{\left(V_{\text {ref }}+V-2 V_{T}\right)\left(V_{\text {ref }}-V\right)} . \tag{4.79}
\end{equation*}
$$

Let us define the rise time $t_{r}$ as the time required for the current to reach $I_{l}=\alpha I_{\text {ref }}$, and the voltage $V_{1}$ as the voltage which produces the current $I_{l .} V_{l}$ is obtained as

$$
V_{1}=\sqrt{\alpha} V_{r e f}+(1-\sqrt{\alpha}) V_{T}
$$

by solving the following equation:

$$
\begin{equation*}
I_{1}=\frac{\beta}{2}\left(V_{1}-V_{T}\right)^{2}=\alpha \frac{\beta}{2}\left(V_{\text {ref }}-V_{T}\right)^{2} . \tag{4.80}
\end{equation*}
$$

Then integrating the left-hand side of eqn. (4.79) from time 0 to $t_{r}$ and integrating the right-hand side from $V_{T}$ to $V_{l}$ yields

$$
\begin{equation*}
\int_{0}^{t_{r}} \frac{\beta}{2 C} d t=\int_{V_{T}}^{V_{1}} \frac{d V_{P}}{\left(V_{r e f}+V-2 V_{T}\right)\left(V_{r e f}-V\right)} . \tag{4.81}
\end{equation*}
$$

Using the following identity:

$$
\begin{equation*}
\frac{1}{\left(V_{r e f}+V-2 V_{T}\right)\left(V_{r e f}-V\right)}=\frac{1}{2\left(V_{r e f}-V_{T}\right)}\left(\frac{1}{\left(V_{r e f}+V-2 V_{T}\right)}+\frac{1}{\left(V_{r e f}-V\right)}\right) \tag{4.82}
\end{equation*}
$$

Eqn. (4.81) is rewritten as

$$
\begin{equation*}
\frac{\beta}{2 C} t_{r}=\frac{1}{2\left(V_{r e f}-V_{T}\right)} \int_{V_{T}}^{V_{1}}\left(\frac{1}{V_{r e f}+V-2 V_{T}}+\frac{1}{V_{r e f}-V}\right) d V_{P} \tag{4.83}
\end{equation*}
$$

Substituting $V_{1}$ into eqn. (4.83) and performing integration yields

$$
\begin{equation*}
\frac{\beta}{2 C} t_{r}=\left.\frac{1}{2\left(V_{r e f}-V_{T}\right)} \ln \frac{V_{r e f}+V-2 V_{T}}{V_{r e f}-V}\right|_{V_{T}} ^{\sqrt{\alpha} V_{r e f}+(1-\sqrt{\alpha}) V_{T}} \tag{4.84}
\end{equation*}
$$

By calculating the right-hand side, the rise time is obtained as

$$
\begin{equation*}
t_{r}=\frac{C}{\beta\left(V_{r e f}-V_{T}\right)} \ln \frac{1+\sqrt{\alpha}}{1-\sqrt{\alpha}} \tag{4.85}
\end{equation*}
$$

or alternatively

$$
\begin{equation*}
t_{r}=\frac{C}{\sqrt{2 \beta I_{r e f}}} \ln \frac{1+\sqrt{\alpha}}{1-\sqrt{\alpha}} . \tag{4.86}
\end{equation*}
$$

The fall time can be obtained almost in the same manner. Since $I_{\text {ref }}=0$ in this case, eqn. (4.76) reduces to

$$
\begin{equation*}
0=\frac{\beta}{2}\left(V-V_{T}\right)^{2}+C \frac{d V}{d t} \tag{4.87}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{-\beta}{2 C} d t=\frac{d V}{\left(V-V_{T}\right)^{2}} . \tag{4.88}
\end{equation*}
$$

Integrating the left-hand side from 0 to $t_{f}$ and integrating the right-hand side from $V_{\text {ref }}$ to $V_{2}$ yields

$$
\begin{equation*}
t_{f}=\left.\frac{2 C}{\beta} \frac{1}{V-V_{T}}\right|_{V_{r e f}} ^{V_{2}} \tag{4.89}
\end{equation*}
$$

where $t_{f}$ is defined as the fall time required for the current to decrease to $I_{2}=\varepsilon I_{\text {ref }}, V_{2}$ is defined as the gate voltage that produces the current $I_{2}$, which is calculated as

$$
\begin{equation*}
V_{2}=\sqrt{\varepsilon} V_{\text {ref }}+(1-\sqrt{\varepsilon}) V_{T} . \tag{4.90}
\end{equation*}
$$

Substituting eqn. (4.90) into eqn. (4.89) gives

$$
\begin{equation*}
t_{f}=\frac{C}{\beta\left(V_{r e f}-V_{T}\right)} \frac{2(1-\sqrt{\varepsilon})}{\sqrt{\varepsilon}} \tag{4.91}
\end{equation*}
$$

or alternatively

$$
\begin{equation*}
t_{f}=\frac{C}{\sqrt{2 \beta I_{r e f}}} \frac{2(1-\sqrt{\varepsilon})}{\sqrt{\varepsilon}} . \tag{4.92}
\end{equation*}
$$

Both eqns. (4.86) and (4.92) have the following term in common:

$$
\begin{equation*}
\frac{C}{\sqrt{2 \beta I_{r e f}}}=\frac{2 \times \frac{2}{3} C_{o x} W L}{\sqrt{2 \mu C_{o x}(W / L) I_{r e f}}}=\sqrt{\frac{8 C_{o x} W L}{9 \mu I_{r e f}}} L, \tag{4.93}
\end{equation*}
$$

which clearly demonstrates the effect of the design parameters on the rise time and the fall time. Larger values of $W$ and $L$ contribute to larger values of the rise time and the fall time. Smaller values of $W$ are preferable for faster operation. However, $L$ should be chosen carefully since smaller values of $L$ lead to faster operation as well as to lower accuracy (see eqn. (4.31)).

The relationship between the reference current and the channel length to satisfy the accuracy and speed requirement is plotted in Figure 4.12. Note that the fall time is used as a measure for specifying the operational speed. It is because the fall time is larger than the rise time ${ }^{9}$ under the condition $\alpha=0.95$ and $\varepsilon=0.05$. The channel width is set to $1.5 \mu \mathrm{~m}(5 \lambda)$ instead of the minimum dimension of $0.9 \mu \mathrm{~m}(3 \lambda)$. The reason of this choice is based on the information obtained from MOSIS for the AMI ABN technology, which says that the analog designs using the scalable CMOS rules for the AMI SCMOS $1.2 \mu \mathrm{~m}(\lambda=0.6 \mu \mathrm{~m})$ run should have a minimum channel width of $5 \lambda$ and a minimum channel length of $3 \lambda$. Transistors with the minimum $W$ and minimum $L$ are not being rendered with sufficiently stable characteristics for analog circuit designs. Although the target process is HP $0.5 \mu \mathrm{~m}$ in the present design, the process is originally intended for digital use and hence the minimum channel width is avoided

[^7]

Figure 4.12 Relationship between the channel length and the reference current to satisfy requirements for accuracy and speed. The left graph and the right graph represent the relationship for PMOS current mirrors and NMOS current mirrors, respectively. The three solid lines represent the relationship between $L$ and $I_{\text {ref }}$ to achieve accuracy given by the value of $s$. The three dotted lines represent the relationship to achieve a speed specified by the fall time. All the plots are made for the channel width of $1.5 \mu \mathrm{~m}$.
to better predict the behavior of the circuit by simulation. For analog blocks including current mirror circuits, the minimum $W$ and $L$ are set to $5 \lambda$ and $3 \lambda$, respectively, while for digital blocks, the minimum $W$ and $L$ are set to $3 \lambda$ and $2 \lambda$, respectively.

Going back to Figure 4.12, it is clear that to achieve higher accuracy, larger values of $L$ and $I_{\text {ref }}$ are required which corresponds to the top right region in the graph. On the other hand, to achieve faster speed, larger values of $I_{r e f}$ and smaller values of $W$ and $L$ are required, which corresponds to the bottom right region in the graph. It should be noted that larger values of $W$ shifts these dashed lines downward, demanding even smaller values of $L$ to achieve the same speed. The allowed region for the design parameters to satisfy the demand for high accuracy and fast speed is indicated as the intersection of the two regions, below the dotted line corresponding to a specified value of $t_{f}$ and above the solid line corresponding to a specified value of $s$. The shaded region in Figure 4.12 satisfies the following two requirements for the feature detection sensor: relative current variation smaller than 0.03 and the fall time smaller
than 10 nsec . The maximum allowed value for the fall time is set to 10 nsec , one tenth of one operational cycle assuming a 10 MHz operation.

It should be mentioned that the shaded region for the NMOS case is shifted rightward compared to that for the PMOS case, requiring more current to obtain the same speed and accuracy. In other words, the PMOS current mirror operates faster than the NMOS current mirror when the accuracy requirement is the same. The reason of this difference in the performance is explained below. The ratio of the fall time for the PMOS transistor with respect to that for the NMOS transistor is represented as

$$
\begin{equation*}
\frac{t_{f p}}{t_{f n}}=\frac{\sqrt{\frac{8 C_{o x} W L_{p}^{3}}{9 \mu_{p} I_{r e f}}}}{\sqrt{\frac{8 C_{o x} W L_{n}^{3}}{9 \mu_{n} I_{r e f}}}}=\sqrt{\left(\frac{\mu_{n}}{\mu_{p}}\right)\left(\frac{L_{p}}{L_{n}}\right)^{3}} \tag{4.94}
\end{equation*}
$$

where the same channel width is assumed for both NMOS and PMOS transistors. Since the same accuracy is assumed for both NMOS and PMOS transistors,

$$
\begin{equation*}
s^{2}=\frac{2 A_{V_{T}}^{2} \mu_{n} C_{o x}}{L_{n}^{2} I_{r e f}}=\frac{2 A_{V_{T}}^{2} \mu_{p} C_{o x}}{L_{p}^{2} I_{r e f}} . \tag{4.95}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\left(\frac{L_{p}}{L_{n}}\right)=\sqrt{\frac{\mu_{p}}{\mu_{n}}} . \tag{4.96}
\end{equation*}
$$

Eqn. (4.96) is substituted in (4.94) to generate

$$
\begin{equation*}
\frac{t_{f p}}{t_{f n}}=\sqrt{\left(\frac{\mu_{n}}{\mu_{p}}\right)\left(\sqrt{\frac{\mu_{p}}{\mu_{n}}}\right)^{3}}=\left(\frac{\mu_{p}}{\mu_{n}}\right)^{\frac{1}{4}}=\left(\frac{\mu_{p} C_{o x}}{\mu_{n} C_{o x}}\right)^{\frac{1}{4}}=\left(\frac{35}{104}\right)^{\frac{1}{4}}=0.76 \tag{4.97}
\end{equation*}
$$

where design parameters specified in eqns. (4.74) and (4.75) are used for calculation.
The above equation clearly indicates that the PMOS transistor operates about $24 \%$ faster than the NMOS transistor. The design procedure imposing the same accuracy requirement for
both NMOS and PMOS transistors, in this sense, is not symmetric in terms of speed. The total performance in terms of accuracy and speed is higher for the PMOS transistor than for the NMOS transistors. However, other design procedures are possible. For example, an NMOS transistor and a PMOS transistor can be designed to have the same speed by the adjustment of $W$ while keeping the same accuracy requirement; or the accuracy can be differently designed for an NMOS transistor and a PMOS transistor while keeping the same speed. The choice of the design procedure depends on the type of the application. In other words, the relative importance of accuracy and speed determines the procedure. In the present design, more emphasis was put on the accuracy requirement, resulting in the difference in the operational speed for the NMOS transistor and the PMOS transistor.

From the shape of the shaded region in Figure 4.12, the use of a larger current is the only way to satisfy both higher accuracy and faster operation. It is of course at the expense of smaller power consumption. In the present design, the current level was chosen as $4 \mu \mathrm{~A}$ to satisfy both speed and accuracy requirements without consuming too much power. For this current, the channel length for the NMOS and PMOS transistors are chosen as $3.7 \mu \mathrm{~m}$ and $2.2 \mu \mathrm{~m}$, respectively, to satisfy the accuracy requirement (see Table 4.3). The fall time is estimated to be smaller than 10 nsec with this dimension for both the NMOS and PMOS transistors. It is possible to further increase the current to realize more accuracy and higher speed. However, in reality, the current must be restricted below some level to keep the transistor in the saturation region, which is described next.

