Optimal Wireless Communications With Imperfect Channel State Information

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***Abstract*—This paper studies optimal transmission over wireless channels with imperfect channel state information available at the transmitter side in the context of point-to-point channels, multiuser orthogonal frequency division multiplexing, and random access. Terminals adapt transmitted power and coding mode to channel estimates in order to maximize expected throughput subject to average power constraints. To reduce the likelihood of packet losses due to the mismatch between channel estimates and actual channel values, a backoff function is further introduced to enforce the selection of more conservative coding modes. Joint determination of optimal power allocations and backoff functions is a nonconvex stochastic optimization problem with infinitely many variables that despite its lack of convexity is part of a class of problems with null duality gap. Exploiting the resulting equivalence between primal and dual problems, we show that optimal power allocations and channel backoff functions are uniquely determined by optimal dual variables. This affords considerable simplification because the dual problem is convex and finite dimensional. We further exploit this reduction in computational complexity to develop iterative algorithms to find optimal operating points. These algorithms implement stochastic subgradient descent in the dual domain and operate without knowledge of the probability distribution of the fading channels. Numerical results corroborate theoretical findings.**

***Index Terms*—Imperfect channel state information, orthogonal frequency division multiplexing, random access, resource allocation, system level optimization.**

I. Introduction

ADAPTING transmission parameters such as power and rate to time-varying channel conditions can significantly improve the performance of wireless communication systems, e.g., [3]. Although accurate channel state information (CSI) is essential to achieve this goal, perfect CSI is rarely available in practice due to estimation errors and, perhaps more fundamentally, to feedback delay. Algorithms to handle imperfect CSI in the transmission over wireless channels are the subject matter of this paper. We focus on three types of channels: single user point-to-point block fading channels [4], multiuser downlink orthogonal frequency division multiplexing (OFDM) [5], and multiuser uplink random access (RA) [6]. In all three cases we develop algorithms adapting to imperfect CSI that maximize ergodic throughputs subject to average power constraints.

As in the case of perfect CSI, transmitters adapt their power and coding mode to channel observations in order to exploit favorable channel conditions. However, due to the inaccuracy of imperfect CSI, channel outages occur when the rate selected turns out too aggressive for the actual channel realization. From a practical perspective it is recognized that to mitigate the negative effect of outages caused by imperfect CSI a channel backoff function is needed to enforce the selection of more conservative coding modes; see e.g., [7]. Instead of selecting a code adapted to the channel estimate, we select a code adapted to a smaller channel realization. This reduces the transmission rate but also reduces the likelihood of a channel outage resulting on overall larger throughput. Ideally, power allocation and rate backoff should be jointly optimized but this results in a nonconvex optimization problem. Since we need to determine power allocation and backoff for each fading state and fading takes on a continuum of values it further follows that the problem is infinite dimensional. Infinite dimensionality compounded with lack of convexity results in computational intractability.

Computational intractability notwithstanding, the problem can be simplified through the imposition of additional restrictions to yield more tractable formulations that lead to the successful development of transmission strategies for various types of wireless channels. Most relevant to the work presented here are works on point-to-point channels, e.g., [8]–[10], broadcast channels [11]–[16] and random access channels [7], [17], [18]. E.g., when power is fixed and only rate adaptation is considered the problem is reduced to the determination of the optimal backoff function; e.g., [11]. A second possibility is to fix a target outage probability and separate the optimization problem into the determination of a backoff function for target outage, followed by optimal power allocation over estimated channels [12]. A third possible restriction is to assume that the backoff function takes a certain parametric form and proceed to optimize the corresponding parameters, e.g., [7]. These different reformulations yield tractable problems but the resulting throughputs are not optimal for the original problem.

Rather than reformulating the original problem into a suboptimal tractable alternative, the contribution of this paper is to develop algorithms that jointly find optimal power allocations and channel backoff functions. Key in achieving this goal is the recognition that the structure of the resulting optimization problem makes it part of a class of problems that despite their

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lack of convexity have null Lagrangian duality gap [19]. The Lagrangian dual problem of the joint power and backoff function optimization is convex, because dual problems are convex regardless of the convexity of the primal problem, and their dimensionality is given by the number of power constraints which is typically equal to the number of terminals. The combination of convexity and small finite dimensionality results in computational tractability that has to be contrasted with the computational intractability that follows from the infinite dimensionality and lack of convexity of the primal problem. Let us emphasize that lack of duality gap makes primal and dual problems equivalent.

