Adaptive Distributed Algorithms for Optimal Random Access Channels

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***Abstract*—We develop adaptive scheduling and power control algorithms for random access in a multiple access channel where terminals acquire instantaneous channel state information but do not know the probability distribution of the channel. In each time slot, terminals measure the channel to the common access point. Based on the observed channel value, they determine whether to transmit or not and, if they decide to do so, adjust their transmitted power. We remark that there is no coordination between terminals and that adaptation is to the local channel value only. It is shown that the proposed algorithm almost surely maximizes a proportional fair utility while adhering to instantaneous and average power constraints. Important properties of the algorithm are low computational complexity and the ability to handle non-convex rate functions. Numerical results on a randomly generated network with heterogeneous users corroborate theoretical results.**

***Index Terms*—Random access, channel state information, adaptive algorithms, optimization, multiuser diversity.**

I. Introduction

THIS paper considers wireless random access channels in which terminals contend for access to a common access point (AP) as is the case in wireless local area networks and cellular systems. To exploit favorable channel conditions terminals adapt their transmitted power and access decisions to the state of the random fading channels linking them to the AP. The challenges in developing this adaptive scheme are that terminals have access to their own channel state information (CSI) only, and that the probability distribution function (pdf) of the fading channel is unknown. The goal of this paper is to develop a distributed learning algorithm to determine optimal transmitted power and channel access decisions relying on local CSI only.

The idea of adapting medium access and power control to CSI has been extensively explored in wireless communications. Early references dealing with power adaptation on the uplink of multiuser systems focus on centralized power control schemes where the AP collects channel states for all terminals to select the one to be scheduled. In, e.g., [2], the AP schedules the terminal with the best channel gain with a power adapted to the channel condition. Similar ideas have also been used for scheduling and resource allocation in broadcast downlink channels, see e.g., [3]–[5]. Although these centralized schemes exploit multiuser diversity, they require significant information exchange between terminals and the AP; a problem exacerbated when the number of users is large. To avoid this overhead, recent work integrates channel adaptation into random access protocols. Exploiting the idea of aligning schedules to good channel opportunities, [6] develops a distributed channel-aware Aloha protocol in which terminals transmit only when their channel gains exceed pre-defined thresholds. This algorithm is shown to be asymptotically optimal in the sense that the only performance loss compared to a centralized scheme is due to user contention.

Under simple collision models, it has been shown that distributed threshold-based schedulers with properly designed thresholds maximize total throughput of a network with homogeneous users and total logarithmic throughput in the case of heterogeneous users [7]. Similar threshold-based decentralized adaptive random access schemes have been investigated for other types of networks with different packet reception models, see e.g., [8]–[14]. To compute the optimal thresholds, however, terminals are assumed to know the probability distribution of their fading channels. This is a restrictive assumption because the channel fading distribution is usually unknown and can only be estimated based on channel observations. Overcoming this limitation motivates the development of adaptive algorithms to learn optimal operating points based on local CSI [15], [16]. The work in [15] proposes a heuristic adaptive algorithm for threshold-based schedulers in which the thresholds are tuned based on local channel realizations in a time window. The work in [16] develops an online learning algorithm for transmission probability and power control under rate constraints using game-theoretic approaches. However, neither [15] nor [16] guarantees global optimality.

The contribution of this paper is the development of an optimal distributed adaptive algorithm for scheduling and power control given that terminals only have access to local CSI and operate independently of each other. At each time slot, terminals observe their channel states and decide whether to transmit or not. If they decide to transmit, they choose a power for their communication attempt. As time progresses, power budgets are satisfied almost surely, while the network almost surely maximizes a weighted proportional fair utility. We remark that terminals operate independently without access to the channel state of other terminals and that the channel pdf is unknown. The proposed algorithm can handle general non-convex, even discontinuous, rate functions with manageable computational complexity. It is worth noting that under the frame work of network utility maximization (NUM)

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algorithms for computing optimal channel access probabilities in random access networks are developed (see e.g. [17]). However, neither fading nor power adaptation is considered in these work.

The presentation begins by formulating optimal adaptive random access as a utility maximization problem whose objective is to maximize a weighted sum of throughput logarithms (Section II). The variables to be determined as a solution of this optimization problem are a scheduling function that determines if a terminal should transmit or not based on its CSI, and a power allocation function that maps a terminal CSI to its transmit power. It is important to remark that: (i) because fading takes on a continuum of values, this optimization problem is infinite-dimensional; (ii) the constraints modeling random access are non-convex; (iii) despite the existence of these non-convex constraints optimization problems of this form are known to have null duality gap [18]; and (iv) since the number of constraints turns out to be finite the optimization problem is finite-dimensional in the dual domain. A further complication is that the original problem formulation yields solutions that require access to global CSI.

We start by overcoming the dependence on global CSI by introducing an equivalent decomposition in per-terminal subproblems whereby nodes maximize local utilities (Section III.A). While this reformulation yields solutions that depend on local CSI only, attempting a solution in the primal domain is difficult because the per-terminal subproblems inherit infinite dimensionality and lack of convexity from the original problem formulation, as well as the need to have access to the channel pdf. We therefore exploit the lack of duality gap to approach their solution through a stochastic subgradient descent algorithm in the dual domain (Section III.B). Based on channel realizations in each time slot, the algorithm computes instantaneous values for the scheduling and power allocation functions and updates Lagrangian multipliers in a direction that can be proven to point towards the set of optimal dual variables in an average sense (Proposition 1). Exploiting this fact we prove that the throughput utility achieved by the algorithm almost surely converges to a value close to the optimal utility. The gap between the optimal and the achieved utility can be made arbitrarily small by reducing a fixed step size (Theorem 1). The paper closes with a numerical evaluation of the proposed algorithm for a randomly generated heterogeneous network (Section IV). To illustrate generality of the proposed approach we consider a system with terminals employing capacity achieving codes (Section IV-A) and a more practical scenario with nodes employing adaptive modulation and coding (Section IV-B). Concluding remarks are presented in Section V.

II. System Model and Problem Formulation

Consider a multiple access channel with n terminals contending to communicate with a common AP. Time is divided in slots identified by an index t. We assume a backlogged system, i.e., all terminals always have packets to transmit in each time slot. The time-varying nonnegative channel $h_i(t) \in \mathbf{R}^+$ between terminal i and the AP at time t is modeled as block fading. Channel gains $h_i(t_1)$ and $h_i(t_2)$ of terminal i at different time slots $t_1 \neq t_2$ are assumed independent and identically distributed (i.i.d.) with pdf $f_{h_i}(\cdot)$. Channel gains $h_i(t)$ and $h_j(t)$ of different terminals $i \neq j$ are also assumed independent. Channels are assumed to have continuous pdf. This latter assumption holds true for models used in practice, e.g., Rayleigh, Rician and Nakagami [19, Ch. 3]. We assume each terminal i has access to its channel gain $h_i(t)$ at each time slot t. While there are various alternatives to obtain channel state information, the simplest would be for the AP to send a beacon signal at the beginning of each time slot. This beacon signal would serve the double purpose of providing a reference for channel estimation as well as a synchronization signal.

Based on its channel state $h_i(t)$, node i decides whether to transmit or not in time slot t by determining the value of a scheduling function $q_i(t) := Q_i(h_i(t)) : \mathbf{R}^+ \to \{0, 1\}$. Node i transmits in time slot t if $q_i(t) = 1$ and remains silent if $q_i(t) = 0$. Notice that each terminal has a different scheduling function and that schedules $q_i(t)$ are determined based on the CSI of each node independently of other terminals. Although each node has access to its local CSI $h_i(t)$, the underlying pdf $f_{h_i}(\cdot)$ is unknown.

Besides channel access decisions, terminals also adapt transmission power to their channel gains through a power control function $P_i(h_i(t)) : \mathbf{R}^+ \to [0, p_i^{\text{inst}}]$, where $p_i^{\text{inst}} \in \mathbf{R}^+$ is a constant representing the instantaneous power constraint of node i. By using this function, terminal i adjusts its transmission power $P_i(h_i(t))$ in response to $h_i(t)$. Similar to $q_i(t)$, we define $p_i(t) := P_i(h_i(t))$, representing the power allocated to node i in time slot t. If node i transmits in time slot t, $p_i(t)$ and $h_i(t)$ jointly determine the transmission rate through a function $C_i(h_i(t)p_i(t)) : \mathbf{R}^+ \to \mathbf{R}^+$. The exact form of $C_i(h_i(t)p_i(t))$ depends on how the signal is modulated and coded at the physical layer. Examples considered later in the paper include capacity-achieving codes and adaptive modulation and coding (AMC). With capacity-achieving codes, $C_i(h_i(t)p_i(t))$ takes the form

$$C_i(h_i(t)p_i(t)) = B \log\left(1 + \frac{h_i(t)p_i(t)}{BN_0}\right), \tag{1}$$

where B and N_0 are the channel bandwidth and the power spectral density of the channel noise, respectively. With AMC, there are M transmission modes available. The mth mode affords communication rate τ_m and is used when the signal to noise ratio (SNR) $h_i(t)p_i(t)/BN_0$ is between η_m and η_{m+1}. The rate function is therefore

$$C_i(h_i(t)p_i(t)) = \sum_{m=1}^{M} \tau_m \mathbb{I}\left(\eta_m \leq \frac{h_i(t)p_i(t)}{BN_0} \leq \eta_{m+1}\right), \tag{2}$$

where $\mathbb{I}(\cdot)$ denotes the indicator function. To keep the analysis general we do not restrict $C_i(h_i(t)p_i(t))$ to take either specific form. It is only assumed that $C_i(h_i(t)p_i(t))$ is a nonnegative increasing function of the product of $h_i(t)$ and $p_i(t)$ that takes finite values for finite arguments. These assumptions are satisfied by (1) and (2) and are likely to hold in practice.