### 4.3.4 Consideration on the operating region

For a given $W / L$, a higher gate voltage is required to produce a large current. A high gate voltage, however, may bring the transistor into the linear region when the drain voltage is determined by other parts of the circuit. It is important to operate the transistor in the saturation
region so that a constant current is always supplied from the transistor. Let us analyze the circuit shown in Figure 3.9. The following situation should be avoided: the bottom NMOS transistors enter into the linear region by the lowering of the voltage of node Nc. This could happen when node Nc is brought down by several current sources. For the feature detection algorithm described before to work correctly, the current as large as three times $I_{r e f}$ has to be accommodated by the PMOS transistor without bringing down the voltage of node Nc too much. The node voltage is given by

$$
\begin{equation*}
V_{C}=V_{D D}-V_{T_{p}}-\sqrt{\frac{2 \times 3 I_{r e f}}{\mu_{p} C_{o x}\left(W_{p} / L_{p}\right)}} . \tag{4.98}
\end{equation*}
$$

This voltage is equal to the drain voltage of the current sinking NMOS transistor when the resistance of NMOS switches is neglected. Therefore, $V_{c}$ has to be higher than the gate voltage of these transistors shifted downward by the threshold voltage $V_{T_{n}}$ for NMOS transistors to remain in the saturation region. This condition is expressed as

$$
\begin{equation*}
V_{C}>V_{T_{n}}+\sqrt{\frac{2 I_{r e f}}{\mu_{n} C_{o x}\left(W_{n} / L_{n}\right)}}-V_{T_{n}} . \tag{4.99}
\end{equation*}
$$

Substituting eqn. (4.98) into (4.99) yields

$$
\begin{equation*}
V_{D D}-V_{T_{p}}-\sqrt{\frac{2 \times 3 I_{r e f}}{\mu_{p} C_{o x}\left(W_{p} / L_{p}\right)}}>\sqrt{\frac{2 I_{r e f}}{\mu_{n} C_{o x}\left(W_{n} / L_{n}\right)}} . \tag{4.100}
\end{equation*}
$$

Since the channel width is the same for both the NMOS and PMOS transistors, i.e., $W_{n}=W_{p}$ $=W$, the above equation reduces to

$$
\begin{equation*}
\left(1+\sqrt{\frac{3\left(L_{p} / L_{n}\right)}{\left(\mu_{p} / \mu_{n}\right)}}\right) \sqrt{\frac{2 I_{r e f}}{\mu_{n} C_{o x}\left(W / L_{n}\right)}}<V_{D D}-V_{T_{p}} . \tag{4.101}
\end{equation*}
$$

Substitution of eqn. (4.96) into eqn. (4.101) and further simplification yields

$$
\begin{equation*}
\left(1+\sqrt{3 \sqrt{\frac{\mu_{n}}{\mu_{p}}}}\right)^{2} \frac{2 I_{r e f} L_{n}}{\mu_{n} C_{o x} W}<\left(V_{D D}-V_{T_{p}}\right)^{2} . \tag{4.102}
\end{equation*}
$$

Finally the constraint on $L_{n}$ as a function of the reference current $I_{r e f}$ is obtained as:

$$
\begin{equation*}
L_{n}<\frac{W \mu_{n} C_{o x}\left(V_{D D}-V_{T_{p}}\right)^{2}}{2 I_{r e f}\left(1+\sqrt{3 \sqrt{\mu_{n} / \mu_{p}}}\right)^{2}} . \tag{4.103}
\end{equation*}
$$

$L_{p}$ can be obtained using eqns. (4.103) and (4.96).
The shaded region in Figure 4.12 satisfies eqn. (4.103), confirming the validity of the previous selection of the reference current and the channel length. The restriction by the operating region becomes more prominent if the narrower channel width is used. This is exemplified in Figure 4.13 for $W=0.9[\mu \mathrm{~m}]$. The maximum value of $L$ decreases, as $I_{r e f}$ becomes larger. For both the PMOS and the NMOS cases, the fan-shaped shaded region originally extended rightward reduces to a very small region. Any point inside this region satisfies the three requirements so far requested: relative current variation smaller than 0.03 ; the fall time smaller than 10 nsec , all transistors operate in the saturation region. If no overlapping region exists, the value of $W$ should be increased so that the requirement on the operating condition is relaxed although the operational speed is slowed down.

It is interesting at this point to see the implication of the restriction by the operating region. The following analysis is partly based on the work conducted by Kinget [4]. Let us define the speed of the circuit as the reciprocal of the fall time, i.e.,

$$
\begin{equation*}
\text { Speed }=\frac{1}{t_{f}}=\frac{1}{k} \sqrt{\frac{9 \mu I_{r e f}}{8 C_{o x} W L^{3}}} \tag{4.104}
\end{equation*}
$$

where

$$
\begin{equation*}
k=\frac{2(1-\sqrt{\varepsilon})}{\sqrt{\varepsilon}} . \tag{4.105}
\end{equation*}
$$



Figure 4.13 Relationship between the channel length and the reference current to ensure the transistor operation in the saturation region. The requirement is plotted as a thick line superimposed on the relationship shown in Figure 4.12. All the plots were made for the channel width of $0.9 \mu \mathrm{~m}$.

Likewise, Accuracy can be defined as the reciprocal of the relative current variation $s$, i.e.,

$$
\begin{equation*}
\text { Accuracy }^{2}=\frac{1}{s^{2}}=\frac{L^{2} I_{r e f}}{2 A_{V_{T}}^{2} \mu C_{o x}} . \tag{4.106}
\end{equation*}
$$

If the total performance of the circuit is defined as the product of eqns (4.104) and (4.106),

$$
\begin{align*}
\text { Speed Accuracy }^{2} & =\frac{1}{k} \sqrt{\frac{9 \mu I_{r e f}}{8 C_{o x} W L^{3}}} \frac{L^{2} I_{\text {ref }}}{2 A_{V_{T}}^{2} \mu C_{o x}}  \tag{4.107}\\
& =\frac{1}{k} \sqrt{\frac{9 \mu}{8 C_{o x}}} \frac{1}{2 A_{V_{T}}^{2} \mu C_{o x}} \sqrt{\frac{I_{\text {ref }}}{W / L}} I_{\text {ref }} .
\end{align*}
$$

Replacing the first $I_{r e f}$ with $\frac{\mu C_{o x}}{2} \frac{W}{L}\left(V_{G S}-V_{T}\right)^{2}$ gives

$$
\begin{align*}
\text { Speed Accuracy }^{2} & =\frac{1}{k} \sqrt{\frac{9 \mu}{8 C_{o x}}} \frac{1}{2 A_{V_{T}}^{2} \mu C_{o x}} \sqrt{\frac{1}{2} \mu C_{o x}\left(V_{G S}-V_{T}\right)^{2}} I_{r e f} \\
& =\frac{1}{k} \sqrt{\frac{9 \mu}{8 C_{o x}}} \frac{1}{2 A_{V_{T}}^{2} \mu C_{o x}} \sqrt{\frac{1}{2} \mu C_{o x}}\left(V_{G S}-V_{T}\right) I_{r e f}  \tag{4.108}\\
& =\frac{3}{8 k A_{V_{T}}^{2} C_{o x}}\left(V_{G S}-V_{T}\right) I_{r e f} .
\end{align*}
$$

This equation states that the total performance in terms of speed and accuracy is determined by the product of $V_{G S}-V_{T}$ and $I_{r e f}$. The result is similar to what is described in ref. [4]. Eqn. (4.108) can be rewritten as

$$
\begin{align*}
\frac{\text { Speed Accuracy }^{2}}{\text { Power }} & =\frac{3}{8 k A_{V_{T}}^{2} C_{o x}}\left(V_{G S}-V_{T}\right) \frac{I_{\text {ref }}}{I_{\text {ref }} V_{D D}}  \tag{4.109}\\
& =\frac{3}{8 k A_{V_{T}}^{2} C_{o x} V_{D D}}\left(V_{G S}-V_{T}\right)
\end{align*}
$$

where only DC power consumption is incorporated in the above formula. This is considered a normalized total performance and is constant as long as the degree of biasing is the same. The only way to increase the normalized total performance is to increase $V_{G S}-V_{T}$ as much as possible while ensuring the transistor operation in the saturation region. It is also clear from eqn. (4.109) that higher performance is obtained for processes characterized by smaller mismatch.

Along the curve shown as a thick line in Figure 4.13, $V_{G S}-V_{T}$ is constant, indicating the same normalized total performance. Moving to the right direction along this curve, the circuit gains higher speed while losing accuracy and suffering from higher power consumption, but still keeping the same normalized total performance. This is the maximum performance obtained since this value of $V_{G S}-V_{T}$ corresponds to the edge of the saturation region. If the power consumption is not much of an issue, the best performance is obtained by choosing the point where the $V_{G S}-V_{T}$ constraint curve meets the accuracy requirement curve.

The difference of the performance between the PMOS and the NMOS transistors can be explained from the point of biasing. The bias voltages for the PMOS and the NMOS transistors for $W=1.5 \mu \mathrm{~m}$ are calculated as

$$
V_{G S}-V_{T}= \begin{cases}0.67[\mathrm{~V}] & \text { for PMOS }  \tag{4.110}\\ 0.51[\mathrm{~V}] & \text { for NMOS }\end{cases}
$$



Figure 4.14 Extracted schematic from Figure 3.9 focusing on the current thresholding node. Expression in a parenthesis represents the variance of the current shown above the parenthesis.

The larger value of $V_{G S}-V_{T}$ for the PMOS transistor than for the NMOS transistor indicates that the total performance is higher for the PMOS transistor than for the NMOS transistor. This observation agrees with the previous discussion that the present design procedure results in the superior performance in the PMOS transistor due to its higher operational speed while maintaining the same accuracy (refer to eqn. (4.97)).

### 4.3.5 Simulation using the modified circuit

In the previous subsection, the design parameters have been determined as follows: $I_{\text {ref }}=4[\mu \mathrm{~A}], \quad W_{n}=W_{p}=1.5[\mu \mathrm{~m}], \quad L_{n}=3.7[\mu \mathrm{~m}], \quad L_{p}=2.2[\mu \mathrm{~m}]$. Using these values, simulations were performed for the circuit shown in Figure 3.9, which is a finalized version of the pixel circuit. The difference between this circuit and the previously used circuit shown in Figure 4.8 lies in the method of current thresholding. The reason of the initial choice of Figure 4.8 is to save an area because only one line is required to control the threshold current compared to the scheme in Figure 3.9, which requires two control lines. However, the new thresholding
scheme is simpler and does not have to distribute an analog signal in an entire chip area. In addition, the new scheme is advantageous in terms of matching since a set of NMOS transistors for current mirroring were removed and the new threshold current has less variation.

