ENERGY DISSIPATION IN CONDUCTIVE POLYMERIC FIBER BUNDLES: SIMULATION EFFORT

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ABSTRACT

This work built a mathematical model and simulation scheme to support a physical experiment on energy dissipation from vibrating conductive polymer fibers. The fundamental idea in the physical experiment is to oscillate a magnetic field from a coil, so a conductive polymeric fiber with a current through it, is excited to oscillations by the resulting Lorentz force. I used Matlab software to graphically represent the functional dependence of the vibrational amplitude on force amplitude, frequency, and quality factor. The model is able to develop three two-dimensional and three three-dimensional graphs. The model lets the user choose a graph and either use the default values or adjust the values of the constants and variables for a particular situation.

1. INTRODUCTION

In this time, when the technology is going in advance and the nature is suffering a lot of changes, because of this reason, the humanity should look for a option that does not cause a lot of damage to the nature. Actually the polymers are in abundance in the world and several of them "have been tested and proved to work in a variety of applications including batteries, capacitors, smart windows, etc"[6], and much more things that are used all the time and for the majority of the people. Those are heavy reasons to study the properties of the conductive polymers. In the physical experiment of our project we want to measure the mechanical module of the polymer fiber because it is not known yet. By other way in this work we want to create a mathematical model and simulation scheme to support the physical experiment. The mathematical model consists in show the functional dependence of the vibrational amplitude of the polymer beam when it is exited by a current through it.

With this experiment we hope to find more information about the properties of the polymers, and in this way help the scientists to find newer ways in the advance of the science and technology.

2. BACKGROUND

The purpose of this work is to build a mathematical model to support a physical experiment on energy dissipation from a vibrating conductive polymer fiber beam. The fibers utilized in this project are obtained with the electrospinning process that is described in the next section. The goal of this project is to investigate the properties of

the polymer fiber in different situations. In particular, we want to measure the mechanical moduli of the polymer fibers, which are not yet known.

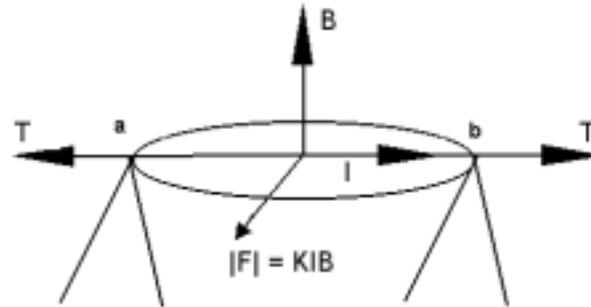To reach our goal we utilized the scheme shown in Figure 1.

Figure 1: Experiment diagram.

a and **b** are the tension points of the beam, B is the magnetic field applied to the beam, and I is the current applied through the beam. The product of B x I is the value of the Lorentz force, the force that makes possible the excitations of the beam that we want to measure. When the beam is excited, are induced vibrations in the polymeric beam. Figure 1 identifies the maximum value of the vibrational amplitude as the curves drawn over and under the beam.

To support the experimental project I developed a computational model, described later, that shows the functional dependence of the vibrational amplitude. I will also discuss electrospinning; because this is the process utilized to obtain the fibers that form the bundle used in the experiment, polymers, and friction; because this situation affect the values of the energy dissipation and decreases the value of vibrational amplitude. Also I will talk about the method used to develop the model; implementation of the model; and our results and conclusions.

2.1.   Electrospinning

"Electrospinning offers unique capabilities for producing novel synthetic fibers of unusually small diameter and good mechanical performance" [1]. The fibers produced in this process, known as nanofibers, "are of substantial scientific and commercial interest, as they may be expected to exhibit morphologies and properties quite different from conventional fibers. Electrospinning is a fast and simple process which is readily implemented as a micro processing technique" [2].