Since terminals contend for channel access, transmission of terminal i in a time slot t is successful if and only if $q_i(t) = 1$ and $q_j(t) = 0$ for all $j \neq i$. If the transmission of terminal i is successful, its transmission rate is determined by $C_i(h_i(t)p_i(t))$. As as consequence, the instantaneous transmission rate for terminal i in time slot t is

$$r_i(t) = C_i\left(h_i(t)p_i(t)\right) q_i(t) \prod_{j=1,j\neq i}^{n} \left[1 - q_j(t)\right]. \tag{3}$$

Assuming an ergodic mode of operation, quality of service is determined by the long term behavior of $r_i(t)$. This implies that system performance is determined by the ergodic limits

$$\begin{aligned} r_i &:= \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} r_i(u) \\ &= \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \left[C_i\left(h_i(u)p_i(u)\right) q_i(u) \prod_{j=1,j\neq i}^{n} \left[1 - q_j(u)\right] \right]. \end{aligned} \tag{4}$$

Assuming ergodicity of schedules $q_i(t) = q_i(h_i(t))$ and power allocations $p_i(t) = p_i(h_i(t))$, the limit r_i can be written as a expected value over channel realizations,

$$r_i = \mathbb{E}_{\mathbf{h}} \left[Q_i(h_i) C_i(h_i P_i(h_i)) \prod_{j=1,j\neq i}^{n} \left[1 - Q_j(h_j)\right] \right], \tag{5}$$

where we have defined the vector $\mathbf{h} = [h_1, \cdots, h_n]^T$ grouping all channels h_i. An important observation here is that since terminals are required to make channel access and power control decisions independently of each other, $Q_i(h_i)$ and $P_i(h_i)$ are independent of $Q_j(h_j)$ and $P_j(h_j)$ for all $i \neq j$. This allows us to rewrite r_i as

$$r_i = \mathbb{E}_{h_i}\left[Q_i(h_i) C_i(h_i P_i(h_i))\right] \prod_{j=1,j\neq i}^{n} \left[1 - \mathbb{E}_{h_j}[Q_j(h_j)]\right]. \tag{6}$$

In addition to instantaneous power constraints $p_i(t) \leq p_i^{\text{inst}}$, terminals adhere to average power constraints $p_i^{\text{avg}} \in \mathbf{R}^+$ as in, e.g., [20]. This average power constraint restricts the long term average of transmitted power that we either write as an ergodic limit or as an expectation over channel realizations,

$$p_i := \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} q_i(u) p_i(u) = \mathbb{E}_{h_i}[Q_i(h_i) P_i(h_i)]. \tag{7}$$

With rates r_i given as in (6), the objective is to maximize a weighted proportional fair (WPF) utility defined as

$$U(\mathbf{r}) = \sum_{i=1}^{n} w_i \log(r_i), \tag{8}$$

where $\mathbf{r} = [r_1, \cdots, r_n]^T$ is the vector of rates and $w_i \in \mathbf{R}^+$ is the weight coefficient for terminal i. Setting $w_i = w_j$ for all $i \neq j$ in a homogenous system with all channels having the same pdf, the WPF utility is equivalent to maximizing the sum of throughputs. In a heterogeneous network where channel pdfs vary among users, maximizing $U(\mathbf{r})$ yields solutions that are fair since it prevents users from having very low transmission rates.

Grouping the objective in (8) with the constraints in (6) and (7), optimal adaptive random access is formulated as the following optimization problem

$$\begin{aligned} \mathsf{P} \ &= \ \max \ U(\mathbf{r}) \\ \text{s.t.} \ \ & r_i = \mathbb{E}_{h_i}\left[Q_i(h_i) C_i(h_i P_i(h_i))\right] \prod_{j=1,j\neq i}^{n} \left[1 - \mathbb{E}_{h_j}[Q_j(h_j)]\right], \\ & \mathbb{E}_{h_i}\left[Q_i(h_i) P_i(h_i)\right] \leq p_i^{\text{avg}}, \\ & Q_i(h_i) \in \mathcal{Q}, P_i(h_i) \in \mathcal{P}_i, \forall i \end{aligned} \tag{9}$$

where $\mathcal{Q}$ is the set of functions $\mathbf{R}^+ \to \{0,1\}$ taking values on $\{0,1\}$ and $\mathcal{P}_i$ represents the set of functions $\mathbf{R}^+ \to [0, p_i^{\text{inst}}]$ taking values on $[0, p_i^{\text{inst}}]$. Notice that the joint optimization across users required to solve (9) introduces *functional* dependence between the actions of different terminals. This is not incongruent with the requirement of *statistically* independent schedules in each time slot. In fact, the notations $Q_i(h_i)$ and $P_i(h_i)$ in (9) stipulates that terminals are required to make channel access and power allocation decisions based on local CSI only. Consequently, although problem (9) requires joint optimization across users, it restricts optimization to policies that result in statistically independent operations.

The goal of this paper is to develop an online algorithm to determine schedules $q_i(t)$ and power assignments $p_i(t)$ having statistics that solve the optimization problem in (9). The algorithm is required to: (i) operate without knowledge of the channel distribution; and (ii) yield functions $q_i(t)$ and $p_i(t)$ that depend on the current and past values of the local channel $h_i(t)$ but are independent of other terminal's channels $h_j(t)$ for $j \neq i$.

Remark 1: In order to allow terminals to know if their transmissions are successful or not, the AP provides feedback on whether the transmission attempt was successful or a collision detected. If a terminal transmits a packet but detects a collision, it can reschedule the packet for retransmission in a subsequent time slot. We remark that feedback does not introduce correlation between the transmission decisions of different terminals. The provided feedback only tells terminals if they should retransmit previous packets or not, but does not enforce them to make channel access or power allocation decisions.

III. Adaptive Algorithms For Decentralized Channel-Aware Random Access

The stated goal is to devise scheduling and power control policies based on local CSI that are globally optimal as per (9). These two objectives, i.e., global optimality while relying on local CSI, seem to contradict each other. Because r_i depends not only on $Q_i(h_i)$ and $P_i(h_i)$ but on $Q_j(h_j)$ for all $j \neq i$, it seems that optimal $Q_i(h_i)$ and $P_i(h_i)$ solving (9) might also be functions of other terminals' CSI. To see that this is not the case, we will show that it is possible to decompose (9) in per terminal subproblems. After introducing this decomposition the complicating fact that the channel pdf $f_{h_i}(h_i)$ is unknown remains. To overcome this complication, we will introduce a stochastic subgradient descent algorithm in the dual domain that is optimal in an ergodic sense.

A. Problem Decomposition and Its Dual

Begin then by separating (9) in per terminal subproblems. To do so, we substitute (6) into (8) and express the logarithm of a product as a sum of logarithms. As a result, the global utility in (8) can be rewritten as

$$U(\mathbf{r}) = \sum_{i=1}^{n} w_i \Bigg[\log \mathbb{E}_{h_i}[Q_i(h_i) C_i(h_i P_i(h_i))] + \sum_{j=1, j\neq i}^{n} \log\Big[1 - \mathbb{E}_{h_j}[Q_j(h_j)]\Big]\Bigg]. \quad (10)$$

Note that each summand in (10) only involves variables related to a particular node. Thus, we can reorder summands in (10) to group all of the terms pertaining to node i. Further defining $\tilde{w}_i := \sum_{j=1, j\neq i}^{n} w_i$, we can rewrite (10) as

$$U(\mathbf{r}) = \sum_{i=1}^{n} \Bigg[w_i \log\Big[\mathbb{E}_{h_i}[Q_i(h_i) C_i(h_i P_i(h_i))]\Big] + \tilde{w}_i \log\Big[1 - \mathbb{E}_{h_i}[Q_i(h_i)]\Big]\Bigg] := \sum_{i=1}^{n} U_i, \quad (11)$$

where we have defined the local utilities U_i. Since U_i only involves variables that are related to terminal i, it can be regarded as a utility function for terminal i. To maximize $U(\mathbf{r})$ for the whole system it suffices to separately maximize U_i for each terminal i. Introducing auxiliary variables $x_i = \mathbb{E}_{h_i}[Q_i(h_i) C_i(h_i P_i(h_i))]$ and $y_i = \mathbb{E}_{h_i}[Q_i(h_i)]$, it follows that (9) is equivalent to the following per terminal subproblems

$$\begin{aligned} \mathsf{P}_i \;=\; & \max \; w_i \log x_i + \tilde{w}_i \log(1 - y_i) \\ & \text{s.t.} \;\; x_i \le \mathbb{E}_{h_i}\left[Q_i(h_i) C_i(h_i P_i(h_i))\right], \\ & \qquad y_i \ge \mathbb{E}_{h_i}\left[Q_i(h_i)\right], \\ & \qquad \mathbb{E}_{h_i}\left[Q_i(h_i) P_i(h_i)\right] \le p_i^{\text{avg}}, \\ & \qquad x_i \ge 0, 0 \le y_i \le 1, Q_i(h_i) \in \mathcal{Q}, P_i(h_i) \in \mathcal{P}_i, \end{aligned} \quad (12)$$

where we relaxed the equality constraints to inequality ones which can be done without loss of optimality. Finding optimal solutions of (12) for all terminals i is equivalent to solving (9). Different from (9), however, there is no coupling between variables of different terminals in (12). This property leads naturally to optimal $Q_i(h_i)$ and $P_i(h_i)$ that are independent of other terminals' CSI as required by problem definition. Alas, (12) inherits the complex structure of (9).

As is the case with (9), solving (12) is difficult because: (i) The optimization space in (12) includes functions $Q_i(h_i)$ and $P_i(h_i)$ that are defined on $\mathbf{R}^+$, implying that the dimension of the problem is infinite. (ii) The rate function $C_i(h_i P_i(h_i))$ is in general non-concave with respect to $h_i P_i(h_i)$, and may be even discontinuous as in (2). (iii) The constraints involve expected values over random variables h_i whose pdfs are unknown.

An important observation is that the number of constraints in (12) is finite. This implies that while there are infinite variables in the primal domain, there are a finite number of variables in the dual domain. This observation suggests that (12) is more tractable in the dual space. Introduce then Lagrange multipliers $\boldsymbol{\lambda}_i = [\lambda_{i1}, \lambda_{i2}, \lambda_{i3}]^T$ associated with the first three inequality constraints in (12); define vectors $\mathbf{x}_i := [x_i, y_i]^T$ and $\mathbf{P}_i(h_i) := [Q_i(h_i), P_i(h_i)]^T$; and write the Lagragian of the optimization problem in (12) as

$$\begin{aligned} &\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i) \\ =& w_i \log x_i + \tilde{w}_i \log(1 - y_i) \\ &+ \lambda_{i1}\left[\mathbb{E}_{h_i}\left[Q_i(h_i) C_i(h_i P_i(h_i))\right] - x_i\right] \\ &+ \lambda_{i2}\left[y_i - \mathbb{E}_{h_i}\left[Q_i(h_i)\right]\right] + \lambda_{i3}\left[p_i^{\text{avg}} - \mathbb{E}_{h_i}\left[Q_i(h_i) P_i(h_i)\right]\right] \\ =& \lambda_{i3} p_i^{\text{avg}} + \left[w_i \log x_i - \lambda_{i1} x_i\right] + \left[\tilde{w}_i \log(1 - y_i) + \lambda_{i2} y_i\right] \\ &+ \mathbb{E}_{h_i}\left[Q_i(h_i)\left[\lambda_{i1} C_i(h_i P_i(h_i)) - \lambda_{i2} - \lambda_{i3} P_i(h_i)\right]\right]. \end{aligned} \quad (13)$$

where the second equality follows after reordering terms in the first equation. Notice that the first term in the second equality in (13) depends on x_i only, the second term on y_i and the third term on $P_i(h_i)$ and $Q_i(h_i)$. This property is exploited later on. The dual function is then defined as the maximum of the Lagrangian over the set of feasible $\mathbf{x}_i$ and $\mathbf{P}_i(h_i)$, i.e.,

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i) := \max \;& \mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i) \\ & \text{s.t. } x_i \ge 0, 0 \le y_i \le 1, Q_i(h_i) \in \mathcal{Q}, P_i(h_i) \in \mathcal{P}_i. \end{aligned} \quad (14)$$

We now can write the dual problem as the minimum of $g_i(\boldsymbol{\lambda}_i)$ over positive dual variables, i.e.,

$$\mathsf{D}_i = \min_{\boldsymbol{\lambda}_i \ge 0} \quad g_i(\boldsymbol{\lambda}_i). \quad (15)$$

In general, the optimal dual value D_i of (15) provides an upper bound for the optimal primal value P_i of (12), i.e., $\mathsf{D}_i \ge \mathsf{P}_i$. While the inequality is typically strict for non-convex problems, for the problem in (12) $\mathsf{P}_i = \mathsf{D}_i$ as long as the fading distribution has no realization with positive probability [18]. Notice that this is true despite the non-convex constraints present in (12). This lack of duality gap implies that the finite dimensional convex dual problem is equivalent to the infinite dimensional nonconvex primal problem. While this affords a substantial improvement in computational tractability, it does not necessarily mean that solving the dual problem is easy because evaluation of the dual function's value requires maximization of the Lagrangian. In particular, this maximization includes an expected value over the unknown channel distribution $f_{h_i}(h_i)$. Still, convexity of the dual function allows the use of descent algorithms in the dual domain because any local optimal solution is a global optimal solution $\boldsymbol{\lambda}_i^* = [\lambda_{i1}^*, \lambda_{i2}^*, \lambda_{i3}^*]^T$. This property is exploited next to develop a stochastic subgradient descent algorithm that solves (15) using observations of instantaneous channel realizations $h_i(t)$.