This is explained next using the simplified circuit shown in Figure 4.14. Due to lack of an additional current mirror operation, the variance of the current $I_{4}$ decreases to $14 s^{2}$ and $19 s^{2}$ when the current level is $I_{r e f}$ and $2 I_{r e f}$, respectively, from the original variance of $18 s^{2}$ and $21 s^{2}$. The variance of the threshold current is calculated as the summation of the variance of each of the three currents $\left(I_{5 a}, I_{5 b}, I_{5 c}\right)$ since there is no correlation between these currents. For $I_{5}=1.5 I_{r e f}$, It is represented as

$$
\begin{align*}
\sigma_{I_{s}}^{2} & =\sigma_{I}^{2}\left(I_{r e f}, L\right)+\sigma_{I}^{2}\left(I_{r e f} / 2,2 L\right) \\
& =\sigma_{I}^{2}\left(I_{r e f}, L\right)+\left(\frac{1}{2}\right)^{3} \sigma_{I}^{2}\left(I_{r e f}, L\right) \\
& =\frac{9}{8} \sigma_{I}^{2}\left(I_{r e f}, L\right)  \tag{4.111}\\
& =\frac{9}{8} s^{2} I_{r e f}^{2} .
\end{align*}
$$

Note that eqn. (4.33) is used to derive the current variation for the third current source (transistor $\mathrm{M}_{5 \mathrm{c}}$ ), which employs half the channel length to produce half the reference current. The probability density function given in eqns. (4.69), (4.70), and (4.71) are rewritten as

$$
\begin{gather*}
p_{2}(x)=N\left(2 I_{r e f}, 14 s^{2} I_{r e f}^{2}\right),  \tag{4.112}\\
p_{1}(x)=N\left(I_{r e f}, 19 s^{2} I_{r e f}^{2}\right), \tag{4.113}
\end{gather*}
$$

and

$$
\begin{equation*}
p_{1.5}(x)=N\left(1.5 I_{r e f},(9 / 8) s^{2} I_{r e f}^{2}\right), \tag{4.114}
\end{equation*}
$$

respectively. Note that the variance is smaller than those obtained in the previous design (eqns. (4.69), (4.70), and (4.71)). The expected error rate for these distributions is plotted as a function of $s$ in Figure 4.15 with the Hspice Monte-Carlo simulation result. In the Hspice simulation,


Figure 4.15 Expected and simulated error rate as a function of the reference current. The dashed line represents the expected error rate, while the solid line represents the Monte-Carlo simulation result using Hspice circuit simulator.
the reference current was varied instead of the channel length, to produce different values of $s$. The simulation results agreed well with the expected value, again demonstrating the validity of the design procedure. The error rate shown in Figure 4.15, either expected one or the one obtained by the Hspice simulation, was smaller than the error rate shown in the top row of Figure 4.10 for any value of $s(s=0.03,0.04,0.05$, or 0.06$)$.

### 4.3.6 Final adjustment of the design parameters

As a final step of the parameter determination, the effect of $\beta$ mismatch, which has been treated negligible, is considered. The effect of $\beta$ mismatch is estimated by comparing the gatedrive voltage $V_{G S}-V_{T}$ with the corner gate-drive voltage $\left(V_{G S}-V_{T}\right)_{m}=2 A_{V_{T}} / A_{\beta}$ (refer to eqn. (4.25)). From Table 4.1, assuming $A_{\beta}=2[\% \mu \mathrm{~m}]$ for both PMOS and NMOS transistors in the HP $0.5 \mu \mathrm{~m}$ process, the corner gate-drive voltage is calculated as

$$
\begin{equation*}
\left(V_{G S}-V_{T}\right)_{m}=1.22[\mathrm{~V}] \tag{4.115}
\end{equation*}
$$

which is almost twice the gate-drive voltage shown in eqn. (4.110). Therefore, the effect of $\beta$ mismatch is smaller than $V_{T}$ mismatch, but the contribution cannot be completely ignored.

The error rate increases from 0.03 to 0.037 and to 0.034 for the PMOS and the NMOS transistors, respectively by incorporating the effect of $\beta$ mismatch. The PMOS transistor is more affected than the NMOS transistor due to its smaller area. This increase should be compensated by the increase in $L$ so that the overall relative current variation remains 0.03 . As a final value, $L$ is chosen as $2.7 \mu \mathrm{~m}$ and $4.2 \mu \mathrm{~m}$ for the PMOS and the NMOS transistors, respectively.

### 4.3.7 Summary of the design procedure

A systematic design procedure that determines the transistor dimension and the reference current to satisfy the given specification in terms of accuracy and speed has been presented in this section. The procedure is briefly summarized below. First, a given accuracy requirement has to be converted to the requirement for the current variation. It is possible by representing the variable concerning accuracy, such as an error rate, as a function of the current variation. The formulas derived in the previous section are necessary for this representation. Next, the minimum value of $L^{2} I_{\text {ref }}$ is determined to satisfy the required current variation. In other words, the allowed region for $L$ and $I_{\text {ref }}$ can be graphically obtained as the region above a constraint curve.

Another requirement concerning speed gives the maximum value of $\sqrt{W L^{3} / I_{r e f}} . W$ is chosen as the minimum dimension as an initial choice. For this value of $W$, the allowed region for $L$ and $I_{\text {ref }}$ can be graphically represented as the region below another constraint curve. At this point, the allowed combinations of $L$ and $I_{\text {ref }}$ can be represented as an intersection of the above two regions, which takes a fan shape extended rightward. Next, the operating region of the transistor has to be examined. The constraint for transistors to stay in the saturation region, which is given as the maximum value of $V_{G S}-V_{T}$ or $\sqrt{I_{\text {ref }} /(W / L)}$, can be also graphically
represented on the same graph. By this constraint, the allowed region may reduce to a triangular shape. If no region exists to satisfy this constraint on the operating region, $W$ has to be increased to relax the condition and the above procedure has to be repeated.

The final selection from the triangular region depends on the type of applications. It should be decided by considering the relative importance of accuracy and speed as well as the power consumption. One of the selection methods when the power consumption is not too high within the allowed region is to choose the values of $L$ and $I_{r e f}$ which correspond to the bottom right vertex of the triangle, since the total performance (not normalized) in terms of speed and power is maximum at this point.

### 4.4 Summary

A systematic design procedure based on transistor mismatch analysis for current-mode processing circuits has been proposed in this section. First, the formula relating the design parameters to the current variation and the other formula describing the increase in the current variation by current mirroring are newly derived. The effect of correlation between multiple currents is also formulated. Using these formulas, the proposed method converts the requirement for accuracy and speed to the requirement for the design parameters $\left(W, L, I_{r e f}\right)$. The condition for ensuring the transistor operation in the saturation region is also converted to the other form of requirement. The design parameters are chosen to satisfy all the above three requirements. It was found that the normalized total performance is kept constant as long as the degree of transistor biasing is the same. The transistor dimension for the current mirror circuit employed for the feature detection sensor has been determined using the proposed method to achieve an error rate smaller than 0.0001 and the fall time smaller than 10 nsec .

### 4.5 References

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## Chapter 5 Implementation and experiments

Design parameters for current mirror circuits have been determined in the previous chapter using the newly proposed systematic design procedure. With these parameters, transient simulations are performed in this chapter. Based on this result, additional mechanisms for achieving faster operation are presented. Other implementation related issues including control circuits are also discussed. Then experimental results using the prototype sensor are presented in detail to demonstrate its functionality. The performance of the sensor is characterized by several important parameters concerning the mismatch and the operational speed.

### 5.1 Transient behavior of the circuit

In the previous chapter, the reference current and the dimension of transistors used for current sources have been determined using a systematic design procedure to satisfy given requirements for accuracy and speed. The speed estimate, however, is valid only for a simple current mirror consisting of two transistors. Hence, the overall operational speed has to be examined for the entire circuit, which contains several variations of current mirror circuit.

From a closer look of the circuit shown in Figure 3.9, it is obvious that the operational speed is mainly limited by the time required for charging and discharging node Nc. This is the most critical node determining the overall operational speed due to its large capacitance since thirteen PMOS gates are connected to this node. The charging and discharging behavior can be analyzed by considering this circuit as a flipped version of Figure 4.11 with additional eleven transistors (thirteen transistors in total). The load capacitance $C$ is estimated as 120 fF for thirteen PMOS transistors. Hence, the rise time and the fall time are calculated using eqns. (4.86) and (4.92) as


Figure 5.1 Expected evolution pattern when the line completion operation is applied for a single isolated point. See text for the specification for op1, op2, and op3 operations.

$$
\begin{equation*}
t_{r}=48.3(\mathrm{nsec}) \quad(\alpha=0.95), \tag{5.1}
\end{equation*}
$$

and

$$
\begin{equation*}
t_{f}=76.9(\mathrm{n} \mathrm{sec}) \quad(\varepsilon=0.05), \tag{5.2}
\end{equation*}
$$

respectively.
To confirm these estimated values and to investigate the dynamic behavior of the circuit, transient simulation was performed using the Hspice simulator. The line completion operation, which has been described in Section 2.6.4, was used in the simulation. The formulas (eqn. (2.42) to (2.48)) for the line completion are rewritten below to explicitly specify memories used for operation:

$$
\begin{align*}
& \left.o p 1 \text { (line elongation in } 0^{\circ}\right): \quad \mathrm{M}_{\mathrm{a}}=T_{L E a}\left(\mathrm{M}_{\mathrm{x}}\right)=T_{A_{\mathrm{a} a}^{\prime}, I_{3 a}^{\prime}}\left(\mathrm{M}_{\mathrm{x}}\right),  \tag{5.3}\\
& o p 2 \text { (linestop detection): } \quad \mathrm{M}_{\mathrm{y}}=T_{L S D 8}\left(\mathrm{M}_{\mathrm{a}}\right)=T_{A_{L S 88}, I_{L S D 8}}\left(\mathrm{M}_{\mathrm{a}}\right),  \tag{5.4}\\
& o p 3 \text { : (removal of the linestops) } \quad \mathrm{M}_{\mathrm{y}}=T_{E, 0.5}\left(\mathrm{M}_{\mathrm{a}}-\mathrm{M}_{\mathrm{y}}\right) \tag{5.5}
\end{align*}
$$

with

$$
\begin{align*}
A_{3 a}^{1}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 0
\end{array}\right), \quad A_{L S D 8} & =\left(\begin{array}{ccc}
-1 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & -1
\end{array}\right), \quad E=\left(\begin{array}{lll}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{array}\right),  \tag{5.6}\\
I_{3 a}^{1} & =I_{L S D 8}=0.5 . \tag{5.7}
\end{align*}
$$

Figure 5.1 shows the expected evolution pattern when the line completion operation is applied for a single isolated point. Figure 5.2 shows the simulation results obtained at the clock


Figure 5.2 Simulation results at the clock frequency of 2.5 MHz . The evolution pattern is shown on the left. The top two waveforms correspond to a two-phase clock ( $\phi_{1}$ and $\phi_{2}$ ). The third, fourth, and fifth waveforms represent the critical node voltage $V_{c}$, the current output of the pixel $I_{c}$, and the voltage at the thresholding node $\mathrm{V}_{\mathrm{o}}$, respectively.
frequency of 2.5 MHz . The evolution pattern shown on the left-hand side in Figure 5.2, which is reconstructed from the voltage stored in the specified memory after each operation, is identical to the one shown in Figure 5.1, indicating the correct operation at this frequency. The waveforms on the right-hand side show the voltages and the current at the important nodes in the center pixel. It should be noted that the voltage stored in the specified memory corresponds to the voltage $\mathrm{V}_{\mathrm{o}}$ at the thresholding node sampled when $\phi_{1}$ goes down. The rise time and the fall time are read as 50 nsec and 86 nsec , respectively, which are in good agreement with the predicted values in eqns. (5.1) and (5.2).

Figure 5.3 shows the simulation result at the clock frequency of 5 MHz . An error is


Figure 5.3 Simulation results at the clock frequency of 5 MHz . An error is observed at the center pixel during op 2 operation. The center pixel is still logic High although it should be classified as logic Low. The voltages and current on the right-hand waveforms represents those at the center pixel.
observed at the center pixel for operation op2. The center pixel is classified as logic High although it should be classified as logic Low. This is because $\mathrm{V}_{\mathrm{o}}$ is sampled before it decreases low enough. The reason of the upward and the subsequent downward movement of the voltage $\mathrm{V}_{\mathrm{o}}$ during $\phi_{1}$ phase is that the discharging of the thresholding node by the neighborhood pixels lags the charging by the pixel itself.

Therefore, it is important to accelerate the process of charging and discharging of the critical node so that the current output from each pixel quickly reaches the steady state. In the next section, two mechanisms employed in the current design, one is for the quick charging and discharging of the critical node and the other for stabilizing a thresholding node voltage, are


Figure 5.4 Pixel circuit schematic modified to accelerate the process of charging and discharging for the critical node $\mathrm{N}_{\mathrm{c}}$.
explained.