"Electrospinning uses electrostatic forces as a driving force to spin ultra thin synthetic fibers."[3] In this process an amount of polymer solution is subjected to an electric field by which an charge is induced to the polymer solution. Then mutual charge repulsion causes a force directly opposite to the surface tension that held the polymer solution. "The intensity of the electric field is increased, the hemispherical surface of the solution at the tip of the capillary tube elongates to form the Taylor Cone."[3] The charged

polymer fibers are leaved behind because the solvent is evaporated when the jet travels in the air. "The resulting fibers have diameters of about 50 nanometers and arbitrary lengths"[4].
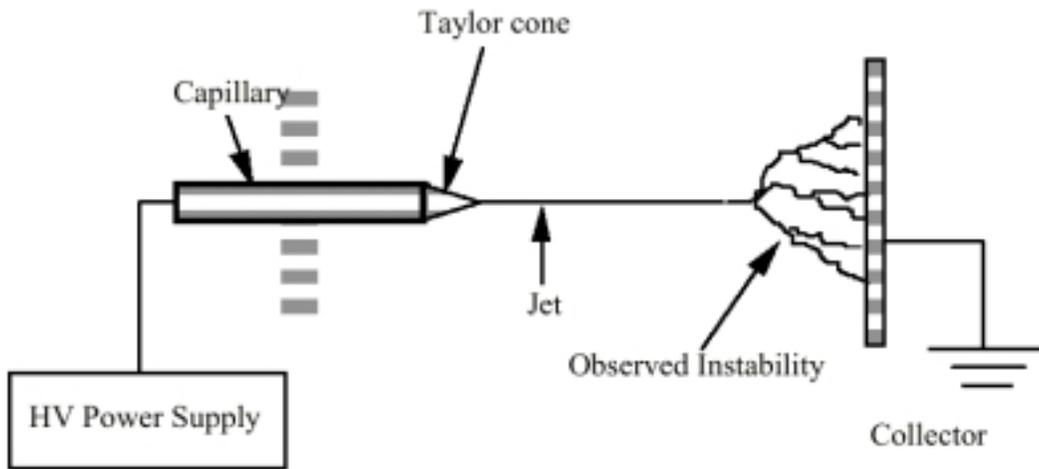
Figure 2. Electrospinner diagram.

2.2   Polymers

Polymers are giant molecules, some up to 1 meter long and others weighing almost 1 kilogram. Polymers have many uses and are produced in huge quantities [5], so their properties are of great interest.

Polymer applications include "batteries, capacitors, smart windows, light emitting, diodes, transistors, photovoltaics, microlithography, corrosion control, conductive adhesives and inks, static dissipation, EMI shielding, radar/microwave absorption, direct plating, electrostatic powder coating, clean room applications, sensors, and drug delivery systems" [6]. The bundle that we are using in our project is formed with fibers of conductive polymers in the Figure 3 is an example of the conductive polymer. The right image is more conductive than the left right because it is ordered and the other one is not ordered.
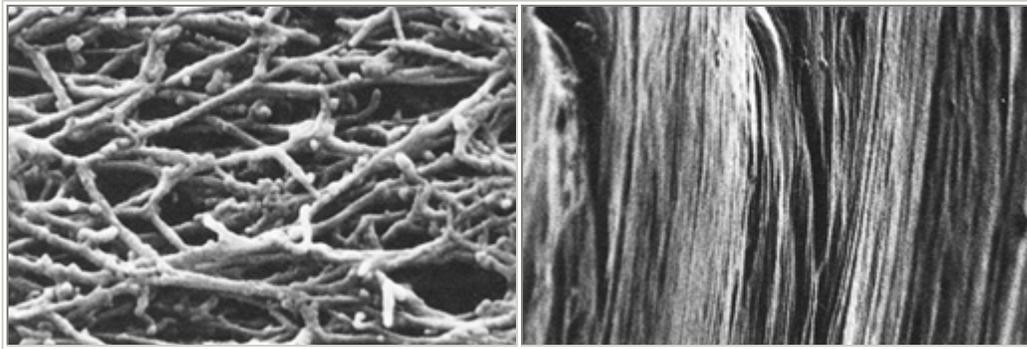
Figure 3. Typical Scanning Electron Micrographs of Conductive Polymers.

2.3   Friction

Our bundle of fibers is formed with a group of polymer fibers, so there is friction or anelasticity between the fibers in the bundle, and between the molecules in the fiber.