B. Adaptive Algorithms Using Stochastic Subgradient Descent

Instead of directly finding optimal x_i, y_i, $Q_i(h_i)$ and $P_i(h_i)$ for the primal problem (12), the proposed algorithm exploits the lack of duality gap to use a stochastic subgradient descent in the dual domain. Starting from given dual variables $\boldsymbol{\lambda}_i(t)$, the algorithm computes instantaneous primal variables $x_i(t)$, $y_i(t)$, $q_i(t)$ and $p_i(t)$ based on channel realization $h_i(t)$ in time slot t, and uses these values to update dual variables $\boldsymbol{\lambda}_i(t+1)$. Specifically, the algorithm starts finding primal variables that optimize the summands of the Lagrangian in (13) (the operator

$[\cdot]^+$ denotes projection in the positive orthant)

$$x_i(t) = \operatorname*{argmax}_{x_i \geq 0} \{w_i \log x_i - \lambda_{i1}(t) x_i\} = \frac{w_i}{\lambda_{i1}(t)}, \tag{16}$$

$$y_i(t) = \operatorname*{argmax}_{0 \leq y_i \leq 1} \{\tilde{w}_i \log(1 - y_i) + \lambda_{i2}(t) y_i\} = \left[1 - \frac{\tilde{w}_i}{\lambda_{i2}(t)}\right]^+, \tag{17}$$

$$\{q_i(t), p_i(t)\} = \operatorname*{argmax}_{q_i \in \{0,1\}, p_i \in [0, p_i^{\text{inst}}]} \{q_i [\lambda_{i1}(t) C_i(h_i(t) p_i) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i]\}, \tag{18}$$

The maximization in (18) determines schedules and transmitted power associated with current channel realization $h_i(t)$. Since q_i in (18) takes values on $\{0,1\}$ the objective is either 0 when $q_i = 0$ or $\lambda_{i1}(t) C_i(h_i(t) p_i) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i$ when $q_i = 1$. Thus, to solve (18) we only need to find the optimal $p_i(t)$ when $q_i(t) = 1$ and see if the resulting objective is greater than 0. Thus, we can rewrite (18) as

$$p_i(t) = \operatorname*{argmax}_{p_i \in [0, p_i^{\text{inst}}]} \{\lambda_{i1}(t) C_i(h_i(t) p_i) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i\},$$
$$q_i(t) = H\big(\lambda_{i1}(t) C_i(h_i(t) p_i(t)) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i(t)\big), \tag{19}$$

where $H(a)$ denotes Heaviside's step function with $H(a) = 1$ for $a > 0$ and $H(a) = 0$ otherwise.

Based on $x_i(t)$, $y_i(t)$, $q_i(t)$ and $p_i(t)$, define the stochastic subgradient $\mathbf{s}_i(t) = [s_{i1}(t), s_{i2}(t), s_{i3}(t)]^T$ with components

$$s_{i1}(t) = q_i(t) C_i(h_i(t) p_i(t)) - x_i(t), \tag{20}$$
$$s_{i2}(t) = y_i(t) - q_i(t), \tag{21}$$
$$s_{i3}(t) = p_i^{\text{avg}} - q_i(t) p_i(t). \tag{22}$$

The algorithm is completed with the introduction of a constant step size ϵ and a descent update in the dual domain along the stochastic subgradient $\mathbf{s}_i(t)$

$$\lambda_{il}(t+1) = [\lambda_{il}(t) - \epsilon s_{il}(t)]^+, \quad \text{for } l = 1, 2, 3. \tag{23}$$

Notice that computation of variables in (16)-(23) does not require information exchanges between terminals. This guarantees $Q_i(h_i)$ and $P_i(h_i)$ to be independent of $Q_j(h_j)$ and $P_j(h_j)$ for all $i \neq j$ as required by problem formulation. The proposed algorithm is summarized in Table I.

To analyze convergence of (16)-(23) let us start by showing that $\mathbf{s}_i(t)$ is indeed a stochastic subgradient of the dual function as stated in the following proposition.

Proposition 1: *Given $\boldsymbol{\lambda}_i(t)$, the expected value of the stochastic subgradient $\mathbf{s}_i(t)$ is a subgradient of the dual function in* (14)*, i.e., $\forall \boldsymbol{\lambda}_i \geq 0$,*

$$\mathbb{E}_{h_i}\left[\mathbf{s}_i^T(t) | \boldsymbol{\lambda}_i(t)\right] (\boldsymbol{\lambda}_i(t) - \boldsymbol{\lambda}_i) \geq g_i(\boldsymbol{\lambda}_i(t)) - g_i(\boldsymbol{\lambda}_i). \tag{24}$$

In particular,

$$\mathbb{E}_{h_i}\left[\mathbf{s}_i^T(t) | \boldsymbol{\lambda}_i(t)\right] (\boldsymbol{\lambda}_i(t) - \boldsymbol{\lambda}_i^*) \geq g_i(\boldsymbol{\lambda}_i(t)) - \mathsf{D}_i \geq 0. \tag{25}$$

Proof: See Appendix A. ■

Proposition 1 states that the average of the stochastic subgradient $\mathbf{s}_i(t)$ is a subgradient of the dual function. We can then think of an alternative algorithm by replacing $\mathbb{E}_{h_i}\left[\mathbf{s}_i(t) \middle| \boldsymbol{\lambda}_i(t)\right]$ for $\mathbf{s}_i(t)$ in the dual iteration step (23), which would amount to a subgradient descent algorithm for the dual function. Since, $\mathbb{E}_{h_i}\left[\mathbf{s}_i(t) \middle| \boldsymbol{\lambda}_i(t)\right]$ points towards $\boldsymbol{\lambda}^*$ – the angle between $\mathbb{E}_{h_i}\left[\mathbf{s}_i(t) \middle| \boldsymbol{\lambda}_i(t)\right]$ and $\boldsymbol{\lambda}_i(t) - \boldsymbol{\lambda}_i^*$ is positive as indicated by (25) –, it is not difficult to prove that $\boldsymbol{\lambda}_i(t)$ eventually approaches $\boldsymbol{\lambda}_i^*$, e.g., [21, Ch. 2]. However, since we assume the pdf of h_i is unknown, the subgradient $\mathbb{E}_{h_i}\left[\mathbf{s}_i(t) \middle| \boldsymbol{\lambda}_i(t)\right]$ can only be approximated using past channel realizations $h_i(1), \ldots, h_i(t)$. While this approach is possible, it is computationally costly.

TABLE I
ADAPTIVE DISTRIBUTED SCHEDULING AND POWER CONTROL ALGORITHM FOR OPTIMAL RANDOM ACCESS

Algorithm 1: Adaptive scheduling and power control at terminal i

Initialize Lagrangian multipliers $\boldsymbol{\lambda}_i(0)$;
for $t = 0, 1, 2, \cdots$ **do**
  Compute primal variables as per (16), (17), and (19):
    $x_i(t) = \frac{w_i}{\lambda_{i1}(t)}$;
    $y_i(t) = \left[1 - \frac{\tilde{w}_i}{\lambda_{i2}(t)}\right]^+$;
    $p_i(t) = \operatorname*{argmax}_{p_i \in [0, p_i^{\text{inst}}]} \{\lambda_{i1}(t) C_i(h_i(t) p_i) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i\}$;
    $q_i(t) = H\big(\lambda_{i1}(t) C_i(h_i(t) p_i(t)) - \lambda_{i2}(t) - \lambda_{i3}(t) p_i(t)\big)$;
  if $q_i(t) = 1$ **then**
    Transmit with power $p_i(t)$;
  end
  Compute stochastic subgradients as per (20)-(22):
    $s_{i1}(t) = q_i(t) C_i(h_i(t) p_i(t)) - x_i(t)$;
    $s_{i2}(t) = y_i(t) - q_i(t)$;
    $s_{i3}(t) = p_i^{\text{avg}} - q_i(t) p_i(t)$;
  Update dual variables as per (23):
    $\lambda_{il}(t+1) = [\lambda_{il}(t) - \epsilon s_{il}(t)]^+$, for $l = 1, 2, 3$;
end

The computation of the stochastic subgradient $\mathbf{s}_i(t)$, on the contrary, is simple because it only depends on the current channel state $h_i(t)$. Furthermore, since $\mathbf{s}_i(t)$ points towards the set of optimal dual variables $\boldsymbol{\lambda}_i^*$ on average [cf. (25)] it is reasonable to expect the stochastic subgradient descent iterations in (23) to also approach $\boldsymbol{\lambda}_i^*$ in some sense. This can be proved true and leveraged to prove almost sure convergence of primal iterates $x_i(t)$, $y_i(t)$, $p_i(t)$ and $q_i(t)$ to an optimal operating point in an ergodic sense [22]. Specifically, Theorem 1 of [22] assumes as hypotheses that the second moment of the norm of the stochastic subgradient $\mathbf{s}_i(t)$ is finite, i.e., $\mathbb{E}_{h_i}\left[\|\mathbf{s}_i(t)\|^2 \middle| \boldsymbol{\lambda}_i(t)\right] \leq \hat{S}_i^2$, and that there exists a set of strictly feasible primal variables that satisfy the constraints in (12) with strict inequality. If these hypotheses are true, primal iterates of dual stochastic subgradient descent are almost surely feasible in an ergodic sense. For the particular case of the problem in (12), [22, Theorem 1] implies that

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} q_i(u) p_i(u) \leq p_i^{\text{avg}} \quad \text{a.s.}, \tag{26}$$

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} x_i(u) \leq \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} q_i(u) C_i(h_i(u) p_i(u)) \quad \text{a.s.}, \tag{27}$$

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} y_i(u) \geq \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} q_i(u) \quad \text{a.s.} \tag{28}$$

It also follows from [22, Theorem 1] that $x_i(t)$ and $y_i(t)$ yield ergodic utilities that are almost surely within $\epsilon\hat{S}_i^2/2$ of optimal, i.e.,

$$\mathsf{P}_i - \lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}\left[w_i\log x_i(u) + \tilde{w}_i\log(1-y_i(u))\right] \leq \frac{\epsilon\hat{S}_i^2}{2} \text{ a.s.} \tag{29}$$

From (26) we can conclude that the ergodic limit of the power allocated by the proposed algorithm satisfies the average power constraint. However, (29) does not imply that the scheduling and power allocation variables $p_i(t)$ and $q_i(t)$ are optimal. The optimality claim in (29) is for the auxiliary variables $x_i(t)$ and $y_i(t)$ but the goal here is to claim optimality of the scheduling and power allocation variables $p_i(t)$ and $q_i(t)$. To prove optimality of the algorithm, we need to show that the ergodic transmission rate r_i of (4), achieved by allocations $q_i(t)$ and $p_i(t)$ is optimal in the sense of maximizing the throughput utility $U(\mathbf{r}) = \sum_{i=1}^{n} w_i\log(r_i)$.