### 5.2 Mechanisms for high speed operation

### 5.2.1 High speed charging/discharging mechanism

In order to speed up the process of the charging and discharging of the critical node Nc , an additional current source and a simple control circuit are implemented as shown in the shaded area in Figure 5.4. The key idea of this modification is to try to always keep the critical node voltage around 1.6 V , corresponding to $I_{r e f}=4 \mu \mathrm{~A}$, by discharging the critical node during $\phi_{2}$ phase. This is achieved by the discharging of node Nc by the current whose amount is $I_{r e f}$ through switch $\mathrm{M}_{4}$. To further speed up the process of discharging, an additional current of $2 I_{\text {ref }}$


Figure 5.5 Simulation results at the clock frequency of 5 MHz using the modified circuit incorporating the high speed charging/discharging mechanism. All the operations were executed correctly. The voltages and the current on the right-hand waveforms represent those at the center pixel.
is used until the current $I_{c}$ reaches $0.75 I_{\text {ref }}(=3 \mu \mathrm{~A})$. This process of conditional discharging is controlled by properly turning transistor $\mathrm{M}_{6}$ on or off: $\mathrm{M}_{6}$ is turned on when $\mathrm{I}_{\mathrm{c}}$ is smaller than $0.75 I_{\text {ref }}\left(\overline{\mathrm{V}_{\text {comp }}}:\right.$ High $)$ and turned off when $\mathrm{I}_{\mathrm{c}}$ is larger than $0.75 I_{\text {ref }}\left(\overline{\mathrm{V}_{\text {comp }}}:\right.$ Low $)$

The action explained above sets the node voltage $\mathrm{V}_{\mathrm{c}}$ to around 1.6 V , and gets transistor $\mathrm{M}_{1}$ to be a "ready" state for starting current distribution. If the content of the memory specified for the operation is logic High, the voltage $\mathrm{V}_{\mathrm{c}}$ does not have to change and the current distribution immediately occurs when $\phi_{2}$ goes Low and the switches in the distribution path get turned on. This is in contrast to the previous design in which transistor $\mathrm{M}_{1}$ always gets turned off during $\phi_{2}$ phase and hence some time to initiate current distribution is needed at the onset of
$\phi_{1}$ phase.
However, if the content of the memory is logic Low, transistor $\mathrm{M}_{1}$ has to be quickly turned off so that current distribution does not happen. The previously explained current comparison scheme for conditional discharging works also for this purpose. As the node voltage $V_{c}$ goes up, the current $I_{c}$ decreases, slowing down the process of turning transistor $M_{1}$ off. When the current $\mathrm{I}_{\mathrm{C}}$ becomes smaller than $0.75 I_{\text {ref }}(=3 \mu \mathrm{~A})$, the voltage at the comparison node is brought down ( $\mathrm{V}_{\text {comp }}$ : Low), which turns PMOS transistor $\mathrm{M}_{10}$ on. This action connects node $N_{c}$ to $V_{D D}$, since $M_{11}$ is turned on during the operation phase ( $\phi_{2}$ is low), leading to the quick voltage rise at $\mathrm{N}_{\mathrm{c}}$, which turns off transistor $\mathrm{M}_{1}$. The same action takes place for turning off transistor $\mathrm{M}_{12}$, which controls the voltage of transistors for generating sinking currents. Transistor $\mathrm{M}_{13}$ gets turned on when $\mathrm{I}_{\mathrm{c}}$ becomes smaller than $0.75 I_{\text {ref }}$ to quickly discharge the gate of transistor $\mathrm{M}_{12}$.

Figure 5.5 shows the effect of the additional charging/discharging mechanism on the operational speed. All the operations were successfully executed. The discharging action of the voltage at node $\mathrm{N}_{\mathrm{c}}$, represented as $\mathrm{V}_{\mathrm{c}}(2,2)$, takes place quickly to set the current level to $I_{\text {ref }}$ at the start of $\phi_{2}$ phase. The reason of the voltage dip and the corresponding current overshoot at around $t=180$ [nsec] is due to the delay between the time when $\mathrm{I}_{\mathrm{c}}$ is equal to $0.75 I_{\text {ref }}$ and the time when transistor $\mathrm{M}_{6}$ really gets turned off. The voltage of the comparison node starts rising at the very time when $I_{c}$ is equal to $0.75 I_{\text {ref }}$, and it takes about 10 nsec for this node voltage to rise high enough to set $\mathrm{V}_{\text {comp }}$ to logic Low to turn off transistor $\mathrm{M}_{6}$. The overshoot phenomenon can be suppressed in future designs by implementing an additional mechanism for an adjustable threshold current. The modified circuit also works nicely for quickly charging node Nc, resulting in the consequent current decrease at around $t=670[\mathrm{nsec}]$. The current decreases to zero in about 30 nsec , which is much faster than the gradually decreasing behavior found in


Figure 5.6 Simulation results at the clock frequency of 10 MHz using the modified circuit incorporating the high speed charging/discharging mechanism. An error was observed for opl operation at the pixel $(1,2)$. The voltages and the current on the right-hand waveforms represent those at the pixel $(1,2)$.

Figure 5.3.
The high speed charging/discharging mechanism fails at the frequency of 10 MHz as shown in Figure 5.6. The failure occurs at the pixel $(1,2)$ and pixel $(3,2)$ instead of the center pixel $(2,2)$ : the pixel output is high after opl operation. The voltage profile at the thresholding node $\mathrm{V}_{\mathrm{o}}(1,2)$ explains the mechanism of the error. The voltage $\mathrm{V}_{\mathrm{o}}$ is initially set to logic High at the onset of $\phi_{1}$ phase. Note that at this point all the pixels including pixels $(1,1),(1,2)$, and $(1,3)$ are set to a "ready" state for current distribution by the previously described high speed charging mechanism. All these pixels contribute to the positive current flow to the thresholding node at pixel $(1,2)$ since the template for opl operation works for horizontal enhancement. Then, the current $I_{c}$ at these pixels decreases to zero by the high speed discharging mechanism


Figure 5.7 Pixel circuit incorporating the second mechanism for high speed operation. The dark shaded block is implemented to keep the voltage of the thresholding node constant before starting the current mode processing.
around 30 nsec after the onset of $\phi_{1}$ phase, which leads to the discharge of the thresholding node $\mathrm{V}_{\mathrm{o}}(1,2)$. When the voltage is sampled at the time of transition from $\phi_{1}$ to $\phi_{2}$, the voltage $\mathrm{V}_{\mathrm{o}}(1,2)$ is still in the middle of the decreasing process and has not reached low enough. Generally speaking, the voltage $\mathrm{V}_{0}$ at the onset of $\phi_{1}$ phase is determined by the type of operation in the previous operational cycle. There is no control mechanism to fix this voltage to a certain value.

### 5.2.2 Node locking mechanism

The observation described above leads to the second mechanism for high-speed operation shown in Figure 5.7. The dark shaded block, which is a self feedback inverter with two switches, keeps the voltage at the thresholding node constant, i.e., the switching voltage where


Figure 5.8 Simulation results at the clock frequency of 10 MHz using the circuit shown in Figure 5.7 with the delay between $\phi_{2 \mathrm{~d}}$ and $\phi_{2}$ set to 30 nsec . All the operations are executed correctly. The voltages and the current shown on the right-hand side represent those at the pixel $(1,2)$.
the input and output of the inverter is the same. The inverter is designed to have a switching voltage of 1.15 V . The voltage is one half of 2.3 V , which appears at the output of an NMOS switch after transfer of logic High $\left(=\mathrm{V}_{\mathrm{DD}}\right)$ signal. The voltage is kept constant during $\phi_{2}$ phase and an extended time period by an additional control signal $\phi_{2 d}$, which is a delayed version of $\phi_{2}$. By this mechanism, the voltage can go up or go down from the initial voltage of 1.15 V depending on the state of the pixel and its neighbors. Just a small change in the voltage drives the next inverter INV1 to either logic High or Low and hence high-speed operation is expected.

The additional duration control by signal $\phi_{2 d}$ is to ensure that all the transistors sourcing a current to or sinking a current from the thresholding node reach the steady state. At the end of $\phi_{2 d}$, the state of all transistors used as current sources should be completely fixed, indicating that


Figure 5.9 Simulation results at the clock frequency of 10 MHz using the circuit shown in Figure 5.7 with the delay between $\phi_{2 d}$ and $\phi_{2}$ set to 5 nsec . An error was observed for opl operation at the pixel $(1,2)$. The voltages and the current shown on the right-hand side represent those at the pixel $(1,2)$.
the thresholding node can be set free to start current-mode operation. Figure 5.8 shows the simulation result with the modified circuit at the frequency of 10 MHz . The delay between $\phi_{2 \mathrm{~d}}$ and $\phi_{2}$ was chosen as 30 nsec , which is the required time for transistor $\mathrm{M}_{1}$ to get turned off completely as explained before (Figure 5.5). All the operations are executed correctly. The node voltage at the thresholding node $\mathrm{V}_{\mathrm{o}}$ is kept at 1.15 V until $\phi_{2 \mathrm{~d}}$ gets low. Immediately after $\phi_{2 \mathrm{~d}}$ gets low, the voltage $\mathrm{V}_{\mathrm{o}}$ decreases during opl operation to produce the correct output.

Effect of the improper selection of the delay on the operation is shown in Figure 5.9, where the delay is chosen as 5 nsec . The operation result is incorrect due to the insufficient delay. Although the initial voltage at the thresholding node is 1.15 V , the voltage starts rising,


Figure 5.10 Modified timing diagram incorporating the delay signal $\phi_{2 d}$. When $\phi_{2}$ is logic Low, the operation is performed; when $\phi_{2}$ is logic High, the precharging of the critical node is performed. The thresholding node is kept to $\mathrm{V}_{\mathrm{DD}} / 2$ while $\phi_{\mathrm{d}}$ and $\phi_{2}$ are both high.
which is a wrong direction, immediately after the node is released at around $t=140$ [nsec], resulting in the incorrect sampling result.

To summarize this section, two mechanisms are implemented to realize high speed operation. The first mechanism enables fast charging and discharging of the critical node by properly controlling additional current sources. The second mechanism keeps the thresholding node voltage to $\mathrm{V}_{\mathrm{DD}} / 2$ until transistors used as current sources get ready for current distribution. With these two mechanisms, the operation up to 10 MHz was confirmed by simulation.

### 5.3 Control Circuits

The two mechanisms for achieving high-speed operation at each pixel was described in


Figure 5.11 Schematic of the timing control circuit.
the previous section in detail. In this section, control circuits that govern the operation of the entire pixels are described based on the modified timing control scheme. Figure 5.10 shows the timing control diagram, which incorporates the additional clock $\phi_{2 \mathrm{~d}}$ and associated changes in other control signals. The control signals were gated with $\overline{\phi_{2} \phi_{2 \mathrm{~d}}}$ to produce truncated control signals (CS1). The new control signals CS1 have the following features. First, the onset of the signals CS1 is synchronized with the time when the operation phase is initiated ( $\phi_{2}$ goes Low). Second, the control signal should be kept fixed for a while after $\phi_{2}$ goes High. During this extended time, the processing result stored in the intermediate node is transferred to the memory. The new timing signals are generated with a set of circuit modules shown in Figure 5.11. Each module constituting the control circuit is explained below.

### 5.3.1 Two-phase clock generator

The two-phase clock generator is the core part of the timing control circuit, which governs the overall operation of the entire chip. The circuit schematic is shown in Figure 5.12. It takes input CLK and generates non-overlapping clocks $\phi_{1}$ and $\phi_{2}$. The circuit also ensures


Figure 5.12 Schematic of the two-phase clock generator.
that both $\phi_{1}$ and $\phi_{2}$ do not become logic High at the same time by self-feedback mechanism. For example, if $\phi_{1}$ is logic High, the NOR gate (NOR1) forces the output $\phi_{2}$ to logic Low, and vice versa. A set of strong inverters (INV3 and INV5) are used at the output stage to drive a large number of signal lines running all through the entire chip and gate capacitance connected to them. The inverter is designed to drive a load capacitance of 10 pF , which is an estimated value for the accumulated load.

