Matlab Exercise II: Simulation of a Rolling/Sliding Disk

Find the position vector of the center \((C^*)\) of the disk in frame \(A\) and express it in terms of triad \((a_1, a_2, a_3)\)

\[
\overrightarrow{OC^*} = \overrightarrow{OQ} + \overrightarrow{QC^*}
\]

\[
= (?) \overrightarrow{a_1} + (?) \overrightarrow{a_2} + (?) \overrightarrow{a_3}
\]

Reference Triads

- Rotate triad \(A\) about \(z\) through \(q_1\) followed by rotation about \(x\) by 90 deg to get \(E\)
- Rotate triad \(E\) about \(-x\) through \(q_2\) to get \(B\)
- Rotate triad \(B\) about \(z\) through \(q_3\) to get \(C\) (not shown)

Imagine \(E\) to be a virtual body that is attached to \(Q\)

Imagine \(B\) to be a virtual body that is attached to \(C^*\)
Angular Velocity: Components

\[ \omega_C = u_1 \mathbf{b}_1 + u_2 \mathbf{b}_2 + u_3 \mathbf{b}_3 \]

- \( u_i \) are the components of the angular velocity of the disk with respect to the reference triad \( B \).

\[ \omega_C = u_1 \mathbf{a}_1 + u_2 \mathbf{a}_2 + u_3 \mathbf{a}_3 \]

- \( u_i \) are the components of the angular velocity of the disk with respect to the reference triad \( A \).

Express the time derivative of each coordinate in terms of the five coordinates and the velocities \( u_i \).

\[ \dot{\omega}^C = \dot{\omega}^E + \omega^B \times R^C + R^C \dot{\omega}^C = \dot{\mathbf{a}}_1 \mathbf{b}_1 + \dot{\mathbf{b}}_1 + \dot{\mathbf{b}}_2 \]

\[ \dot{\omega}^C = u_1 \dot{\mathbf{b}}_1 + u_2 \dot{\mathbf{b}}_2 + u_3 \dot{\mathbf{b}}_3 = -\dot{q}_1 \mathbf{b}_1 + \dot{q}_1 \cos q_3 \mathbf{b}_3 + (\dot{q}_1 \sin q_2 + \dot{q}_3) \mathbf{b}_3 \]

- \( u_i = -\dot{q}_2 \)
- \( u_2 = \dot{q}_1 \cos q_3 \)
- \( u_3 = \dot{q}_3 \sin q_2 \)
- \( u_4 = \dot{q}_4 \)
- \( u_5 = \dot{q}_5 \)

\[ \dot{q}_1 = \frac{u_4}{\sin q_3}, \quad \dot{q}_2 = \frac{u_3}{\cos q_3} \]

Two constraints which reduce the disk's degrees of freedom from 5 to 3...

Homework

Program Structure

- **disk_simulation.m**: Input: \( u \) (3x1 rolling, 5x1 sliding) \( q_0 \) (5x1 vector) \( t_{stop} \) (stopping time) Output: \( \dot{q}_0 \)
- **get_qdot_from_u.m**: Input: \( u \) Output: \( \dot{q}_0 \)
- **disk_animation.m**: Input: \( R \) Output: \( \dot{R} \)
- **get_R_r.m**: Input: \( q \) Output: \( \dot{R} \) & \( \dot{q} \)