If the constraints in (12) were satisfied for all times t, i.e., if $x_i(t) \leq q_i(t)C_i(h_i(t)p_i(t))$ and $y_i(t) \geq q_i(t)$, transforming (29) into an almost sure near optimality claim for the ergodic limit r_i is a simple matter of substitution and algebraic manipulation. However, these inequalities do not necessarily hold for all times t. They hold in an ergodic sense as stated in (27) and (28). This subtle yet fundamental mismatch is addressed in the proof of the following theorem.

Theorem 1: *Consider a random multiple access channel with n terminals using schedules $q_i(t)$ and power allocations $p_i(t)$ generated by the algorithm defined by* (16)-(23) *resulting in instantaneous transmission rates $r_i(t)$ as given by* (3) *and ergodic rates r_i as defined by* (4)*. Define vector $\mathbf{r} := [r_1,\ldots,r_n]^T$, and let $U(\mathbf{r})$ be the weighted proportional fair utility in* (8)*. Assume that the second moment of the norm of the stochastic subgradient $\mathbf{s}_i(t)$ with components as in* (20)-(22) *is finite* [1]*, i.e., $\mathbb{E}_{h_i}\left[\|\mathbf{s}_i(t)\|^2\middle|\boldsymbol{\lambda}_i(t)\right] \leq \hat{S}_i^2$, and that there exists a set of strictly feasible primal variables that satisfy the constraints in* (12) *with strict inequality. Then, the average power constraint is almost surely satisfied*

$$\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} q_i(u)p_i(u) \leq p_i^{avg} \quad a.s., \tag{30}$$

and the utility of the ergodic limit of the transmission rates almost surely converges to a value within $\epsilon/2\sum_{i=1}^{n}\hat{S}_i^2$ of optimality,

$$\mathsf{P} - U(\mathbf{r}) := \mathsf{P} - \sum_{i=1}^{n} w_i\log\left(\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} r_i(u)\right) \leq \frac{\epsilon}{2}\sum_{i=1}^{n}\hat{S}_i^2. \tag{31}$$

Proof: The hypotheses of Theorem 1 are chosen to satisfy the hypotheses guaranteeing convergence of ergodic stochastic optimization algorithms [22, Theorem 1]. Thus, almost sure feasibility and almost sure near optimality of iterates $x_i(t)$, $y_i(t)$, $p_i(t)$ and $q_i(t)$ follows in the sense of (26)-(29). To establish almost sure satisfaction of average power constraints as per (30) just notice that this inequality coincides with the one in (26). To establish (31) start by rearranging terms in (29) to conclude that $\mathsf{P}_i - \epsilon\hat{S}_i^2/2 \leq \lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}\left[w_i\log x_i(u) + \tilde{w}_i\log(1-y_i(u))\right]$. Due to continuity and concavity of the logarithm function we can further bound $\mathsf{P}_i - \epsilon\hat{S}_i^2/2$ as

$$\begin{aligned}\mathsf{P}_i - \frac{\epsilon\hat{S}_i^2}{2} \leq & w_i\log\left[\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} x_i(u)\right] \\ & + \tilde{w}_i\log\left[1 - \lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} y_i(u)\right].\end{aligned} \tag{32}$$

The limits in (32) are equal to the limits in the left hand sides of the inequalities in (27) and (28). Thus, using this almost sure ergodic feasibility results $\mathsf{P}_i - \epsilon\hat{S}_i^2/2$ is bounded as

$$\begin{aligned}\mathsf{P}_i - \frac{\epsilon\hat{S}_i^2}{2} \leq & w_i\log\left[\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} q_i(u)C_i(h_i(u)p_i(u))\right] \\ & + \tilde{w}_i\log\left[1 - \lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t} q_i(u)\right].\end{aligned} \tag{33}$$

Ergodicity, possibly restricted to an ergodic component, allows replacement of the ergodic limits in (34) by the corresponding expected values, leading to the bound

$$\begin{aligned}\mathsf{P}_i - \frac{\epsilon\hat{S}_i^2}{2} \leq & w_i\log\mathbb{E}_{h_i}\left[Q_i(h_i)C_i(h_iP_i(h_i))\right] \\ & + \tilde{w}_i\log\mathbb{E}_{h_i}[1 - Q_i(h_i)].\end{aligned} \tag{34}$$

Recall that $\mathsf{P} = \sum_{i=1}^{n}\mathsf{P}_i$ per definition, and consider the sum of the inequalities in (34) for all terminals i so as to write

$$\begin{aligned}\mathsf{P} - \sum_{i=1}^{n}\frac{\epsilon\hat{S}_i^2}{2} \leq & \sum_{i=1}^{n} w_i\log\mathbb{E}_{h_i}\left[Q_i(h_i)C_i(h_iP_i(h_i))\right] \\ & + \tilde{w}_i\log\mathbb{E}_{h_i}[1 - Q_i(h_i)] \\ \leq & \sum_{i=1}^{n} w_i\log\Bigg[\mathbb{E}_{h_i}\left[Q_i(h_i)C_i(h_i(t)P_i(h_i))\right] \\ & \prod_{j=1,j\neq i}^{n}\mathbb{E}_{h_j}[1 - Q_j(h_j)]\Bigg],\end{aligned} \tag{35}$$

where the second inequality follows by using the definition $\tilde{w}_i := \sum_{j=1,j\neq i}^{n} w_i$, reordering terms in the sum, and rewriting a sum of logarithms as the logarithm of a product.

The fundamental observation in this proof is that the scheduling function $Q_i(h_i)$ and the power allocation function $P_i(h_i)$ are independent of the corresponding $Q_j(h_j)$ and $P_j(h_j)$ of other terminals. This is not a coincidence, but the intended goal of reformulating (9) as (12). Using this independence, the product of expectations in (35) can be written as single expectation over the vector channel $\mathbf{h}$ to yield

$$\begin{aligned}&\mathsf{P} - \sum_{i=1}^{n}\frac{\epsilon\hat{S}_i^2}{2} \leq \\ &\sum_{i=1}^{n} w_i\log\left[\mathbb{E}_{\mathbf{h}}\left(Q_i(h_i)C_i(h_iP_i(h_i))\prod_{j=1,j\neq i}^{n}(1 - Q_j(h_j))\right)\right].\end{aligned} \tag{36}$$

To finalize the proof use ergodicity, possibly restricted to an ergodic component, to substitute the expectation in (36) by an

[1]The finite assumption of the second moment of the subgradients is necessary for the proof of almost sure near optimality of the ergodic stochastic optimization algorithm [22].

ergodic limit to yield

$$\mathsf{P} - \sum_{i=1}^{n} \frac{\epsilon \hat{S}_i^2}{2} \leq \sum_{i=1}^{n} w_i \log \left[\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} q_i(u) C_i(h_i(u)p_i(u)) \prod_{j=1, j\neq i}^{n} (1 - q_j(u)) \right] := U(\mathbf{r}), \tag{37}$$

where we have used the definitions of the ergodic rate in (4) and of the utility in (8). The result in (31) follows after reordering terms in (37). ■

Theorem 1 states that the stochastic dual descent algorithm in (16)-(23) computes schedules $q_i(t)$ and power allocations $p_i(t)$ yielding rates $r_i(t)$ that are almost surely near optimal in an ergodic sense [cf. (31)]. It also states that $p_i(t)$ satisfies the average power constraint with probability 1. Notice that the stochastic dual descent algorithm in (16)-(23) does not compute the optimal scheduling and power control functions for each terminal. Rather, it draws schedules $q_i(t)$ and power allocations $p_i(t)$ that are close to the optimal functions. This is not a drawback because the latter property is sufficient for a practical implementation. Further note that the use of constant step sizes ϵ endows the algorithm with adaptability to time-varying channel distributions. This is important in practice because wireless channels are non-stationary due to user mobility and environmental dynamics. The gap between $U(\mathbf{r})$ and P can be made arbitrarily small by reducing ϵ.

Remark 2: The desired optimal schedules $Q^*(h(t))$ and power allocations $P^*(h(t))$ as prescribed in Section II are functions of the current channel realizations only. The proposed online policy, however, computes schedules $q_i(t)$ and power allocations $p_i(t)$ based on the current channel $h_i(t)$ and dual variables $\boldsymbol{\lambda}_i(t)$. In each time slot the iterative policy updates $\boldsymbol{\lambda}_i(t)$ using $\boldsymbol{\lambda}_i(t-1)$ and stochastic subgradients $\mathbf{s}_i(t)$ which depend on $q_i(t)$, $p_i(t)$ and $h_i(t)$. As a result, the dual variable $\boldsymbol{\lambda}_i(t)$ depends on all previous channel gains from $h_i(0)$ up to $h_i(t)$. Since $q_i(t)$ and $p_i(t)$ are functions of $\boldsymbol{\lambda}_i(t)$, they depend on all previous channel gains as well. This is not a contradiction because as the algorithm progresses, $\boldsymbol{\lambda}_i(t)$ approaches the optimal multiplier $\boldsymbol{\lambda}_i^*$, implying that the time-dependent variables $q_i(t), p_i(t)$ converge towards the optimal policy $P^*(h(t)), Q^*(h(t))$. As a matter of fact, $\boldsymbol{\lambda}_i(t)$ does not converge to $\boldsymbol{\lambda}_i^*$, but to a neighborhood of $\boldsymbol{\lambda}_i^*$. This results in some residual time dependence in the variables $q_i(t), p_i(t)$ that accounts for the algorithm's (arbitrarily small) optimality penalty as stated in Theorem 1.