Anelasticity is summarized in three postulates:

  a.   "For every stress there is a unique equilibrium value for strain, and vice versa.
  b.   The equilibrium response is achieved only after the passage of sufficient time.
  c.   The stress strain relationship is linear" [7].

Friction is a problem in this work because it decreases the energy dissipation of the bundle and could change the values of the vibrational amplitude. One example of this situation is shown in Figure 6.
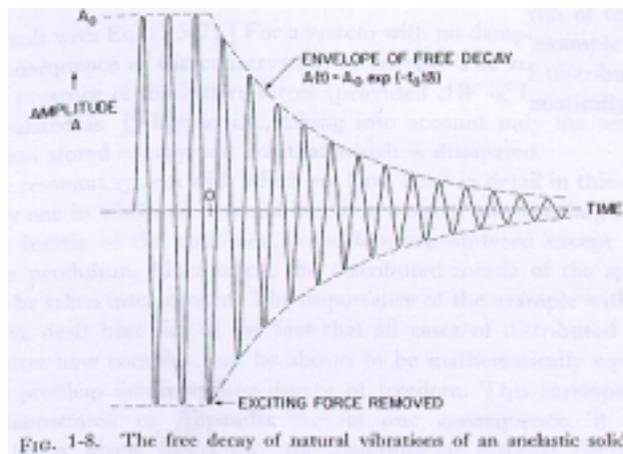

Figure 4: Example of decreasing amplitude of an anelastic solid [7].

This graph shows how the value of the amplitude of an anelastic solid decreases as a function of time. This same phenomenon can exist when we are working with the vibrational amplitude or energy dissipation of a double clamped polymer beam.

3.   METHOD

The model consists of developing graphs that represent the functional dependence of vibrational amplitude in a double clamped polymer beam. The following equation is the differential equation for the vibrational amplitude of a double clamped beam.

$$\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}; C = \sqrt{\frac{T}{P}} \qquad (1)$$

The initial conditions to solve this differential equation are:

$$U(\omega,0) = f(x); \frac{\partial U}{\partial t}(x,0) = g(x), \qquad (2)$$

and the boundary conditions are:

$$U(0,t) = U(l,t) = 0 \qquad (3)$$

When we solved the differential equation for our problem we found the following equation for vibrational amplitude as dependent of ω

$$U(\omega) = \frac{F(\omega)/Keff}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 - \left(\frac{\omega * \omega_0}{Q}\right)^2}} \qquad (4)$$

where $F(\omega) = A * \cos(\omega t)$; $\omega = 2\pi f$. F represents the force, A represents the force amplitude, $Keff$ is the effective spring constant, and Q represents the quality factor.

Then we began to develop the mathematical model using Matlab 5.3. In the model the user has the opportunity to decide what graph to develop and whether to use the default values or other values. In the latter case the program will ask for the values of quality factor, force amplitude, initial frequency, frequency, and spring constant. For the variables values the program will ask for the initial value, last value, and the steps. Then the program proceeds to develop the graph. If the user decides to use the default values the program automatically produces the graph.

4.   IMPLEMENTATION

The computational model was developed "from scratch" with Matlab 5.3. In the initial stage of the implementation it was applied to a two-dimensional graph as shown in Figures 7-9. This profile provided the opportunity to test and debug the code for a simple

case and gave us some hints about possible difficulties to be encountered in more complex cases.

5.   RESULTS

In our simulation we developed six different graph that represent the vibrational amplitude of a polymer fiber beam as dependent on frequency, force amplitude, and quality factor. Those graphs help us to know what is happening during the physical experiment.
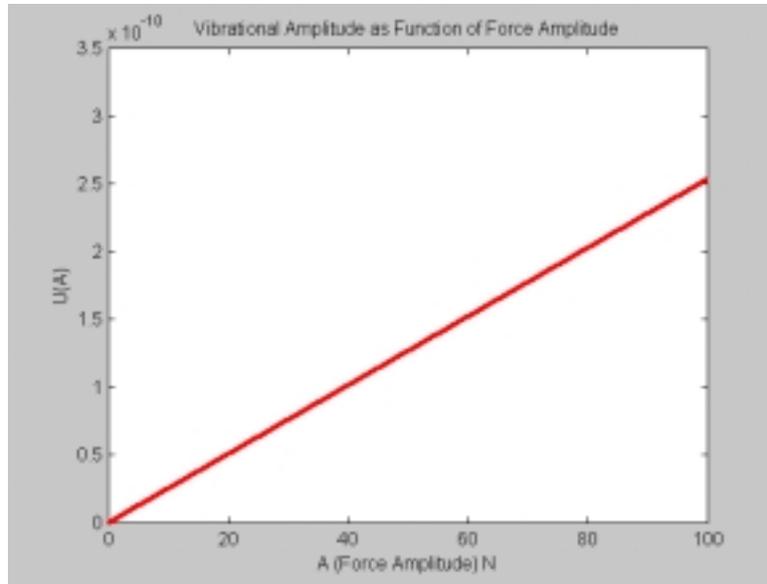