### 5.3.2 Programmable delay module

From the simulation results in the previous section, the delay of $\phi_{2 d}$ with respect to $\phi_{2}$ should be around 30 nsec, which corresponds to the time for the current sourcing PMOS transistors to get turned off. This estimate, however, may be altered by some unexpected reasons in real experiments. Therefore, a delay module is designed to produce various delays according to the external settings. The circuit is shown in Figure 5.13. It consists of a series of seven delay units, eight transmission gates, and a multiplexer. The delay unit consists of two


Figure 5.13 Circuit diagram of the programmable delay generator. One of the multiplexer outputs is set to logic Low to turn on the transmission gate and hence to connect the output of the selected delay unit to the OUT terminal. Each delay unit consists of two inverters.
cascaded inverters. The multiplexer sets one of its eight outputs to logic Low, turning on the transmission gate to connect one of the delay module outputs to the OUT terminal. The transmission gate employs both PMOS and NMOS transistors to ensure complete signal transfer for both $\mathrm{V}_{\mathrm{DD}}$ and GND. The delay unit is designed to produce a delay of around 5 nsec . This will give the maximum delay of 35 nsec from the final delay unit.

The dimensions for the PMOS and the NMOS transistors are chosen as $(W / L)=(4.5 / 3.6)$ and $(W / L)=(1.5 / 3.6)$, respectively, based on the following procedure. Let the channel length of the NMOS and the PMOS transistors be the same. The ratio of the channel width of the PMOS transistor to that of the NMOS transistor is chosen as three to compensate the difference in the driving capability, which should result in the same rise time and the fall time. The channel width does not significantly affect the speed since the increase in the channel width results both in the increase in the driving capability and the increase in the load capacitance, which is


Figure 5.14 Schematic of the scanning circuit. The pixel is connected to the output through two switches: the black pixel is connected to the output through a thick line above. The switches are controlled by the output of the shift register, which transfers logic Low. Scanning is carried out in an autonomous way by automatically generating logic Low when the outputs of all shift registers become logic High.
actually the capacitance of the next stage inverter. These two variables, the driving capability and the capacitance, cancel each other unless $W$ is not very large when the contribution of the drain-bulk capacitance of the driving inverter becomes significant to produce a larger delay than expected. Therefore, the channel width of the NMOS transistor is chosen as the minimum dimension of the analog design, which is $5 \lambda=1.5 \mu \mathrm{~m}$. To obtain the same driving capability, the channel width of the PMOS transistor is chosen as $4.5 \mu \mathrm{~m}$. The channel length was determined to be $3.6 \mu \mathrm{~m}$ by simulation to produce a delay of about 5 nsec per delay unit.

### 5.3.3 Scanner module

To visualize the processing result, the state of each pixel (logic High or Low) has to be


Figure 5.15 Schematic of the shift register. clk1 and clk2 are non-overlapping two phase clocks. Data transfer occurs when Reset is logic Low and Set ${ }^{\wedge}$ is logic High. Actions for other switch settings are described in the table.
read off-chip by the scanning circuit. Figure 5.14 shows the schematic of the scanning circuit, which is based on the design by Mead and Delbrück [1]. The scanner circuit consists of a series of shift registers placed in both horizontal and vertical directions. The shift register transfers logic Low to the next shift register at every clock (Clk). When transfer is complete and no logic Low exists as an output of shift registers, the NAND circuit automatically generates logic Low as an input for the first shift register to repeat the process of data transfer. The synchronization between $x$-direction and $y$-direction is automatically established by generating the driving clock for $y$-direction based on the data transfer in $x$-direction.

A pixel is connected to the output when logic Low is present in the corresponding positions in shift registers in both $x$ - and $y$-directions. Then, the current (designated as $\mathrm{I}_{\mathrm{c}}$ in Figure 3.9) flows to the output if the pixel has value 1, i.e., the content of the specified memory is logic High. The autonomous scanning mechanism connects pixels in a sequential fashion and generates a sequence of current outputs, which changes in a stepwise function. The current output is converted to a voltage by an Opamp to be read into a PC for further analysis. Signals

Newscan and Trigger are used for synchronization with the data acquisition system. Signal Newscan produces a High pulse, indicating the beginning of one frame, followed by a sequence of Trigger pulses. The data sampling is performed at the rising edge of Trigger pulses.

The schematic of the shift register is shown in Figure 5.15. It is a modified version of a basic configuration that consists of two semi-static memories connected in a series. Suppose for the moment that Set^ is logic High and Reset is Low. In this case, the second half of the circuit reduces to the normal semi-static memory. A pair of clock (clk1, clk2), where clk1 and clk2 are mutually non-overlapping clocks, transfers data from IN to OUT.

The output can be forced to logic High or logic Low by a combination of Reset and Set^ signals. When both Reset and Set^ are logic Low, signal OUT is logic High. When Reset is logic High, signal OUT is logic Low regardless of the value of Set^. This resetting function, which forces the output of all shift registers to logic Low, selects all pixels (remember that logic Low is an active signal for the selection of a pixel) and hence the current from each pixel is summed at the output. Therefore, the output current is proportional to the number of pixels of value 1. This type of output is useful when the position of pixels of value 1 is not important and the number of these pixels is important instead.

### 5.4 Layout and chip fabrication

After the entire circuit operation was confirmed by the Hspice simulator, all the circuits were laid out. Figure 5.16 shows the layout of the pixel. Care was taken to make the pixel shape square-like to have an equal number of pixels in both $x$ - and $y$-directions. The pixel size measures $154.5 \mu \mathrm{~m} \times 153.3 \mu \mathrm{~m}$. The phototransistor, which is implemented at the top left corner of the pixel, measures $99.6 \mu \mathrm{~m} \times 29.7 \mu \mathrm{~m}$ (base area) and accounts for $12.5 \%$ of the entire pixel area. The transistor accounts for $32.4 \%$ of the pixel area. The rest of the pixel, which consumes more than half the entire area, was used for the common signal lines, which


Figure 5.16 Layout of the pixel.
were laid out both horizontally and vertically. The first and the second metal layers were used for signal lines while the third metal was used for shielding the circuit from light illumination.

An array of $16 \times 16$ pixels were placed on a chip area of $3.2 \mathrm{~mm} \times 3.2 \mathrm{~mm}$. Figure 5.17 shows the layout of the entire chip. Based on this layout, the sensor was fabricaed using HP $0.5 \mu \mathrm{~m}$ technology through MOSIS.

### 5.5 Experiments

This section describes several experimental results obtained using the fabricated sensor. A testing environment is presented first. Experimental results concerning each circuit module are presented next in the following order: phototransduction circuit; programmable delay module, and scanner module. Then experimental results concerning the functionality of the


Figure 5.17 Layout of the entire chip.
entire sensor chip are presented. These results include the measurement results of the important parameters characterizing the sensor performance such as mismatch proportionality constant, maximum operating frequency, and total execution time. Several problems concerning the memory configuration are also mentioned. Finally, the sensor responses to various letter images are shown to demonstrate the functionality of the sensor.

### 5.5.1 Testing environment

The fabricated sensor was mounted on a test board. The board determines several operating conditions. These conditions include the amount of the reference current, the delay generated by the programmable delay module ( 3 bits), the amount of the threshold current (4 bits). The board also serves as an interface between the test chip and a PC equipped with a digital I/O board and an A/D converter. The connection schematic is shown in Figure 5.18. All the control signals (31 bits) are provided by the digital I/O board (PCI-DIO-32HS, National


Figure 5.18 Schematic of the connection between the test board which contains the test chip and the PC equipped with a digital I/O board and an A/D converter.

Semiconductor). The output current from the sensor $\left(\mathrm{I}_{\text {out }}\right)$ is converted into a voltage by an Opamp installed on the board. The voltage is then converted to a digital representation by an A/D converter (PCI-MIO-16E-4, National Semiconductor) to be read into the PC for further analysis. The control of the test chip with these two boards was carried out by Labview (National Semiconductor) software.

### 5.5.2 Phototransduction

Various images were projected on the test chip through a lens mounted above the chip. The amount of the threshold current ( $\mathrm{I}_{\text {out }}$ in Figure 3.7) for image digitization was measured. When the input current $\mathrm{I}_{\mathrm{IN}}$ is about $1.5 \mu \mathrm{~A}$, $\mathrm{I}_{\text {Out }}$ was $27.6 \mathrm{nA}, 7.1 \mathrm{nA}, 2.8 \mathrm{nA}$, and 1.1 nA , for the selection of $\mathrm{SW}_{\mathrm{a}}, \mathrm{SW}_{\mathrm{b}}, \mathrm{SW}_{\mathrm{c}}$, and $\mathrm{SW}_{\mathrm{d}}$, respectively. The measured current is smaller than the expected current shown in Figure 3.8; the discrepancy is larger for smaller current settings. Even when the smallest threshold current of 1.1 nA was selected, the result of image digitization was all value 0 , indicating that the photogenerated current was below 1.1 nA . This result indicates that the estimate of 4.8 nA (Section 3.2.2) for a photogenerated current was too high


Figure 5.19 Measurement result of the delay of $\phi_{2 \mathrm{~d}}$ with respect to $\phi_{2}$ for two power supply voltages $\left(\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}\right.$ and $\left.\mathrm{V}_{\mathrm{DD}}=3.3 \mathrm{~V}\right)$.
possibly because the amount the incident light from an object is over-estimated $\left(E=0.1\left[\mathrm{~W} / \mathrm{m}^{2}\right]\right.$ in Section 3.1.1). To increase the amount of the photogenerated current above 1.1 nA , an additional light source (halogen lamp) was used to illuminate the image. With this additional light source, an image was successfully digitized.

### 5.5.3 Programmable delay module

Figure 5.19 shows the measurement result of the delay between $\phi_{2}$ and $\phi_{2 d}$ for different number of delay units. The delay was measured as the time difference between $\phi_{2}$ and $\phi_{2 d}$. The measurement was performed for $\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}$ in addition to $\mathrm{V}_{\mathrm{DD}}=3.3 \mathrm{~V}$ for the reasons described below. In both cases, the delay increased almost linearly as a function of the delay units. For $\mathrm{V}_{\mathrm{DD}}=3.3 \mathrm{~V}$, the measured delay was slightly larger than the one obtained by simulation. The delay decreased for $\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}$ due to increased driving capability of each inverter constituting the delay module.

### 5.5.4 Scanner module

The scanner circuit was operating correctly. Synchronization signals Newscan and Trigger were correctly generated up to the clock frequency of 2 MHz , which was the maximum frequency of the signal generator used in the experiment. However, A/D conversion was carried out at a frequency of 35 kHz due to the limitation of the $\mathrm{A} / \mathrm{D}$ board.

### 5.5.5 Problems associated with memory configuration

During the course of the experiment, it became apparent that there are several problems associated with the memory configuration. First of all, six memories are not enough to compute all the five features of interest, T-type junction (JCT-T); X-type junction (JCT-X); Y-type junction (JCT-Y); corners (trueC); linestops (trueLS), in one sequence of operations. This is due to the shortage of the memory for temporal storage of the computation result. An additional memory would be required to compute and store all these five features in one sequence of operations. If both JCT-Y and trueC are treated as one feature (totalC in Figure 2.7), which was the situation at the time of chip submission ${ }^{10}$, six memories are sufficient. For this reason, the testing was performed by applying five separate sequences of operations, each corresponding to one of the five features.

The second problem was an unstable operation: the result of feature detection varied depending on the instruction and the operational frequency. It was observed that the phenomena of this unstable operation happened less and less frequently as $\mathrm{V}_{\mathrm{DD}}$ was increased, and completely disappeared at $\mathrm{V}_{\mathrm{DD}}=4[\mathrm{~V}]$. This observation supports the idea that the unstable operation is probably due to the undesirable voltage drop associated with NMOS switches. The possible mechanism is explained using Figure 5.20, which is an extracted version from Figure 3.9 focusing on the memory configuration. When logic High $\left(=\mathrm{V}_{\mathrm{DD}}\right)$ passes through an NMOS

[^8]

Figure 5.20 Schematic showing the signal transfer from the thresholding node to the memory. This figure is an extracted version from Figure 3.9 focusing on the memory configuration.
switch ( $\mathrm{M}_{4 \mathrm{a}}$, for example), a voltage drop of about 1 V occurs, decreasing the level of logic High from 3.3 V to 2.3 V for $\mathrm{V}_{4 \mathrm{a}}$. This voltage further decreases when $\mathrm{M}_{4 \mathrm{a}}$ gets turned off due to the inflow of electrons expelled from under the gate. The reduced voltage $\mathrm{V}_{4 \mathrm{a}}$ is transferred into the memory. If the voltage at the input of the inverter becomes below 1.15 V , which is the threshold voltage of the inverter, an erroneous result is produced. The problem of the voltage drop can be avoided in future designs by putting a PMOS transistor in parallel with a NMOS transistor for the switch.