C. Structure of the Optimal Primal Solution

While the algorithm in (16)-(23) provides a method to find the optimal operating point for the random multiple access channel, it does not provide intuition on the properties of this operating point. This section studies structural properties of the optimal primal solution.

In convex optimization problems optimal primal variables are obtained as the Lagrangian maximizers for optimal dual variables. The optimization problem in (12) is not convex. This is not a hindrance because the recovery of optimal primals from optimal duals through Lagrangian maximization follows from the lack of duality gap, which is a property that (12) does possess [18]. Let us then begin by showing that the optimal primal variables $\mathbf{x}_i^* = [x_i^*, y_i^*]^T$ and $\mathbf{P}_i^*(h_i) = [Q_i^*(h_i), P_i^*(h_i)]^T$ of the primal problem in (12) can be obtained from the maximizers of the Lagrangian $\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i^*)$. From the definition of the dual function in (14), the optimal dual value can be written as

$$\mathsf{D}_i = g_i(\boldsymbol{\lambda}_i^*) = \max \mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i^*) \quad \text{s.t. } x_i \geq 0, 0 \leq y_i \leq 1, Q_i(h_i) \in \mathcal{Q}, P_i(h_i) \in \mathcal{P}_i. \tag{38}$$

Since the maximization in (38) is with respect to all primal variables satisfying the stated constraints and the optimal variables $\mathbf{x}_i^*$ and $\mathbf{P}_i^*(h_i)$ satisfy these constraints, it must be

$$\mathsf{D}_i \geq \mathcal{L}_i(\mathbf{x}_i^*, \mathbf{P}_i^*(h_i), \boldsymbol{\lambda}_i^*). \tag{39}$$

Consider now the explicit expression of $\mathcal{L}_i(\mathbf{x}_i^*, \mathbf{P}_i^*(h_i), \boldsymbol{\lambda}_i^*)$ as it follows from the definition in (13)

$$\begin{aligned}
&\mathcal{L}_i(\mathbf{x}_i^*, \mathbf{P}_i^*(h_i), \boldsymbol{\lambda}_i^*) \\
&= w_i \log x_i^* + \tilde{w}_i \log(1 - y_i^*) \\
&+ \lambda_{i1}^* \left[\mathbb{E}_{h_i}\left[Q_i^*(h_i) C_i(h_i P_i^*(h_i))\right] - x_i^*\right] \\
&+ \lambda_{i2}^* \left[y_i^* - \mathbb{E}_{h_i}\left[Q_i^*(h_i)\right]\right] + \lambda_{i3}^* \left[p_i^{\text{avg}} - \mathbb{E}_{h_i}\left[Q_i^*(h_i) P_i^*(h_i)\right]\right].
\end{aligned} \tag{40}$$

Since $\mathbf{x}_i^*$ and $\mathbf{P}_i^*(h_i)$ are solutions of (12), they are feasible, i.e., they satisfy the inequalities in (12). Thus, the terms $\mathbb{E}_{h_i}\left[Q_i^*(h_i) C_i(h_i P_i^*(h_i))\right] - x_i^* \geq 0$, $y_i^* - \mathbb{E}_{h_i}\left[Q_i^*(h_i)\right] \geq 0$, and $p_i^{\text{avg}} - \mathbb{E}_{h_i}\left[Q_i^*(h_i) P_i^*(h_i)\right] \geq 0$ are all nonnegative. Since the Lagrange multipliers $\lambda_{i1} \geq 0$, $\lambda_{i2} \geq 0$, and $\lambda_{i3} \geq 0$, are also nonnegative, it holds

$$\mathsf{D}_i \geq \mathcal{L}_i(\mathbf{x}_i^*, \mathbf{P}_i^*(h_i), \boldsymbol{\lambda}_i^*) \geq w_i \log x_i^* + \tilde{w}_i \log(1 - y_i^*) = \mathsf{P}_i, \tag{41}$$

where the first inequality follows from (39) and the last equality from the fact that $\mathbf{x}_i^*$ is optimal. Since the duality gap is null, i.e., $\mathsf{D}_i = \mathsf{P}_i$, the inequalities in (41) must hold with equality. It then must be that $\mathbf{x}_i^*$ and $\mathbf{P}_i^*(h_i)$ are a solution to the maximization in (38). Further note that because $\mathbf{x}_i$ and $\mathbf{P}_i(h_i)$ appear in different terms in $\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i^*)$, the joint maximization with respect to $\mathbf{x}_i$ and $\mathbf{P}_i(h_i)$ can be carried out as separate maximizations with respect to $\mathbf{x}_i$ and $\mathbf{P}_i(h_i)$ [cf. (49)]. In particular, for $\mathbf{P}_i^*(h_i)$ we have

$$\{Q_i^*(h_i), P_i^*(h_i)\} \in \operatorname*{argmax}_{Q_i(h_i), P_i(h_i)} \mathbb{E}_{h_i}\left[Q_i(h_i)\left[\lambda_{i1}^* C_i(h_i P_i(h_i)) - \lambda_{i2}^* - \lambda_{i3}^* P_i(h_i)\right]\right]. \tag{42}$$

where the relation is belong to ($\in$) rather than equality ($=$) because there might be more than one argument that maximizes the expression in (42).

Due to linearity of the expectation operator $\mathbb{E}_{h_i}[\cdot]$, to maximize the expected value with respect to the functions $Q_i(h_i) \in \mathcal{Q}$ and $P_i(h_i) \in \mathcal{P}_i$ it is equivalent to maximize with respect to individual values. Therefore, it must be for all $h_i > 0$,

$$\{Q_i^*(h_i), P_i^*(h_i)\} \in \operatorname*{argmax}_{q_i \in \{0,1\}, p_i \in [0, p_i^{\text{inst}}]} \left\{q_i\left[\lambda_{i1}^* C_i(h_i p_i) - \lambda_{i2}^* - \lambda_{i3}^* p_i\right]\right\}. \tag{43}$$

Using the expression in (43) it is possible to infer that the optimal scheduling function $Q_i^*(h_i)$ is a threshold rule as stated in the following theorem.

Theorem 2: *The optimal scheduling function $Q_i^*(h_i)$ solving* (12) *is a threshold rule. I.e., there exists a constant h_0 such that $Q_i^*(h_i) = H(h_i - h_0)$.*

Proof: Let us start by elaborating on the implications of (43). Define $u_i(p_i, h_i) := \lambda_{i1}^* C_i(h_i p_i) - \lambda_{i2}^* - \lambda_{i3}^* p_i$ as the part of the maximand of (43) that depends on p_i and let $v_i(h_i) := \max_{p_i \in [0, p_i^{\text{inst}}]} \{u_i(p_i, h_i)\}$ be the maximum of $u_i(p_i, h_i)$ over allowed p_i. If for given h_i, we have $v_i(h_i) > 0$ it then must be $Q_i^*(h_i) = 1$ because $q_i = 1$ is the sole argument maximizing the expression in (43). Likewise, if $v_i(h_i) < 0$ it must be $Q_i^*(h_i) = 0$. When $v_i(h_i) = 0$ the value of $Q_i^*(h_i)$ cannot be inferred from (43) because both $q_i = 0$ and $q_i = 1$ are maximizing arguments. We then conclude the following two implications pertaining to $Q_i^*(h_i) = 1$: (i) if $v_i(h_i) > 0$ then $Q_i^*(h_i) = 1$; and (ii) if $Q_i^*(h_i) = 1$ then $v_i(h_i) \geq 0$.

To prove that the optimal schedule is a threshold rule it suffices to prove that if $Q_i^*(h_i) = 1$ for some given h_i then $Q_i^*(h_i') = 1$ for any $h_i' > h_i$. We will prove that for h_i' it must be $v_i(h_i') > 0$ from where $Q_i^*(h_i') = 1$ follows as per implication (i) of the previous paragraph. To prove that $v_i(h_i') > 0$ let p_0 denote a maximizer of $u_i(p_i, h_i)$ so that $v_i(h_i) = u_i(p_0, h_i)$. Since $Q_i^*(h_i) = 1$ it follows from implication (ii) in the previous paragraph that $u_i(p_0, h_i) = v_i(h_i) \geq 0$. Observing that for $p_i = 0$ we have $u_i(0, h_i) = -\lambda_{i2}^* < 0$ it follows that it must be $p_0 > 0$. Define now power $p_0' = (h_i/h_i')p_0$. With this selection it follows $h_i p_0 = h_i' p_0'$ and as a consequence $C(h_i p_0) = C(h_i' p_0')$. We can then write the difference $u_i(p_0', h_i') - u_i(p_0, h_i)$ as

$$
\begin{aligned}
&u_i(p_0', h_i') - u_i(p_0, h_i) \\
&= \Big[C(h_i' p_0') - \lambda_{i2}^* - \lambda_{i3}^* p_0'\Big] - \Big[C(h_i p_0) - \lambda_{i2}^* - \lambda_{i3}^* p_0\Big] \\
&= \lambda_{i3}^* p_0 \left(1 - \frac{h_i}{h_i'}\right) > 0
\end{aligned} \tag{44}
$$

where the inequality indicating a strictly positive difference follows from the fact that $h_i' > h_i$ and that $p_0 \neq 0$. Since $u_i(p_0, h_i) \geq 0$ it follows from (44) that $u_i(p_0', h_i') > 0$ and as a consequence that the maximum value $v_i(h_i') \geq u_i(p_0', h_i') > 0$. From implication (i) it then follows that $Q_i^*(h_i') = 1$ and that the optimal schedule is a threshold rule as already argued. ■

When there is no power control function and the rate function is continuous, the optimality of threshold-based schedulers has been proved in [7]. This result is extended here to general cases allowing for power control and the use of discontinuous rate functions. It is worth emphasizing that the optimality of a threshold-based scheduler is independent of the specific form of the rate function $C_i(h_i P_i(h_i))$. Recall that the sole constraint on the function $C_i(h_i P_i(h_i))$ is that is must be finite for finite argument.

If the form of the transmission rate function $C_i(h_i P_i(h_i))$ is known, it is also possible to infer functional forms for the optimal power control functions $P_i^*(h_i)$. If AMC is used at the physical layer the rate function takes the form in (2). In this case it is possible to find unique maximizers of (43) that as a consequence determine the form of the optimal power allocation $P_i^*(h_i)$. The corresponding functional form requires finding the AMC mode $m^* = \text{argmax}_{m=\{1,\ldots,M\}}\{\lambda_1^* \tau_m - \lambda_2^* - \lambda_3^* \frac{\eta_m N_0 B}{h_i}\}$ and setting the transmitted power to

$$
P_i^*(h_i) = \frac{\eta_{m^*} N_0 B}{h_i} Q_i(h_i), \tag{45}
$$

With capacity achieving codes used at the physical layer the rate function takes the form in (1). The optimal power control function then takes the form

$$
P_i^*(h_i) = \left(\frac{\lambda_{i1}^*}{\lambda_{i3}^*} - \frac{N_0}{h_i}\right) B Q(h_i), \tag{46}
$$

because the $P_i^*(h_i)$ in (46) are the unique arguments maximizing (43). The expression in (46) implies the optimality of power waterfilling across fading states.