Figure 5: Vibrational amplitude as dependent on force amplitude.

In Figure 7 the constant values were quality factor = 10^5, frequency = 10^5 Hz, spring constant = 1 N/m, and initial frequency = 70 Hz. The values of the force amplitude varied from 0 N to 100 N in steps of 0.2 N. This graph demonstrates that the vibrational amplitude of a double clamped beam is proportional to the force amplitude.
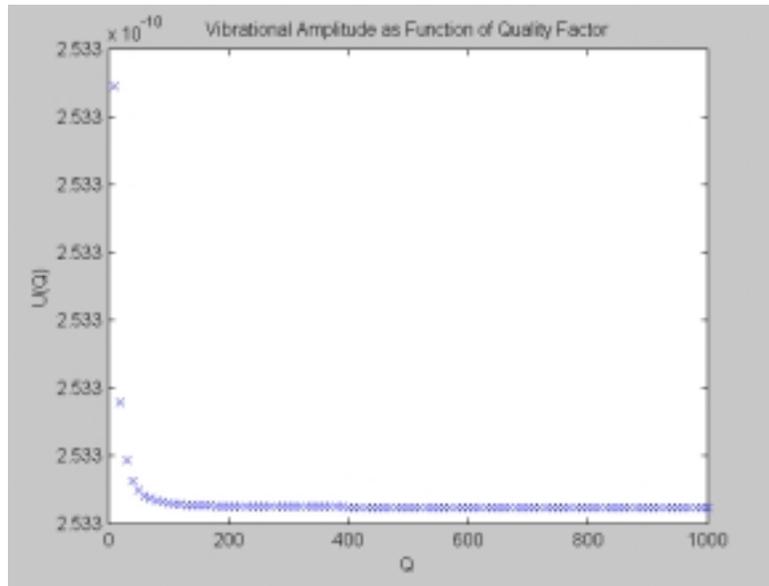
Figure 6: Vibrational amplitude as dependent on quality factor.

The constant values for the graph in Figure 8 were spring constant = 1 N/m, initial frequency = 70 Hz, force amplitude = 0.1 N, and frequency = 10^5 Hz. The value of quality factor varies from 0 to 1000 in steps of 10 units. The graph shows a very small value for vibrational amplitude. Also shows that the value of vibrational amplitude is higher when the value of quality factor is near 0.
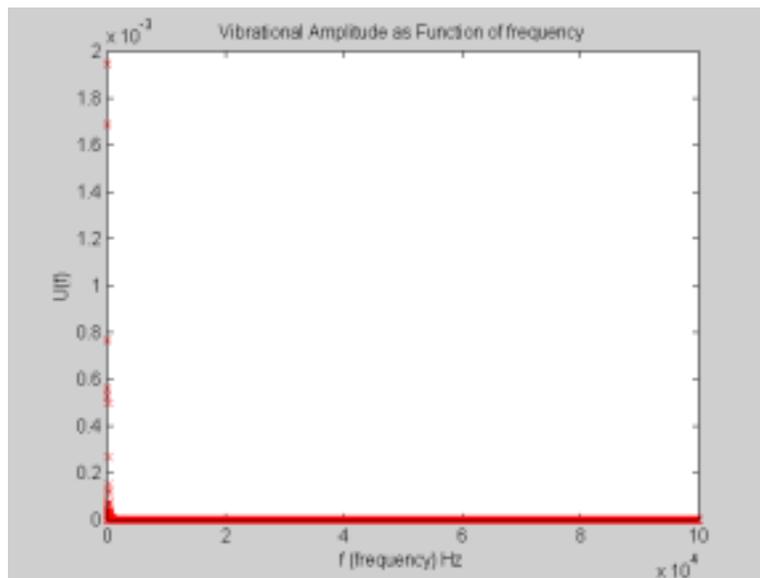

Figure 7: Vibrational amplitude as dependent on frequency.