The third problem is related to the use of y 2 . Consider the following case: the content of $M_{a}$ is used for a certain operation and the operational result $\left(V_{0}\right.$ or equivalently $\left.V_{2}\right)$ is saved to memory $\mathrm{M}_{\mathrm{y}}$ by setting y 2 to logic High. The content of memory $\mathrm{M}_{\mathrm{a}}$ should not be altered by this action. This is achieved by asserting $y 2$, which disconnects $M_{a}$ from voltage $V_{2}$. However in reality, memory $M_{a}$ is updated with $V_{3}$, which is the previous operation result, since al is
logic High. There is no guarantee that the logic value of $\mathrm{V}_{3}$ is equal to the content of $\mathrm{M}_{\mathrm{a}}$. This problem is solved by performing a dummy operation before the intended operation. The dummy operation reads the content and writes it back to memory $\mathrm{M}_{\mathrm{a}}$ by asserting signals a1 and RE (shown in Figure 3.9), automatically presetting $\mathrm{V}_{3}$ to the logic value in memory $\mathrm{M}_{\mathrm{a}}$.

An even more serious problem associated with the use of y 2 was signal mixing when multiple memories are selected as the source of the operation. One of the examples of this type of operation is the OR operation:

$$
\begin{equation*}
\mathrm{M}_{\mathrm{y}}=T_{E, 0.5}\left(\mathrm{M}_{\mathrm{a}}+\mathrm{M}_{\mathrm{b}}+\mathrm{M}_{\mathrm{c}}+\mathrm{M}_{\mathrm{d}}\right) . \tag{5.8}
\end{equation*}
$$

The voltages $\mathrm{V}_{4 \mathrm{a}}, \mathrm{V}_{4 \mathrm{~b}}, \mathrm{~V}_{4 \mathrm{c}}, \mathrm{V}_{4 \mathrm{~d}}$, corresponding to the memory content, get mixed with each other to generate a new voltage $\mathrm{V}_{3}$, which is then written into all these four memories. The signal mixing occurs due to the presence of the common node for the four memories. The above mentioned method of presetting $V_{3}$ is effective only when a single memory is addressed and cannot be used for the case of addressing multiple memories. To get around this problem, operations addressing multiple memories were executed in several steps, addressing a single memory in each step, using $\mathrm{M}_{\mathrm{x}}$ as a temporary memory.

Although the problems described above concerning the memory configuration necessitated the use of $\mathrm{V}_{\mathrm{DD}}=4[\mathrm{~V}]$ and slight modification for the operational sequence, they were not considered fatal to prevent the evaluation of the fabricated sensor.

### 5.5.6 Pattern generation

For the evaluation of the sensor performance, capability of generating an arbitrary image pattern by some means is desirable since optically generating an arbitrary image is not a very flexible method. The ideal method would be to edit an image in a PC and load it into the sensor. However, the present sensor does not have this function and hence the alternative method was implemented.


Figure 5.21 Method of generating a grid pattern. It is explained for a small matrix of $7 \times 7$ instead of $16 \times 16$ for simplicity. The operation necessary for image transformation is shown below the arrow.

Figure 5.21 explains the method for generating a grid pattern. The procedure starts with image digitization, in which the digitized result is stored into memory $\mathrm{M}_{\mathrm{x}}$. The only restriction for the image is that at least one pixel is under bright illumination to be classified as a pixel of value 1 (shown black in the figure). Then the digitized image is expanded into eight neighbors (exp8Mx, eqns. (2.88) and (2.89)) as many as six times to fill the entire pixel array. Once the entire array is filled, the top left point is detected (topleftMx) using the following template and the threshold:

$$
A=\left(\begin{array}{ccc}
-1 & -1 & 0  \tag{5.9}\\
-1 & 2 & 0 \\
0 & 0 & 0
\end{array}\right), \quad I=1.5 .
$$

Then the pixel is shifted toward east ( $\operatorname{shiftM}_{\mathrm{x}_{-}} \mathrm{E}$ ) and subsequently toward south ( $\operatorname{shift}_{\mathrm{x}_{-}} \mathrm{S}$ ). The shifting is performed in three steps: (1) copy $M_{x}$ to $M_{y}$; (2) extend $M_{x}$ to the specified direction; (3) suppress $M_{x}$ by $M_{y}$. The content of memory $M_{x}$ is copied to $M_{a}\left(M_{x} T o M_{a}\right)$. Then
shifting $\mathrm{M}_{\mathrm{x}}$ toward east twice followed by taking OR of $\mathrm{M}_{\mathrm{x}}$ and $\mathrm{M}_{\mathrm{a}}\left(\mathrm{ORM}_{\mathrm{ax}} \mathrm{ToM}_{\mathrm{ax}}\right)$ copies the pixel of value 1 to the position two pixels east of the present position. This operation is performed several times to place pixels of value 1 in every other pixel position in the second row. The same procedure is applied for generating the same row image in every other position below the second row to finally obtain the grid image.

As is obvious in the above explanation, this method is not able to generate any arbitrary pattern. The pattern is a repetition of a fundamental pattern, resulting in a periodical structure in both $x$ - and $y$-directions. Although the variety of the pattern is limited, it is still useful for the evaluation of the basic performance of the sensor, which is presented below.

### 5.5.7 Mismatch measurement

The degree of transistor mismatch was estimated by measuring the current output from all pixels, which are initially set to logic High. The current output ( $\mathrm{I}_{\mathrm{C}}$ in Figure 3.9) should be ideally equal to the reference current $I_{r e}$. However, the output varies from pixel to pixel due to the effect of transistor mismatch. The measurement was performed for different values of $I_{r e f}$, i.e., $0.5 \mu \mathrm{~A}, 1 \mu \mathrm{~A}, 2 \mu \mathrm{~A}, 4 \mu \mathrm{~A}$, and $8 \mu \mathrm{~A}$. For each measurement, the mean and the variance of the 256 output currents were calculated. Based on the measurement result, the relative variation of the output current was plotted as a function of the reference current $I_{\text {ref }}$, which is shown in Figure 5.22. The measurement result demonstrates that the relative current variation decreases as the reference current becomes larger. This behavior is nicely fit by the following equation:

$$
\begin{equation*}
\left(\frac{\sigma_{I_{C}}}{I_{r e f}}\right)^{2}=\frac{0.00235}{I_{r e f}} . \tag{5.10}
\end{equation*}
$$

Since $I_{C}$ is obtained as a result of PMOS current mirroring, its variance is represented as

$$
\begin{equation*}
\sigma_{I_{C}}^{2}=3 s^{2} I_{r e f}^{2} \tag{5.11}
\end{equation*}
$$

with the help of eqn. (4.50). Substituting eqn. (5.11) into (5.10) yields


Figure 5.22 Relative variation of the current as a function of the reference current.

$$
\begin{equation*}
3 s^{2}=\frac{0.00235}{I_{r e f}} \tag{5.12}
\end{equation*}
$$

Expressing s in terms of design parameters (eqn. (4.31)) gives

$$
\begin{equation*}
3 \frac{2 A_{V_{T}}^{2} \mu C_{o x}}{L^{2} I_{r e f}}=\frac{0.00235}{I_{r e f}} . \tag{5.13}
\end{equation*}
$$

By substituting numerical values for corresponding parameters, the above equation can be solved for $A_{V_{T}}$ as

$$
\begin{equation*}
A_{V_{T}}=7.0 \quad[\mathrm{mV} \mu \mathrm{~m}] . \tag{5.14}
\end{equation*}
$$

This value is almost one half of the value assumed in the simulation ( $15 \mathrm{mV} \mu \mathrm{m}$ ). However, the assumed value was a rather conservative estimate, and the measured value of $A_{V_{T}}=7.0[\mathrm{mV} \mu \mathrm{m}]$ agrees well with the reported result shown in Table 4.1 extrapolated toward smaller feature sizes. (Recall that as the feature size decreases, the proportionality constant $A_{V_{T}}$ decreases.) As a result of this rather conservative estimate for $A_{V_{T}}$, the error should happen less frequently than expected. To investigate the effect of transistor mismatch on


Figure 5.23 Schematic explaining the detection error for the linestop detection (LSD) operation. The output image shown in the right-hand half is one of the possible output images for an input image shown in the left-hand half. In this case, 65 pixels out of 70 pixels are correctly detected as linestops, indicating the error rate of 5/70.
the error rate, the error rate was measured for different values of $I_{r e f}$, corresponding to different values of $s$. Figure 5.23 shows the test pattern on the left-hand side, which consists of a set of adjacent pixels of value 1 . This pattern is generated by a method similar to that described in the previous section. All these pixels of value 1 are linestops of each line segment of length 2 , and should be detected by the previously described linestop detection (LSD) operation (see eqn. (2.68)). In other words, the result of the operation is the same as the input image when no error occurs. When the detection error occurs, the result may look like the one shown on the righthand side in Figure 5.23.

Table 5.1 shows the expected and measured error rate for different values of $I_{r e f}$. The expected error rate is computed using the procedure introduced in Section 4.3 with the following distributions:

$$
\begin{gather*}
p_{1}(x)=N\left(I_{r e f}, 17 s^{2} I_{r e f}^{2}\right),  \tag{5.15}\\
p_{0}(x)=N\left(0,22 s^{2} I_{r e f}^{2}\right), \tag{5.16}
\end{gather*}
$$

and

Table 5.1 Expected and measured error rate for different values of $I_{r e}$.

| $I_{\text {ref }}(\mu \mathrm{A})$ | 0.2 | 0.25 | 0.3 | 0.35 |
| :---: | :---: | :---: | :---: | :---: |
| $s$ | 0.063 | 0.056 | 0.051 | 0.047 |
| error (\%) <br> (expected) | 2.7 | 1.5 | 0.9 | 0.5 |
| error (\%) <br> (measured) | 2.1 | 1.4 | 0.7 | 0 |

$$
\begin{equation*}
p_{0.5}(x)=N\left(0.5 I_{r e f},(1 / 8) s^{2} I_{r e f}^{2}\right) . \tag{5.17}
\end{equation*}
$$

where $p_{1}(x), p_{0}(x)$, and $p_{0.5}(x)$ represent the distribution of the current for a mean value of $I_{r e f}, 0$, $0.5 I_{\text {ref }}$, respectively. Note that the error corresponds to that in the right side, $p_{\text {right }}$, which misclassifies a pixel of value 1 as a pixel of value 0 . Errors were not observed when the reference current was larger than or equal to $0.35 \mu \mathrm{~A}$. The measured error rate was defined as the ratio of the number of pixels that are not detected as linestops to the total number of pixels of value 1. Expected and measured error rate agreed to each other reasonably well. This agreement validates the theory for predicting an error rate based on the mismatch parameter $A_{V_{T}}$. The dependence would have been more clearly observed with less accurate design.