Remark 3: Since the optimal policy is a function of the channels' probability distribution, it seems that these distributions have to be estimated in order to design the optimal policy. However, the proposed algorithm in Table I only maintains three Lagrange multipliers $\lambda_{i1}(t)$, $\lambda_{i2}(t)$ and $\lambda_{i3}(t)$. The reason for this is that as can be seen in (43) the optimal solution can be uniquely determined by the optimal Lagrange multipliers λ_{i1}^*, λ_{i2}^* and λ_{i3}^*. Thus, instead of learning the channels' probability distribution it suffices to learn the optimal dual variables $\boldsymbol{\lambda}_i^*$. Learning $\boldsymbol{\lambda}_i^*$ is, in effect, the purpose of the algorithm in Table I. This is an important simplification. Whereas the unknown channel distributions are infinite-dimensional, the dual variables $\boldsymbol{\lambda}_i^*$ are 3-dimensional.

Remark 4: It is possible to interpret (43) in economic terms. Consider λ_{i1}^* as the reward for transmitting a unit of information, while regarding λ_{i2}^* and λ_{i3}^* as the prices for accessing the channel once and for consuming a unit of transmit power, respectively. With these interpretations, $u_i(p_i, h_i)$ represents the profit generated by transmitting with power p_i when the channel state is h_i, and $v_i(h_i)$ is the maximum profit that can be obtained while satisfying the instantaneous power constraint. Consequently, (43) can be interpreted as stating that terminals are allowed to transmit if and only if their maximum possible profits are positive.

IV. Numerical Results

To illustrate performance of the proposed algorithms, we conduct numerical experiments on a network with $n = 20$ terminals randomly placed in a square with side $L = 100$ m and a common AP located at the center of the square. Numerical experiments here utilize the realization of this random placement shown in Fig. 1. Communication between terminals and the AP is over a bandlimited Gaussian channel with bandwidth B and noise power spectral density N_0. We set $B = 1$ so that capacities are measured in bits per second per Hertz (b/s/Hz) and $N_0 = 10^{-10}$ W. Channel gains $h_i(t)$ are Rayleigh distributed with mean $\bar{h}_i$ and are independent across terminals and time. The average channel gain $\bar{h}_i := \mathbb{E}[h_i]$ follows an exponential pathloss law, $\bar{h}_i = \alpha d_i^{-\beta}$ with $\alpha = 10^{-6}\text{m}^{-1}$ and $\beta = 2$ constants and d_i denoting the distance in meters between terminal i and the AP. All weights in the proportional fair utility in (8) are set to $w_i = 1$. Throughout, the performance metric of interest is the average transmission

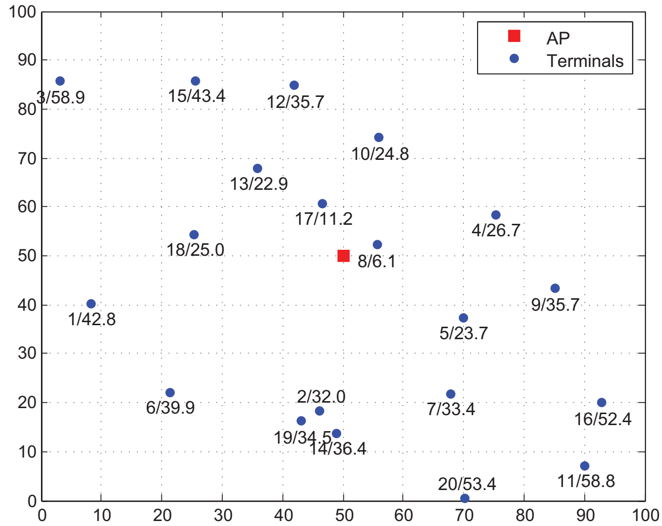

Fig. 1. An example multiple access channel with $n = 20$ nodes communicating with a common access point (AP). Nodes are randomly placed in a 100 m $\times$ 100 m square and the AP is located at the center of the square. Nodes' labels represent indexes and distances to the AP. Subsequent numerical experiments use this realization of the random placement.

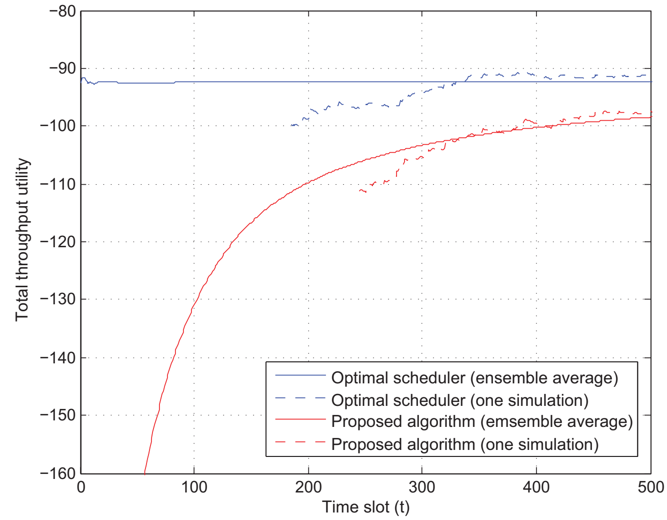

Fig. 2. Convergence of the proposed algorithm to near optimal utility with instantaneous power constrains but no average power constraints. Throughput utility of the proposed adaptive algorithm and of the optimal offline scheduler are shown as functions of time for one realization and for the ensemble average of realizations. In steady state the adaptive algorithm operates with minimal performance loss with respect to the optimal offline scheduler. A utility gap smaller than 10 is achieved in about 350 iterations. Power constraint $p_i^{\text{inst}} = 100$ mW, step size $\epsilon = 0.1$, capacity achieving codes.

rate $\bar{r}_i(t)$ of terminal i at time t defined as

$$\bar{r}_i(t) = \frac{1}{t}\sum_{u=1}^{t} r_i(u), \tag{47}$$

where $r_i(u)$ is normalized so that it represents bits/s/Hz. The system's throughput utility by time t is then defined in terms of $\bar{r}_i(t)$ as $\bar{U}(t) := \sum_{i=1}^{n} w_i \log(\bar{r}_i(t))$.

The algorithm in (16)-(23) is first tested in a network where nodes use capacity achieving codes and have instantaneous power constraints but do not have average power constraints; see Section IV-A. We then consider nodes that have average as well as instantaneous power constraints using AMC; see Section IV-B.

A. System with Instantaneous Power Constraint

Assume the use of capacity achieving codes so that the rate function for terminal i takes the form in (1). Further assume that there is an instantaneous power constraint $p_i^{\text{inst}} = 100$ mW for each terminal, but that there is no average power constraint. Since the rate function is a nonnegative increasing function of power it is optimal for each terminal to transmit with its maximum allowed instantaneous power every time it decides to transmit. Therefore, the power control function is a constant $p_i(t) = p_i^{\text{inst}}$ and the system's performance depends solely on the terminals' scheduling functions $q_i(t)$. In this simplified setting, a closed form solution for $q_i(t)$ is known if the channel pdf is available [7]. Our interest in this simplified problem is that it allows a performance comparison between the schedules yielded by (16)-(23) and those of the optimal offline scheduler.

Convergence of (16)-(23) to a near optimal operating point is illustrated in Fig. 2 for step size $\epsilon = 0.1$. The ergodic utility $\bar{U}(t)$ is shown through 500 iterations and is compared with the utility of the optimal offline scheduler. When using (16)-(23) the total throughput utility converges to a value with negligible optimality gap with respect to the offline scheduler. Observe that convergence is fast as it takes less than 180 iterations to reach a utility with optimality gap smaller than 20 and 360 iterations to get an optimality gap smaller than 10. Figs. 3 and 4 respectively show average rates and transmission probabilities after 500 iterations for each terminal. Observe in Fig. 3 that all terminals achieve average rates that are very close to the optimal ones. Further observe that even though terminals experience different channel conditions, fair schedules are obtained as a consequence of the use of a logarithmic utility. Indeed, as seen in Fig. 4, average transmission probabilities are close for all terminals. Note, however, that the achieved rates shown in Fig. 3 are different because terminals have different average channels.

To test how the optimality gap changes as the step size ϵ varies, we ran the algorithm (16)-(23) with different step sizes. Fig. 5 shows the optimality gap when the step size ϵ varies between 10^{-2} to 10^{-1}. The optimality gap indeed decreases as the step size ϵ is reduced. This corroborates the result of Theorem 1 that ensures a vanishing optimality gap as $\epsilon \to 0$. Using smaller step size, however, leads to slower convergence. This tradeoff between convergence speed and optimality gap determines the choice of ϵ for practical implementations.

B. System with Average Power Constraint

For the same network in Fig. 1, consider now the case in which each terminal adheres to both, instantaneous and average power constraints. We also deviate from Section IV-A in the use of AMC instead of capacity achieving codes at the physcial layer, so that the rate function for terminal i takes the form in (2). Each terminal has $M = 4$ AMC modes with respective rates $\tau_1 = 1$ bits/s/Hz, $\tau_2 = 2$ bits/s/Hz, $\tau_3 = 3$ bits/s/Hz, and $\tau_4 = 4$ bits/s/Hz. The transitions between AMC modes are at SNRs $\eta_1 = 1$, $\eta_2 = 4$, $\eta_3 = 8$, and $\eta_4 = 16$. The instantaneous power constraint is set to $p_i^{\text{inst}} = 100$ mW and the average power constraint to $p_i^{\text{avg}} = 5$ mW for all terminals i.

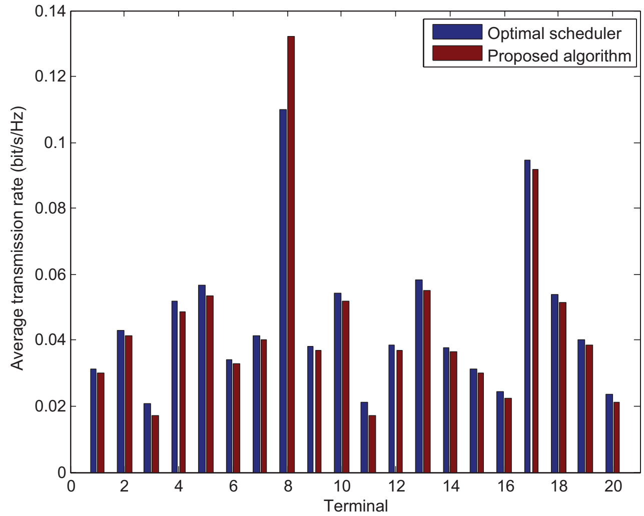

Fig. 3. Average transmission rates (bits/s/Hz) in 500 time slots, i.e., $\bar{r}_i(500)$ as defined in (47), for all terminals. The optimal offline scheduler and the proposed adaptive algorithm yield similar close to optimal average rates. The variation in achieved rates is commensurate with the variation in average signal to noise ratios (SNRs) due to different distances to the access point. For the network in Fig.1 and the pathloss and power parameters used here, average signal to noise ratios vary between 0.4 and 10. Instantaneous power constraint $p_i^{\text{inst}} = 100$ mW, step size $\epsilon = 0.1$, capacity achieving codes.