In Figure 9 the constant values were spring constant = 1 N/m, quality factor = 10^5, initial frequency = 70 Hz, and force amplitude = 0.1 N. The values of frequency varied from 0

Hz to 1000 Hz in steps of 5 Hz. Like the previous graph this graph also has small values for vibrational amplitude, and the highest value for the vibrational amplitude appears when the system is driven at low frequencies.

The next three graphs (Figures 8–10) represent vibrational amplitude in a three-dimensional function.
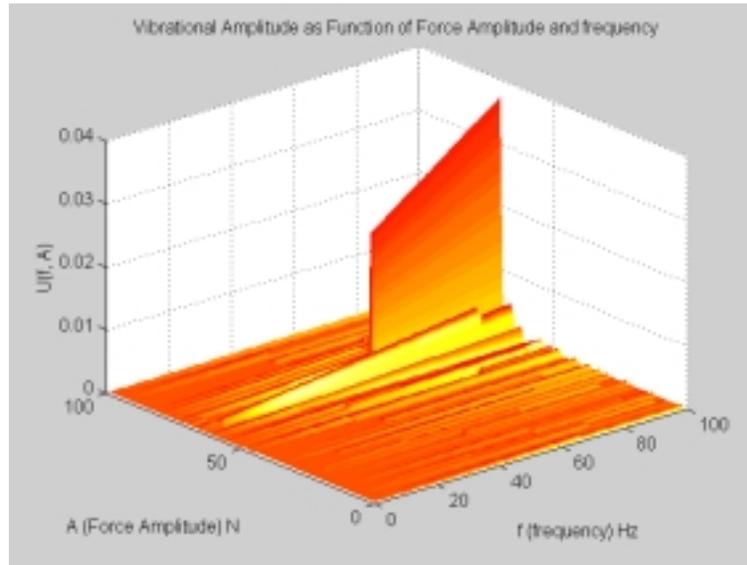

Figure 8. Vibrational amplitude as dependent on frequency and force amplitude.

The constant values for this graph were quality factor = 10^5, spring constant = 1 N/m, initial frequency = 70 Hz, the values of force amplitude varied from 1 N to 100 N in steps of 1 N and the values for frequency varied from 1 Hz to 100 Hz in steps of 1 Hz. This graph shows the highest value of vibrational amplitude when the force amplitude is near to the initial value of frequency and in the maximun value of frequency.
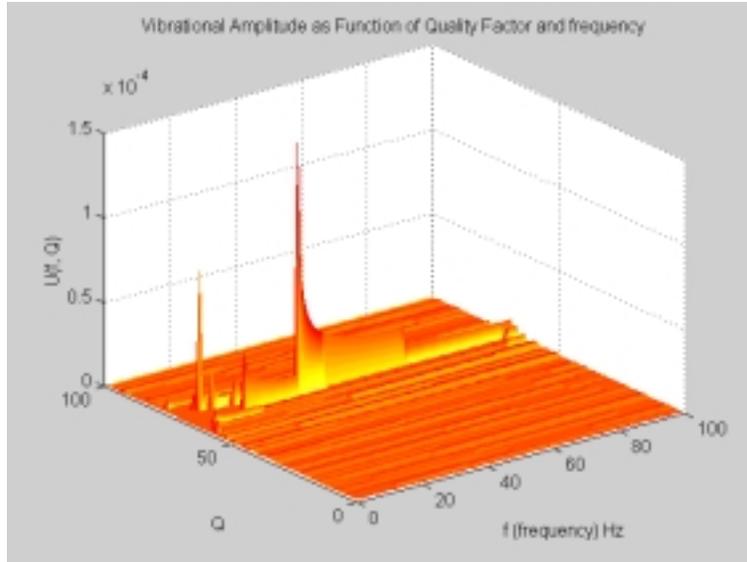
Figure 9. Vibrational amplitude as dependent of frequency and quality factor.