Another experiment was performed using the test pattern that consists of a set of line segment whose length is three to investigate the occurrence of the other type of error. This type of error corresponds to that in the left-hand side, $p_{\text {left }}$, and misclassifies a pixel of value 0 as a pixel of value 1. Applying the LSD operation, it was found that the center pixel was always correctly deleted even with the smallest reference current ( $I_{\text {ref }}=0.2 \mu \mathrm{~A}$ ): no error was observed. This phenomenon is inconsistent with the expected result from the above distributions, which expects a higher error rate ( $p_{\text {left }}$ ) for the deletion of the center pixel than the detection of linestops $\left(p_{\text {right }}\right)$. One of the possible mechanisms explaining this result is stated
below.
Consider a chain of current mirror circuits: an input current is copied to an output current by a first pair of NMOS transistors; the output current is fed into a second pair of PMOS transistors for further copying; the output current is copied again by a third pair of NMOS transistors, and so on. When the current level decreases, the gate voltage that is common for input and output NMOS transistors becomes lower. The drain voltage of the input NMOS transistor, which is tied to the gate, becomes lower as well, while the drain voltage of the output NMOS transistor becomes higher since this node is connected to the gate voltage of PMOS transistors in the subsequent current mirror circuit. The above observation indicates that the difference between the drain and the gate voltage increases for smaller currents. Therefore, every time a current is copied, the output becomes larger than the input current. A multiplication factor is larger for smaller currents.

As a result of this action, for the convolution computation using current distribution, a current used for the negative weight (sinking current) is larger than the current for the positive weight (sourcing current) due to an extra current mirror operation. Therefore, the convolution result for the LSD operation for the pixel at the center of a line tends to become smaller than the expected value of $0(=2 \times 1-1 \times 2)$ as the amount of the reference current $I_{\text {ref }}$ becomes smaller. The percentage of this decrease is larger for smaller values of $I_{r e f}$. The threshold current, however, stays at $0.5 I_{\text {ref. }}$. Therefore, the pixel becomes easier to be deleted for smaller values of $I_{r e f}$, indicating a lower error rate for $p_{\text {left }}$. On the other hand, an error rate becomes higher for the detection of linestops $\left(p_{\text {right }}\right)$ since the convolution result also tends to become smaller than the expected value of $I_{\text {ref }}$, moving closer to the threshold of $0.5 I_{\text {ref }}$. The result shown in Table 5.1 may partly reflect this effect.

In order to prove the validity of the above mechanism for explaining a higher value of $p_{\text {left }}$


Figure 5.24 Maximum operating frequency as a function of the reference current for different settings of internal delay between $\phi_{2}$ and $\phi_{2 d}$. The experiments were carried out to find the minimum required current for certain frequencies. The maximum frequency was 5 MHz due to device limitation.
than that of $p_{\text {right }}$, and to investigate the relationship between the mismatch parameter and the error rate more accurately, a dedicated chip for mismatch analysis is needed. The chip is supposed to produce a reasonable number of errors with currents of $\mu \mathrm{A}$ order.

### 5.5.8 Speed measurement

The maximum operating frequency was investigated as a function of the reference current. The image shown in Figure 5.23 was used as a test pattern. The maximum operating frequency is defined as the frequency below which no detection error occurs (error rate of $0 \%$ ). Since the operating frequency can be set only to certain discrete values, a minimum required current for the correct operation was determined for each of these frequencies. This experiment was repeated for different settings of internal delay between $\phi_{2}$ and $\phi_{2 d}$.

Figure 5.24 shows the relationship between the maximum operating frequency and the


Figure 5.25 Variation of the operating frequency for different error rates. The experiment was conducted for the maximum delay of 35.4 nsec .
reference current for different settings of internal delay. This plot is obtained by representing the operating frequency as a function of the reference current. It is obvious from the graph that the maximum frequency almost linearly increases as a function of the reference current. Although the increase in the maximum frequency for larger reference currents is expected, the linear dependence does not completely agree well with the square root dependence expressed in eqn. (4.104). The reason is not very clear. It may be because the speed is largely determined by the speed of charging and discharging of the thresholding node, which is proportional to the reference current.

The effect of internal delay is demonstrated in the graph. As the delay increases, the maximum frequency increases. This behavior agrees with the simulation result shown in Figure 5.8 and Figure 5.9, which also indicates higher operating frequencies for larger delays. It should be noted that the degree of improvement of the speed becomes less and less as the delay increases, and seems to be reaching the point close to saturation when the delay is 35.4 nsec .


Figure 5.26 Maximum operating frequency for three types of operations. The experiment was conducted for the maximum delay of 35.4 nsec . The error for the Exp8 operation is defined as the generation of a pixel of value 1 by the application of this operation for an entire array of pixels of value 0 .

This observation indicates that the time for sourcing and sinking currents to decreases to 0 is around this value, which is close to the estimate of 30 nsec obtained in the simulation (Section 5.2). No further improvement in the operational speed would be expected for larger delays. On the contrary, larger values of delay, in turn, put a limit for the maximum operating frequency.

Figure 5.25 shows the variation of the operating frequency for different error rates. For a given reference current, the range of the maximum operating frequency for $0 \%$ to $100 \%$ error rate is more than 1 MHz . Note that this variation comes from mismatch in transistor characteristics. Note also that the experimental result cannot be directly compared to the simulation result shown in section 5.2 , which proved that the LSD operation at 10 MHz is possible.

The simulation did not take the effect of transistor mismatch into effect. Some pixels may operate correctly above 10 MHz , while other pixels may not operate correctly below 10

MHz , depending on the degree of mismatch within a $3 \times 3$ region including itself and neighbors. The other difference between the simulation and the experiment is the edge effect. The linestops detected in the simulation is located at the edge. Since three pixels are missing in its neighbors, the detection should be easier for pixels on the edge than those at the center ${ }^{11}$, which is actually the case for experiment.

The effect of the type of operations on the maximum operating frequency was also investigated. These operations include the LSD operation ( $T_{L S D 4}$ ) on the edge and the Exp8 operation $\left(T_{\text {expp }}\right)$. The error for the Exp8 operation is defined as the generation of pixels of value 1 by the application of this operation for an entire array of pixels of value 0 . Figure 5.26 shows the maximum operating frequency for these operations as well as the LSD operation at the center, which is already shown in Figure 5.24. The maximum frequency is lowest for the LSD operation at the center: this operation needs a largest current at a given frequency for correct operation. The maximum frequency for the LSD operation at the edge is higher than the LSD operation at the center. The maximum frequency for the Exp8 operation is significantly higher than these two LSD operations.

For the other type of error for the Exp8 operation, which produces pixels of value 0 in the $3 \times 3$ neighbors of a pixel of value 1 , the maximum operating frequency was even higher. It seems that the maximum frequency is higher for simpler operations and is lower for complex operations. It is the expected result since complex operations use more current sources, which can be either positive or negative, and hence take longer time to obtain the correct result. For the characterization of the sensor, the maximum operating frequency is defined as 5 MHz with the reference current of $4.5 \mu \mathrm{~A}$, using the LSD operation at the center as the worst case.

[^9]

Figure 5.27 Sensor responses to various letter images. For an input image shown on the top row, a set of features is obtained as shown in the bottom row. These features are superimposed on the thinned image. The center of a closed rectangle indicates the position of the detected feature.

### 5.5.9 Responses to letter images

Various letter images were projected on the feature detection sensor. The experiments were conducted under the following conditions: $\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}, I_{\text {ref }}=4.5 \mu \mathrm{~A}$, the delay between $\phi_{2}$ and $\phi_{2 \mathrm{~d}}$ set to 35.4 nsec , the operating frequency of 5 MHz . Figure 5.27 shows detected features by the sensor for five letter images. The images shown in the bottom row are reconstructed by superimposing each feature at the detected position on the thinned image. All the important features were detected in a discriminative fashion. Comparison of these results with the simulation result (as was done in Section 2.7) showed that the sensor response was completely identical to the one predicted by the simulation. Prediction by the simulation was carried out using a digitized image $\left(\mathrm{X}_{0}\right)$. Note that this process is necessary since the result of image digitization can be slightly different from the originally prepared image due to illumination inhomogeneity and edge effects. The identity between the detected results and the simulation demonstrates that the sensor is functioning as designed.

It was found that some of the results are confusing. For example, for the slanted letter " R ", two pixels are detected as linestops instead of corners. This is because the $45^{\circ}$ line

Table 5.2 Execution time required for each operation. The result of each processing is shown as $\mathrm{X}_{\mathrm{i}}$ in Figure 5.28. The processing time is estimated for the operation of 5 MHz .

| order | operation | resultant image | no. of steps | time (usec) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | binarize | X0 | 1 | 0.2 |
| 2 | thinning (1 cycle) |  | 24 | 4.8 |
| 3 | thinning (1 cycle) | X1 | 24 | 4.8 |
| 4 | OD | X2a, X2b, X2c, X2d, | 25 | 5 |
| 5 | LC |  | 26 | 5.2 |
| 6 | EIP |  | 20 | 4 |
| 7 | LI | X 5 a, $\mathrm{X}_{5 \mathrm{~b}}, \mathrm{X}_{5 c}, \mathrm{X}_{5 \mathrm{~d}}$ | 8 | 1.6 |
| 8 | EIP | X6a, $\mathrm{X}_{6 \mathrm{~b}, \mathrm{X}} \mathrm{c}$ c, $\mathrm{X}_{6 \mathrm{~d}}$ | 20 | 4 |
| 9 | LT |  | 2 | 0.4 |
| 10 | LE |  | 4 | 0.8 |
| 11 | LT | $\mathrm{X}_{7 \mathrm{a}}, \mathrm{X}_{7 \mathrm{~b}}, \mathrm{X}_{7 \mathrm{c}}, \mathrm{X}_{7 \mathrm{~d}}$ | 2 | 0.4 |
| 12 | AND2MabcdToMx | X10 | 29 | 5.8 |
| 13 | LSD | X8a, $\mathrm{X}_{8 \mathrm{~b}, \mathrm{X}} \mathrm{Xc}$, $\mathrm{X}_{8 \mathrm{~d}}$ | 4 | 0.8 |
| 14 | ORMabcdToMy |  | 8 | 1.6 |
| 15 | ORMxyToMy | X13 | 1 | 0.2 |
| 16 | rectangleMy |  | 4 | 0.8 |
| 17 | shrinkMy |  | 4 | 0.8 |
| 18 | connectMy |  | 12 | 2.4 |
| 19 | rectangleMy |  | 4 | 0.8 |
| 20 | rectangleMy | X14 | 4 | 0.8 |
| 21 | shrinkMy | X15 | 4 | 0.8 |
| 22 | LSD1 | X9a, \9b, $^{\text {X9c, }}$ X9d | 8 | 1.6 |
| 23 | ORMabcdToMy | X16 | 8 | 1.6 |
| 24 | AND2MabcdToMabcd | X17 | 1 | 0.2 |
| 25 | rectangleMx | X11 | 4 | 0.8 |
| 26 | shrinkMx | X12 | 4 | 0.8 |
| 27 | feature_calc | X18, X19, X20, X21, X22 | 16 | 3.2 |
|  | total |  | 271 | 54.2 |

segment constituting the loop is removed by the EIP operation due to its length being two pixels. This type of removal of short line segments also explains why only seven corners are detected instead of eight for the letter "O" and the unexpected linestops along the stroke for the letter " $S$ ". This problem originates from the low resolution of the sensor: an array of $16 \times 16$ is not large enough for some letters to apply the present feature detection algorithm. Hence, the problem would be solved if the larger number of pixels were used in future.

Table 5.2 shows the execution time for each operation used for the detection of the following four features: JCT-T, JCT-X, totalC (OR of trueC and JCT-Y), and trueLS. Note that trueC and JCT-Y are treated together as totalC in this example. Recall that due to lack of an


Figure 5.28 Detail of the processing flow for the letter "R".
additional memory as mentioned in Section 5.5.5, the five features cannot be computed in one sequence of operations. The operation listed in the table faithfully follows the processing described in Section 2.6. The number of steps required for each operation is converted to the execution time by multiplying $200 \mathrm{nsec} /$ operation for the operational frequency of 5 MHz . The thinning operation is performed twice although in reality it depends on the original thickness of the line.

The resultant image for each operation is shown in Figure 5.28, which is the detailed processing flow for the letter " R " in Figure 5.27, with the notation $X_{i}$ indicating the correspondence to the operation. The total execution time is about $50 \mu \mathrm{~s}$, which is pretty fast.

If the memory configuration is modified to solve the problems described in Section 5.5.5, the twelfth and the fourteenth operations (shaded region in the figure) can be executed in one step for each of them, which would further reduce the processing time.

