Deciding intentional equality of total-inductive functions.

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Martin-Löf's propositional equality is a substitutive equivalence relation, but unlike definitional equality, it is not normally presented as being decidable. The difficulty of making it decidable lies in deciding the equality of functions, for which the only eliminator is application. I have instead defined a built-in decision procedure of intentional (not extensional) equality for all types in a total language with inductive types, including function types. I use a canonical terms presentation of the type theory supporting this decision procedure, and we will see the changes to canonical terms that I have made to accomplish this.

One application that I have in mind for this decision procedure is matching the dependent function codomain part of Π types. This brings us one step closer to defining tactics as generic programs internal to the dependent type theory that they operate on, by considering Π types as telescopes representing the context and the goal.

Finally, I should stress that the type theory supporting this decision procedure becomes inconsistent if combined with any form of functional extensionality, whether it is postulated or accessible via an alternative propositional equality such as Observational Equality. Thus it represents an interesting intentional extreme on the spectrum of propositional equalities, to be contrasted with current research pushing towards the extensional extreme such, as OTT and HoTT.