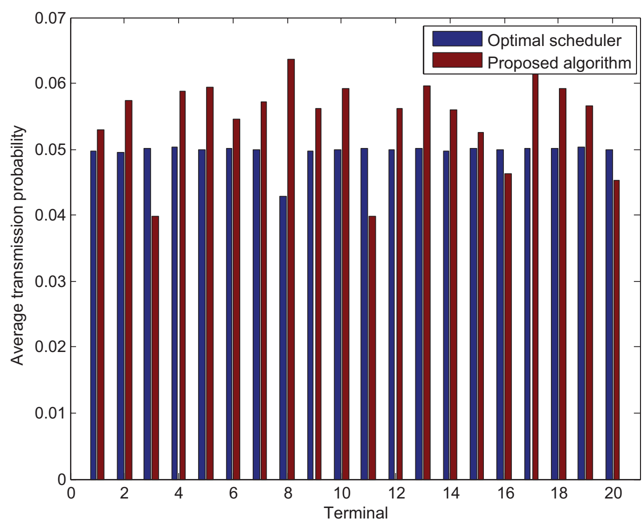

Fig. 4. Average transmission probabilities in 500 time slots for all terminals. Offline and adaptive optimal schedulers shown. Despite different channel conditions all terminals transmit with a similar probability close to $1/n = 0.05$. This is consistent with the use of a logarithmic, i.e., proportional fair, utility. Instantaneous power constraint $p_i^{\text{inst}} = 100$ mW, step size $\epsilon = 0.1$, capacity achieving codes.

To demonstrate optimality of the proposed algorithm, we compute the primal objective $\bar{U}(t)$, the dual value $D(t) = \sum_{i=1}^{n} g_i(\boldsymbol{\lambda}_i(t))$, and examine the duality gap between them. Fig. 6 shows $\bar{U}(t)$ and $D(t)$ for 10^3 time slots. As time grows, the duality gap decreases and eventually approaches a small positive constant, implying near optimality of the proposed algorithm.

To test the satisfaction of the average power constraint, define the average power consumption of terminal i by time t as

$$\bar{p}_i(t) = \frac{1}{t} \sum_{u=1}^{t} p_i(u). \tag{48}$$

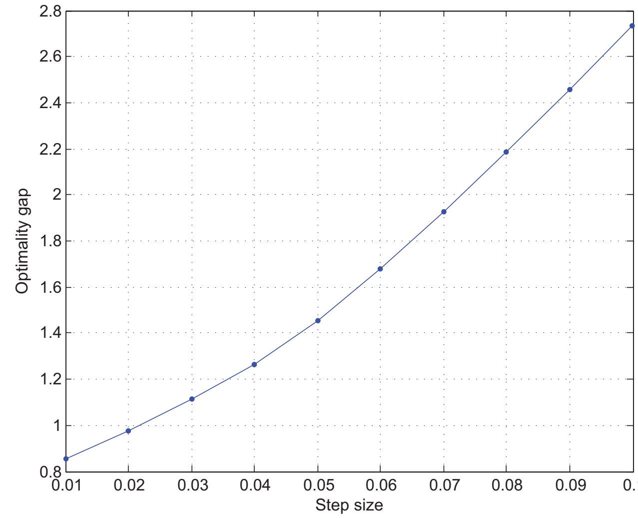

Fig. 5. Steady state optimality gap between proposed adaptive algorithm and optimal offline scheduler as a function of step size ϵ. Values of ϵ between 10^{-2} and 10^{-3} shown. As the step size decreases, the optimality gap decreases. The optimality gap can be made arbitrarily small by reducing ϵ. Instantaneous power constraint $p_i^{\text{inst}} = 100$ mW, capacity achieving codes.

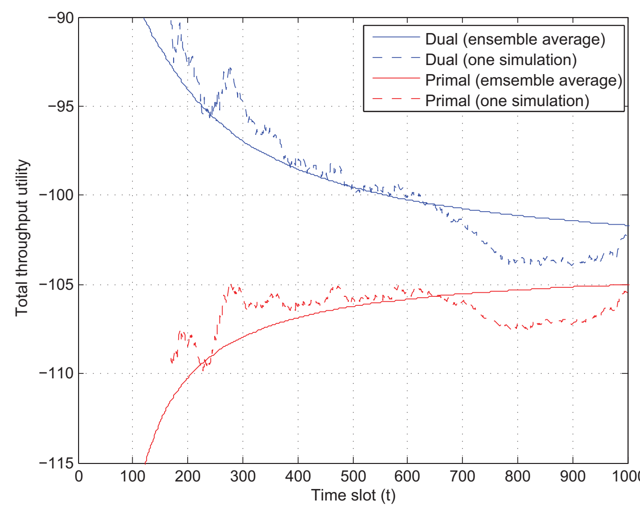

Fig. 6. Primal and dual objectives when instantaneous and average power constraints are in effect. One realization and ensemble average of realizations shown. As time grows the duality gap decreases, eventually approaching a small positive constant and implying near optimality of the achieved rates. Instantaneous power constraint $p_i^{\text{inst}} = 100$ mW, average power constraint $p_i^{\text{avg}} = 5$ mW, step size $\epsilon = 0.1$, adaptive modulation and coding with $M = 4$ modes with rates $\tau_1 = 1$ bits/s/Hz, $\tau_2 = 2$ bits/s/Hz, $\tau_3 = 3$ bits/s/Hz, and $\tau_4 = 4$ bits/s/Hz and transitions at SNRs $\eta_1 = 1$, $\eta_2 = 4$, $\eta_3 = 8$, and $\eta_4 = 16$.

Average power consumptions $\bar{p}_3(t)$ and $\bar{p}_{13}(t)$ for terminals 3 and 13 are shown in Fig. 7. Observe that in both cases the average power constraints are satisfied as time increases. For Terminal 3, $\bar{p}_3(t)$ is always smaller than p_3^{avg} since channel conditions are unfavorable, resulting in Terminal 3 utilizing only mode 1 for communication to the AP. Finally, notice that the average power consumed by Terminal 3 is smaller than the available budget $p_3^{\text{avg}} = 5$ mW. For Terminal 13, $\bar{p}_{13}(t)$ falls below p_{13}^{avg} after 600 iterations. This is as expected due to the almost sure feasibility result of Theorem 1.

Fig. 8 illustrates the relationship between instantaneous power allocations $p_i(t)$ and instantaneous channel gains $h_i(t)$

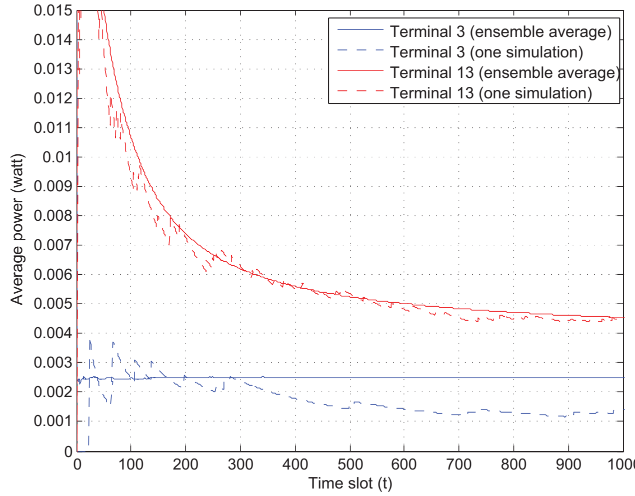

Fig. 7. Average power consumption for terminals 3 and 13, i.e., $\bar{p}_3(t)$ and $\bar{p}_{13}(t)$ as defined in (48). Average power constraints $p_i^{\text{avg}} = 5$ mW are satisfied as time grows. Power $\bar{p}_3(t)$ consumed by Terminal 3 is smaller than the allowed budget p_3^{avg} due to unfavorable channel conditions. Terminal 13 adheres to its power budget after approximately 600 iterations. Parameters as in Fig. 6.

for terminals 3 and 13. Consistent with the fact that the optimal power allocation is a threshold rule, no power is allocated when channel realizations are bad. Further note that Terminal 3 only uses the AMC mode with the lowest rate $\tau_1 = 1$ bits/s/Hz while Terminal 13 uses two modes with rates $\tau_2 = 2$ bits/s/Hz and $\tau_3 = 3$ bits/s/Hz. This happens because terminal 13, being closer to the AP, has a better average channel than terminal 3.

V. Conclusion

We developed optimal adaptive scheduling and power control algorithms for random multiple access channels. Terminals are assumed to know their local channel state information but have no access to the probability distribution of the channel or the channel state of other terminals. In this setting, the proposed online algorithm determines schedules and transmitted powers that maximize a global proportional fair utility. The global utility maximization problem was decomposed in per-terminal utility maximization subproblems. Adaptive algorithms using stochastic subgradient descent in the dual domain were then used to solve these local optimizations. Almost sure convergence and almost sure near optimality of the proposed algorithm was established. Important properties of the algorithm are low computational complexity and the ability to handle non-convex rate functions. Numerical results for a randomly generated network under different physical layer settings corroborated theoretical results.

Future research will provide extensions to multi-packet reception models, multi-carrier systems, and to general multihop random access networks.