### 5.5.10 Discussion

The fabricated feature detection sensor operated successfully with a fairly high speed. The operation was confirmed up to 5 MHz with the reference current of $4.5 \mu \mathrm{~A}$ and the power supply voltage of 4 V . With this setting, the sensor was able to detect features (junctions of three types, corners, and linestops) in a discriminative fashion for various types of letter images in about $50 \mu \mathrm{sec}$. The detection result was exactly the same as that predicted by simulation. The mechanism of high speed charging/discharging of the critical node was necessary as well as the mechanism of temporarily keeping the thresholding node voltage constant in order to achieve this operational speed. Since the maximum operating frequency is almost linearly dependent on the reference current, further improvement of the speed is possible by increasing the reference current within the region for transistors to operate in saturation.

The present sensor has a larger computational power than the one obtained by the sensor architecture using a single conventional signal processor. Suppose that the processor operates at the clock frequency of 1 GHz and that each arithmetic computation requires 1 nsec . Also, suppose that the weighted sum is computed in one clock cycle. These assumptions lead to the computation time for template matching for $3 \times 3$ neighbors of 9 nsec , which further lead to the total computation time of $9 N^{2}$ nsec for a pixel array of size $N \times N$. If $N$ is equal to 16 , which is the size of the present sensor, the computation time is $2.3 \mu \mathrm{sec}$. This is ten times larger than the operational speed of 200 nsec obtained from the fabricated sensor at the operating frequency of 5 MHz . The higher speed of the present sensor comes from the parallel computing due to the incorporation of the processing element per each pixel. The discrepancy between the two
schemes becomes even larger for larger $N$. The computational time required for a single processor architecture is proportional to $N^{2}$, while that required for the present sensor is independent of $N$.

The mismatch proportionality constant $A_{V_{T}}$ was estimated as 7 mV from the variation of the current output. One of the experimental results suggested the validity of the design procedure based on the mismatch analysis: the measured error rate was close to the theoretical prediction. Since the transistor dimension was determined based on the conservative estimate of $A_{V_{T}}$, the transistor channel length could be even reduced to satisfy the original accuracy requirement. The reduced channel length leads to an even faster operation.

The power consumption of the sensor was measured only as a DC current flowing out of the power supply to GND. The AC current was not measured due to the limitation of the measurement instrument. The current consumption depends on the number of current paths between $V_{D D}$ and GND, which is affected by the content of the memories and the specified operation. The current consumption was always below 20 mA under the normal operating conditions ( $\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}, I_{\text {ref }}=4.5 \mu \mathrm{~A}$ ). It was found that the DC current consumption consists of two parts: the constant current consumption independent of the reference current and the other one that is proportional to the reference current. The constant current is consumed by the inverter while keeping the thresholding node voltage constant during $\phi_{2}$ and $\phi_{2 d}$ phases. Although it is not clear how much AC current is consumed, it is reasonable to say that the total power consumption is around 100 mW .

Table 5.3 summarizes the specifications of the sensor. Employment of the mixed-mode approach enabled compact implementation of the pixel circuit. Use of technology of smaller feature sizes enables higher pixel densities in future versions. If the $0.35 \mu \mathrm{~m}$ technology is used, the pixel size reduces to $100 \mu \mathrm{~m} \times 100 \mu \mathrm{~m}$, enabling an array of $64 \times 64$ pixels in a chip

Table 5.3 Specifications of the prototype chip.

| technology | HP $0.5 \mu \mathrm{~m}$ |
| :--- | :--- |
| chip area | $3.2 \mathrm{~mm} \times 3.2 \mathrm{~mm}$ |
| pixel size | $16 \times 16$ |
| pixel area | $154.5 \mu \mathrm{~m} \times 153.3 \mu \mathrm{~m}$ |
| number of transistors | $147 /$ pixel |
| max operating freq. | 5 MHz |
| fill factor (photosensor) | $12.5 \%$ |
| ( ${ }^{*}$ measured for $\mathrm{V}_{\mathrm{DD}}=4 \mathrm{~V}$ and $I_{\text {ref }}=4.5 \mu \mathrm{~A}$ ) |  |

area of $7.2 \mathrm{~mm} \times 7.2 \mathrm{~mm}$. This is a practically useful resolution and is technically feasible to implement. With the shrinking of the pixel size, the photosensor would have to be implemented in the integration type since the amount of a photogenerated current from the reduced size of phototransistor is too small. The change in the configuration of the photosensor would also expand the application area of the feature detection sensor since it opens up the possibility of directly dealing with gray-level images.

There are several issues that need future modifications. Insertion of a complementary PMOS switch is necessary for complete voltage transfer. This would enable chip operation with the supply voltage of 3.3 V . The problem of signal mixing at the common node for memories $M_{a}, M_{b}, M_{c}$, and $M_{d}$, can be solved by replacing the present $y 2$ switch with four separate switches and placing each of them between a1 and $M_{a}, b 1$ and $M_{b}, c 1$ and $M_{c}, d 1$ and $M_{d}$, respectively.

### 5.6 Summary

Experimental results using the prototype sensor has been presented as well as some implementation issues in this chapter. The analysis of the transient behavior of the internal node led to the implementation of two mechanisms for high-speed operation. The first mechanism quickly charges and discharges the critical node for current distribution. The second mechanism
keeps the voltage at the thresholding node constant for an additional time period.
A prototype sensor has been fabricated. The sensor contains an array of $16 \times 16$ pixels, each measuring $150 \mu \mathrm{~m} \times 150 \mu \mathrm{~m}$ in a chip area of $3.2 \mathrm{~mm} \times 3.2 \mathrm{~mm}$. The sensor operated successfully with the power supply voltage of 4 V . The mismatch proportionality constant $A_{V_{T}}$ was determined as $7 \mathrm{mV} \mu \mathrm{m}$. With the reference current set to $4.5 \mu \mathrm{~A}$, the operation was confirmed up to the frequency of 5 MHz . With these settings, discriminative detection of the features (junctions of three types, corners, and linestops) was possible in about $50 \mu \mathrm{sec}$. No classification error was observed due to what turned out to be a conservative design. One of the experimental results suggested the validity of the proposed theory that relates the mismatch parameter to the error rate.

### 5.7 References

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## Chapter 6 Conclusion

This thesis has explored the design and implementation of a new type of VLSI computational sensor. The sensor detects important image features including corners, three types of junctions (T-type, X-type, Y-type), and linestops for a binary image in a discriminative fashion. The sensor contains an array of $16 \times 16$ processing elements, each measuring $150 \mu \mathrm{~m}$ $\times 150 \mu \mathrm{~m}$ in a chip area of $3.2 \mathrm{~mm} \times 3.2 \mathrm{~mm}$. Since these features are detected on-chip in about $50 \mu \mathrm{sec}$, the sensor can be used for various types of applications requiring high speed. For the realization of this sensor, several aspects including an algorithm, architectural design, design procedure based on transistor mismatch, implementation details are investigated and discussed in the thesis.

In Chapter 2, an algorithm for the feature detection has been described. The algorithm is inspired by the biological vision system, which performs detection and hierarchical integration of features. The proposed algorithm first decomposes an input image into four orientations and detects linestops for line segments in each orientation plane. Corners and junctions are detected as a result of interaction between line segments and linestops in different orientation planes. The algorithm is formulated on the basis of template matching which iteratively performs a $3 \times 3$ neighborhood processing.

In Chapter 3, an architectural design of the feature detection sensor has been presented. An analog/digital mixed-mode configuration is employed to achieve the advantages of both analog and digital implementations. An input image is digitized first by comparing the steadystate output of a phototransistor with a threshold current. Then the $3 \times 3$ neighborhood processing is carried out by the current distribution according to a proper kernel and the thresholding operation. The computation result is stored into a specified memory to be used for
the next operation. The operational sequence can be programmed in a digital fashion.
In Chapter 4, a systematic design procedure for current-mode processing circuits based on transistor mismatch analysis has been proposed. First, a set of formulas is derived to represent the current variation as a function of the design parameters (transistor dimension and the reference current). The formulas also take the configuration of the current mirror circuit into account. Using these expressions, the proposed method converts the specifications for accuracy, speed, and the transistor operating condition to the requirement for the design parameters. These parameters are chosen to satisfy all the above three requirements.

In Chapter 5, implementation details and experimental results using the prototype sensor have been presented. Two mechanisms for high-speed operation, one for the quick charging and discharging of the critical node and the other one for keeping the thresholding node voltage constant for an additional time period, are explained. The response obtained from the sensor was identical to the simulated result when operated with the power supply voltage of 4 V and with the operating frequency up to 5 MHz , confirming the functionality of the sensor.

The confirmation of the successful operation of the sensor completes the objective of the thesis. The sensor is unique by incorporating some flavor of the hierarchical feature integration, which enables the discriminative detection of the five features on chip. This is the first feature detection sensor of this kind.

The work presented in this thesis has considerable room for future extensions. Further improvement of the algorithm is possible by incorporating several methods proposed in the Discussion section of Chapter 2 However, the fundamental limitation of an interaction area of $3 \times 3$ neighborhood, which is required for feasible wiring in VLSI implementation, would still exist. The algorithm sometimes suffers from local noise. A certain type of preprocessing which filters out unnecessary branches or image smoothing may be necessary to further increase the robustness.

As far as the sensor is concerned, incorporation of the edge detection mechanism as a first-step processing makes the sensor directly applicable to gray-level images, which should significantly expand the application area of the sensor. Another interesting area to investigate is the method of scanning the pixel activity. The scanning procedure implemented in the present sensor sequentially scans pixels, resulting in a large number of values 0 carrying no information. To fully exploit the advantage of on-chip feature detection, only meaningful information (pixels of value 1) should be transferred to a host CPU by some mechanism. One of the possible mechanisms is to skip a pixel of value 0 by modification of the shift register.

Further investigation for the relationship between the transistor mismatch and the accuracy of the circuit is of interest. A dedicated chip, which is designed to produce large current variations in a reasonable amount of current, is necessary for this purpose. An even more challenging but important work is the analysis of the accuracy in a dynamic sense. The analysis presented in the thesis has represented accuracy in a static sense, which does not completely reflect reality. Ideally, accuracy should be represented as a function of time. The analysis gets even more complicated when the speed variation due to transistor mismatch is taken into account.

As a concluding remark, we hope that the presented work will stimulate further research activities in the field of computational sensors.

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[^0]:    ${ }^{1}$ The receptive field of a neuron is defined as the area on the retina that affects the signaling of the neuron.

[^1]:    ${ }^{2}$ Each element of a set is a $2-D$ vector which represents the row and the column position of a black (by convention) pixel in the image.

[^2]:    ${ }^{3}$ If $-b$ is replaced with $b$, this expression becomes identical to the Minkowski subtraction of two sets.

[^3]:    ${ }^{4}$ The subscript $i$ can be related to the discrete instant $n$ in eqn. (2.1), but is not necessarily numbered according to the order of computation.

[^4]:    ${ }^{5}$ Two pixels are said to be 4-connected or to have 4-connectivity if they are adjacent in either horizontal or vertical direction.
    ${ }^{6}$ Two pixels are said to be 8 -connected or to have 8 -connectivity if they are adjacent to each other in one of the four orientations (horizontal, vertical, right diagonal, left diagonal).

[^5]:    ${ }^{7}$ The stem indicates the vertical line segment of T. The horizontal line segment is called the bar.

[^6]:    ${ }^{8}$ The irradiance of typical office lighting is $1 \mathrm{~W} / \mathrm{m}^{2}$. The value of $0.1 \mathrm{~W} / \mathrm{m}^{2}$ is used as an estimate of the intensity of the light reflected from an object.

[^7]:    ${ }^{9}$ Comparison of eqn. (4.86) and (4.92) shows that the fall time is larger than the rise time when $\alpha>0.85$ under the condition $\alpha=1-\varepsilon$, which imposes the same criteria for the rise time and the fall time.

[^8]:    ${ }^{10}$ The difference between these two features is the number of corners (see Section 2.4 for details).

[^9]:    ${ }^{11}$ This phenomenon happens due to the precharging of each pixel during preparation phase before the operation starts. Without this mechanism for high speed operation, there is no difference between the LSD operation at the edge and the LSD operation at the center.