In this case the constant values were initial frequency = 70 Hz, force amplitude = 0.1 N, spring constant = 1 N/m. The values of quality factor varied from 1 to 100 in steps of 0.5 units and frequency varied form 1 Hz to 100 Hz in steps of 0.5 Hz. In this graph we can see some peaks, but the peak with the highest value of vibrational amplitude appears when the quality factor has a value approximated to the initial frequency. Also we can appreciate that the peaks are around the same value of quality factor.

The last graph that we created shows the vibrational amplitude as a function of force amplitude and quality factor.
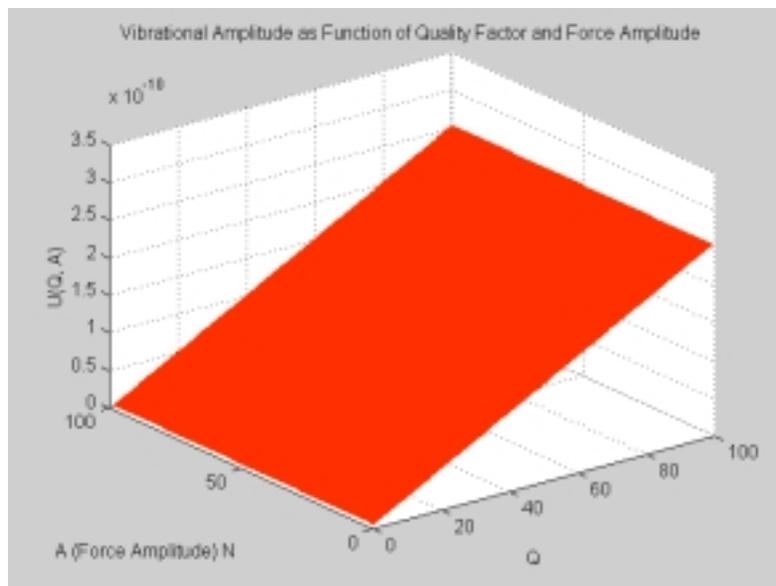

Figure 10: Vibrational amplitude as dependent on quality factor and force amplitude.

The constant values for the graph in Figure 12 were initial frequency = 70 Hz, spring constant = 0.1 N/m, and frequency = 10^5 Hz. The values for quality factor and force amplitude varied from 1 to 100 in steps of 1 unit. There are very small values for vibrational amplitude, but the higher value of vibrational amplitude appears in the higher value of quality factor.

6.   DISCUSSION AND CONCLUSIONS

This project developed a computational model that generates graphs about the functional dependence of vibrational amplitude. The model was based on an equation that represents the vibrational amplitude of a double clamped beam as a function of force amplitude, quality factor, and frequency.

In the case of vibrational amplitude as dependent on force amplitude, the model developed a graph that shows that vibrational amplitude is proportional to force amplitude.

In the case of vibrational amplitude as dependent on quality factor or frequency, the model developed graphs that show the maximum value of vibrational amplitude when their values are near to 0.

In the cases of vibrational amplitude as dependent on frequency and force amplitude or frequency and quality factor, the model developed graphs that show higher values of vibrational amplitude when the force amplitude or quality factor are near the value of the initial frequency.

In the case of vibrational amplitude as dependent on quality factor and force amplitude, the model developed a graph that resembled the slope of a hill. It implies that the vibrational amplitude in this cases increases periodically with respect to quality factor value.

7.   RECOMMENDATIONS

The recommendations for this work are to develop a graphic interface. Where the user can enter the data in different windows. For example, that the program show three different windows. One for enters the data like the parameters and values for the graph that the user wants. Another window that show the graph that the user want. And another window that show an image about what is happening while the graph is running. This image should be an animation that shows how is the beam vibrating, in the way that the user can has an idea about what is happening in the experiment.

8. ACKNOWLEDGMENTS

9. REFERENCES

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