Appendix A: Proof of Proposition 1

Proof: To show that the expected value of the stochastic subgradient $\mathbf{s}_i(t)$ given $\boldsymbol{\lambda}_i(t)$ is a subgradient of the dual function $g_i(\boldsymbol{\lambda}_i)$, we have to establish the validity of the relationship in (24). To do so start noticing that in the Lagrangian $\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i(t))$ the terms involving $\mathbf{x}_i$ and

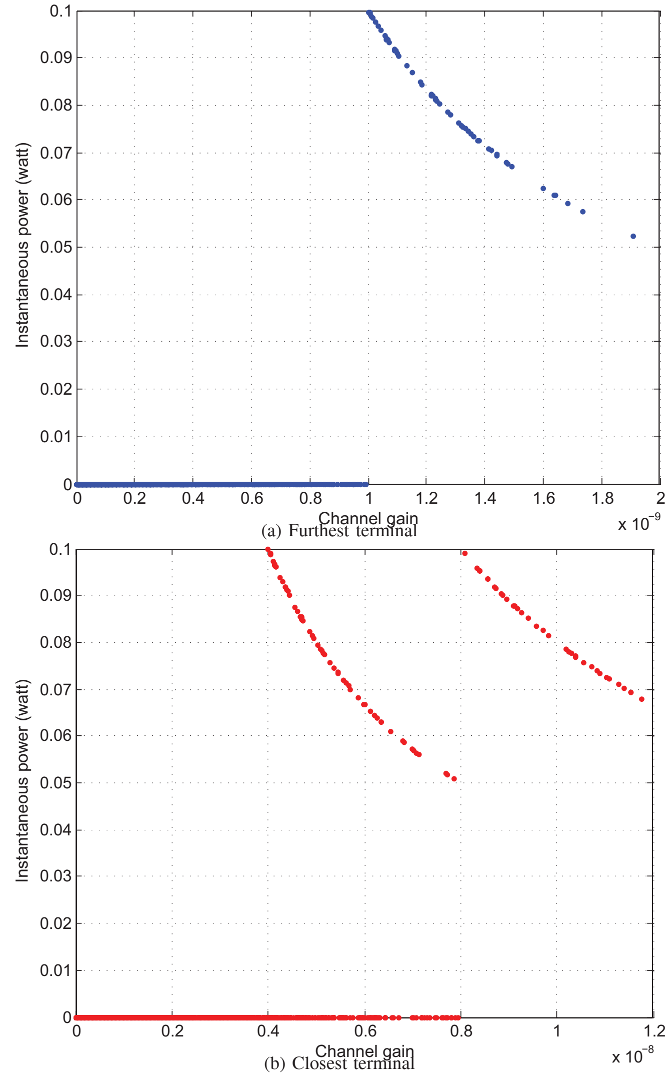

Fig. 8. Instantaneous power allocations $p_i(t)$ for terminals $i = 3$ and $i = 13$ plotted against the channel realization $h_i(t)$. Notice that the channel axes scales are different in (a) and (b). In both cases, no power is allocated when channel realizations are bad. Terminal 3 uses only the AMC mode with the lowest rate $\tau_1 = 1$ bits/s/Hz, while Terminal 13 uses two modes with rates $\tau_2 = 2$ bits/s/Hz and $\tau_3 = 3$ bits/s/Hz. This happens because Terminal 13, being closer to the AP, has a better average channel than Terminal 3. Parameters as in Fig. 6.

$\mathbf{P}_i(h_i)$ are decoupled [cf. (13)]. Consequently, the maximization of $\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i(t))$ in (14) required to evaluate the dual function's value $g_i(\boldsymbol{\lambda}_i(t))$ can be undertaken as maximizations of separate terms with respect to $\mathbf{x}_i$ and $\mathbf{P}_i(h_i)$. Therefore, $g_i(\boldsymbol{\lambda}_i(t))$ can be written as

$$
\begin{aligned}
g_i(\boldsymbol{\lambda}_i(t)) &= \lambda_{i3}(t)p_i^{\text{avg}} + \max_{x_i \geq 0} \{w_i \log x_i - \lambda_{i1}(t)x_i\} \\
&+ \max_{0 \leq y_i \leq 1} \{\tilde{w}_i \log(1-y_i) + \lambda_{i2}(t)y_i\} \\
&+ \max_{Q(h_i), P(h_i)} \mathbb{E}_{h_i(t)} \left[\Gamma_i(Q_i(h_i), P_i(h_i), h_i, \boldsymbol{\lambda}_i(t)) \Big| \boldsymbol{\lambda}_i(t)\right], \qquad (49)
\end{aligned}
$$

where for notational simplicity we defined $\Gamma_i(q_i, p_i, h_i, \boldsymbol{\lambda}_i) := q_i\left[\lambda_{i1}C_i(p_i, h_i) - \lambda_{i2} - \lambda_{i3}p_i\right]$. The expected value is conditional with respect to $\boldsymbol{\lambda}_i(t)$ because $\boldsymbol{\lambda}_i$ is deterministic in (14) but random in (49).

The last summand on the right hand side of (49) is the maximum over the set of functions taking values $Q(h_i) \in \mathcal{Q}$ and $P(h_i) \in \mathcal{P}_i$. Due to linearity of the expectation operator $\mathbb{E}_{h_i(t)}[\cdot]$, this maximum over functions is equal to the expected value of maxima with respect to individual function values. This allows rewriting of (49) as

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i(t)) &= \lambda_{i3}(t)p_i^{\text{avg}} + \max_{x_i \geq 0} \{w_i \log x_i - \lambda_{i1}(t)x_i\} \\ &+ \max_{0 \leq y_i \leq 1} \{\tilde{w}_i \log(1-y_i) + \lambda_{i2}(t)y_i\} \\ &+ \mathbb{E}_{h_i(t)}\left[\max_{q_i\in\{0,1\}, p_i \in [0, p_i^{\text{inst}}]} \Gamma_i(q_i, p_i, h_i(t), \boldsymbol{\lambda}_i(t)) \Big| \boldsymbol{\lambda}_i(t)\right]. \end{aligned} \tag{50}$$

Notice that the maximizations over x_i, y_i, and $\{q_i, p_i\}$ in (50) coincide with the primal iteration maximizations in (16)-(18). Therefore, $x_i(t)$, $y_i(t)$, $q_i(t)$, and $p_i(t)$ obtained from (16)-(18) maximize the right hand side of (50) implying that (50) is equivalent to

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i(t)) =& \lambda_{i3}(t)p_i^{\text{avg}} + [w_i \log x_i(t) - \lambda_{i1}(t)x_i(t)] \\ &+ [\tilde{w}_i \log(1-y_i(t)) + \lambda_{i2}(t)y_i(t)] \\ &+ \mathbb{E}_{h_i(t)}\left[\Gamma_i(q_i(t), p_i(t), h_i(t), \boldsymbol{\lambda}_i(t)) \Big| \boldsymbol{\lambda}_i(t)\right]. \end{aligned} \tag{51}$$

Because $x_i(t)$ and $y_i(t)$ are deterministic functions of $\boldsymbol{\lambda}_i(t)$ it follows that $x_i(t) = \mathbb{E}_{h_i(t)}[x_i(t)|\boldsymbol{\lambda}_i(t)]$ and $y_i(t) = \mathbb{E}_{h_i(t)}[y_i(t)|\boldsymbol{\lambda}_i(t)]$. Use this fact and rearrange terms in (51) to obtain

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i(t)) =& [w_i \log x_i(t) + \tilde{w}_i \log(1-y_i(t))] \\ &+ \lambda_{i1}(t)\mathbb{E}_{h_i(t)}\left[q_i(t)C_i(h_i(t)p_i(t)) - x_i(t)\Big|\boldsymbol{\lambda}_i(t)\right] \\ &+ \lambda_{i2}(t)\mathbb{E}_{h_i(t)}\left[y_i(t) - q_i(t)\Big|\boldsymbol{\lambda}_i(t)\right] \\ &+ \lambda_{i3}(t)\mathbb{E}_{h_i(t)}\left[p_i^{\text{avg}} - q_i(t)p_i(t)\Big|\boldsymbol{\lambda}_i(t)\right]. \end{aligned} \tag{52}$$

According to the definitions in (20)-(22) the terms inside the expectations in (52) are the components $s_i(t)$ of the stochastic subgradient. It then follows

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i(t)) =& w_i \log x_i(t) + \tilde{w}_i \log(1-y_i(t)) \\ &+ \mathbb{E}_{h_i(t)}\left[\mathbf{s}_i^T(t)\big|\boldsymbol{\lambda}_i(t)\right]\boldsymbol{\lambda}_i(t). \end{aligned} \tag{53}$$

Consider now an arbitrary dual variable $\boldsymbol{\lambda}_i \geq 0$ and the corresponding value of the dual function $g(\boldsymbol{\lambda}_i)$ given by the maximum of the Lagrangian $\mathcal{L}_i(\mathbf{x}_i, \mathbf{P}_i(h_i), \boldsymbol{\lambda}_i)$ [cf. 14]. As was done for $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}(t)$ repeat the steps in (49) and (50) to express $g_i(\boldsymbol{\lambda}_i)$ as

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i) =& \lambda_{i3}p_i^{\text{avg}} + \max_{x_i \geq 0}\{w_i \log x_i - \lambda_{i1}x_i\} \\ &+ \max_{0 \leq y_i \leq 1}\{\tilde{w}_i \log(1-y_i) + \lambda_{i2}y_i\} \\ &+ \mathbb{E}_{h_i(t)}\left[\max_{q_i\in\{0,1\}, p \in [0, p_i^{\text{inst}}]} \Gamma_i(q, p, h_i(t), \boldsymbol{\lambda}_i)] \Big| \boldsymbol{\lambda}_i(t)\right], \end{aligned} \tag{54}$$

where the conditioning on $\boldsymbol{\lambda}_i(t)$ is irrelevant because all variables are independent of $\boldsymbol{\lambda}_i(t)$ but will be exploited later on. Since the expression in (54) involves maximizations with respect to x_i, y_i, and $\{q_i, p_i\}$ a lower bound of $g_i(\boldsymbol{\lambda}_i)$ is obtained by evaluating the maximands at $x_i = x_i(t)$, $y_i = y_i(t)$ and $\{q_i, p_i\} = \{q_i(t), p_i(t)\}$. Thus

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i) \geq& \lambda_{i3}p_i^{\text{avg}} + [w_i \log x_i(t) - \lambda_{i1}x_i(t)] \\ &+ [\tilde{w}_i \log(1-y_i(t)) + \lambda_{i2}y_i(t)] \\ &+ \mathbb{E}_{h_i(t)}\left[\Gamma_i(q_i(t), p_i(t), h_i(t), \boldsymbol{\lambda}_i)\Big|\boldsymbol{\lambda}_i(t)\right]. \end{aligned} \tag{55}$$

Reordering terms as when obtaining (52) from (51) we rewrite the bound in (55) as

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i) \geq& [w_i \log x_i(t) + \tilde{w}_i \log(1-y_i(t))] \\ &+ \lambda_{i1}\mathbb{E}_{h_i(t)}\left[q_i(t)C_i(h_i(t)p_i(t)) - x_i(t)\Big|\boldsymbol{\lambda}_i(t)\right] \\ &+ \lambda_{i2}\mathbb{E}_{h_i(t)}\left[y_i(t) - q_i(t)\Big|\boldsymbol{\lambda}_i(t)\right] \\ &+ \lambda_{i3}\mathbb{E}_{h_i(t)}\left[p_i^{\text{avg}} - q_i(t)p_i(t)\Big|\boldsymbol{\lambda}_i(t)\right]. \end{aligned} \tag{56}$$

Using the definition of the stochastic subgradient as when going from (52) to (53) it finally follows

$$\begin{aligned} g_i(\boldsymbol{\lambda}_i) \geq& w_i \log x_i(t) + \tilde{w}_i \log(1-y_i(t)) \\ &+ \mathbb{E}_{h_i(t)}\left[\mathbf{s}_i^T(t)|\boldsymbol{\lambda}_i(t)\right]\boldsymbol{\lambda}_i. \end{aligned} \tag{57}$$

Subtracting (57) from (53) yields (24). Eq. (25) is a particular case of (24) with $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_i^*$ and $g(\boldsymbol{\lambda}_i) = g(\boldsymbol{\lambda}_i^*) = \mathsf{D}_i$. ■

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