Kinematics - II

Velocity and Acceleration Analysis
Relationship between Velocities and Accelerations in A of Two Points fixed to B

- Points \( P \) and \( Q \) fixed to (in) \( B \)
- \( O \) is a point fixed in \( A \)
- Position vectors for \( P \) and \( Q \) in \( A \) are denoted by \( \mathbf{p} \) and \( \mathbf{q} \)
- Velocities for \( P \) and \( Q \) in \( A \)
  \[
  A\mathbf{v}^P = \frac{A}{dt} \mathbf{d} \mathbf{p}, \quad A\mathbf{v}^Q = \frac{A}{dt} \mathbf{d} \mathbf{q}
  \]
- Accelerations for \( P \) and \( Q \) in \( A \)
  \[
  A\mathbf{a}^P = \frac{A}{dt} \frac{A\mathbf{v}^P}{dt}, \quad A\mathbf{a}^Q = \frac{A}{dt} \frac{A\mathbf{v}^Q}{dt}
  \]
Velocities of $P$ and $Q$

- Triangle law of vector addition for points $P$ and $Q$
  \[ \mathbf{q} = \mathbf{p} + \mathbf{r} \]
- Differentiate both sides
  \[ \frac{A}{dt} \mathbf{d} \mathbf{q} = \frac{A}{dt} \mathbf{d} \mathbf{p} + \frac{A}{dt} \mathbf{d} \mathbf{r} \]
- Substitute definitions of velocities
  \[ A \mathbf{v} Q = A \mathbf{v} P + \left( \frac{B}{dt} \mathbf{d} \mathbf{r} + A \mathbf{\omega}_B \times \mathbf{r} \right) \]
- Velocities for $P$ and $Q$ in $A$
  \[ A \mathbf{v} Q = A \mathbf{v} P + A \mathbf{\omega}_B \times \mathbf{r} \]
Accelerations of \( P \) and \( Q \)

- Velocities for \( P \) and \( Q \) in \( A \)

\[
A_{vQ} = A_{vP} + A \omega^B \times \mathbf{r}
\]

- Differentiate both sides

\[
\frac{d}{dt}(A_{vQ}) = \frac{d}{dt}(A_{vP}) + \frac{d}{dt}(A \omega^B \times \mathbf{r})
\]

\[
A_{aQ} = A_{aP} + \frac{d}{dt}(A \omega^B \times \mathbf{r}) + A \omega^B \times \frac{d\mathbf{r}}{dt}
\]

- Accelerations for \( P \) and \( Q \) in \( A \)

\[
A_{aQ} = A_{aP} + A \alpha^B \times \mathbf{r} + A \omega^B \times (A \omega^B \times \mathbf{r})
\]

- tangential acceleration
- centripetal (normal) acceleration
Relationship between Velocities and Accelerations in $A$ of Points described in $B$

- Point $P$ fixed to (in) $B$
- Point $Q$ moving in $B$ (but easily described in $B$)
Velocities of $P$ and $Q$

- Triangle law of vector addition for points $P$ and $Q$
  \[ q = p + r \]

- Differentiate both sides
  \[ \frac{A}{dt}dq = \frac{A}{dt}dp + \frac{A}{dt}dr \]

- Substitute definitions of velocities
  \[ A v_Q = A v_P + \left( \frac{B}{dt}dr + A \omega^B \times r \right) \]

- Velocities for $P$ and $Q$ in $A$
  \[ A v_Q = A v_P + B v_Q + A \omega^B \times r \]
Velocity and Acceleration of $Q$ in $B$

- Point $Q$ moving in $B$ has position vector $s$ in $B$
  \[ s = \overline{QQ} \]

- Velocity, acceleration
  \[ \frac{B}{dt} \frac{ds}{dt} = B \mathbf{v}_Q, \quad B \mathbf{a}_Q = \frac{B}{dt} \left( B \mathbf{v}_Q \right) \]
Acceleration of $P$ and $Q$

- Velocities for $P$ and $Q$ in $A$

$$A v_Q = A v_P + B v_Q + A \omega_B \times r$$

- Differentiate in $A$

$$A a_Q = \frac{A d}{dt}(A v_Q) = \frac{A d}{dt}(A v_P) + \frac{A d}{dt}(B v_Q) + \frac{A d}{dt}(A \omega_B \times r)$$

$$\{A a_P\}$$

$$\left\{\begin{array}{l}
\frac{A d}{dt}(A \omega_B) \times r + A \omega_B \times \left(\frac{B dr}{dt} + A \omega_B \times r\right) \\
\frac{B d}{dt}(B v_Q) + A \omega_B \times B v_Q
\end{array}\right.$$
Acceleration of $P$ and $Q$

\[ \mathbf{a}^Q = \mathbf{a}^P + \mathbf{a}^Q + A_\alpha \times \mathbf{r} + A_\omega \times (A_\omega \times \mathbf{r}) + 2A_\omega B \times \mathbf{v}^Q \]

- Tangential acceleration
- Centripetal (normal) acceleration
- Coriolis acceleration

Special case: $\mathbf{r} = 0$

\[ \mathbf{a}^Q = \mathbf{a}^\overline{Q} + B \mathbf{a}^Q + 2A_\omega B \times \mathbf{v}^Q \]