We begin by studying optimal transmission over a single user point-to-point channel with imperfect CSI to illustrate the methodology we will later generalize to multiuser OFDM and RA channels (Section II). In the case of point-to-point channels there is only one constraint and consequently the dual problem is one-dimensional. Lack of convexity is leveraged to show that the optimal power allocation and channel backoff functions are uniquely determined by the optimal dual variable. With the optimal multiplier available, determination of optimal power allocation and channel backoff decomposes into two-dimensional per-fading state optimization subproblems (Section II.B). We further develop a stochastic subgradient descent algorithm in the dual domain that converges to the optimal Lagrange multiplier and yields the optimal power allocation and channel backoff function as a byproduct (Section II.C). This algorithm operates based on instantaneous channel estimates and does not require access to the channel's probability distribution function (pdf).

We then consider optimal transmission over a downlink multiuser OFDM channel with imperfect CSI (Section III). The objective is to maximize a convex utility of the ergodic rates of all users subject to an average sum power budget. In addition to power allocations and channel backoffs, the algorithm for OFDM needs to determine subcarrier assignments for each channel realization. Similar to the case of single-user channels, jointly optimal backoff and frequency and power allocations are uniquely determined by a finite number of Lagrange multipliers equal to the number of users served plus one. With the optimal multiplier available, the problem of determining optimal operating points decomposes into two-dimensional per-frequency, per-terminal, and per-fading state subproblems (Section III.A). Stochastic subgradient descent algorithms to find optimal operating points are developed as well (Section III.B).

We finally investigate uplink multiuser RA channels whereby users contend for communication with a common receiver (Section IV). In this case, terminals do not coordinate their transmission attempts and make transmission decisions based on estimates of their own channels only. If they decide to transmit, they choose a power and a rate for their communication attempt. The objective is to maximize proportional fair utility of ergodic rates subject to individual power constraints at each terminal. Decompositions and stochastic subgradient descent algorithms analogous to those derived for single user and OFDM channels are derived (Sections IV.A and IV.B).

Numerical results are presented in Section V and concluding remarks in Section VI.

II. Point-to-Point Channels

Consider a wireless channel with time slots indexed by t. The channel at time t is denoted as $h(t)$. The channel is assumed to be block fading—for this to be true the length of a time slot has to be comparable to the coherence time of the channel. As a result, $h(t)$ remains constant within a time slot and changes randomly in subsequent time slots. Corresponding channel gains are defined as $\gamma(t) := |h(t)|^2$ and are independent realizations of a random variable that we denote as γ. The pdfs $m_h(h)$ of the fading coefficient h and $m_\gamma(\gamma)$ of its gain are unknown. We assume channels have continuous pdf. This assumption holds true for models used in practice, e.g., Rayleigh, Rician and Nakagami ([10], Ch. 3). In each time slot the transmitter computes an estimate $\hat{\gamma}(t)$ of the current gain $\gamma(t)$ to adapt transmitted power and code selection to the channel state. The accuracy of estimates $\hat{\gamma}(t)$ is characterized through the conditional probability distribution $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ that determines the probability of the actual channel being γ when the estimate is $\hat{\gamma}$. The probability distribution $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ depends on the channel estimation method and is assumed known, although we make no assumptions on its specific form—see Remark 1 below.

Based on the value of the channel estimate $\hat{\gamma}$, the transmitter decides on a power allocation $P = P(\hat{\gamma}) : \mathbf{R}_+ \rightarrow [0, P_{\max}]$, where $P_{\max} > 0$ is the maximum instantaneous power the transmitter can use. The communication rate through the channel is a function of the transmitted power $P(\hat{\gamma})$ and the *actual* channel gain γ that we generically denote as $C(P(\hat{\gamma}), \gamma)$. The function $C(P(\hat{\gamma}), \gamma)$ depends on how signals are coded and modulated at the physical layer. E.g., if capacity-achieving codes are used,

$$C(P(\hat{\gamma}), \gamma) = \log\left(1 + \frac{P(\hat{\gamma})\gamma}{N_0}\right), \tag{1}$$

where N_0 is the noise power at the receiver. If adaptive modulation and coding (AMC) with K transmission modes is considered, communication rate τ_k is supported when the received signal to noise ratio (SNR) $P(\hat{\gamma})\gamma/N_0$ is between η_k and η_{k+1}. The channel capacity function is therefore

$$C(P(\hat{\gamma}), \gamma) = \sum_{k=1}^{K} \tau_k \mathbb{I}\left[\eta_k \leq \frac{P(\hat{\gamma})\gamma}{N_0} \leq \eta_{k+1}\right], \tag{2}$$

where $\mathbb{I}[\eta_k \leq P(\hat{\gamma})\gamma/N_0 \leq \eta_{k+1}]$ stands for the indicator function of the event $\eta_k \leq P(\hat{\gamma})\gamma/N_0 \leq \eta_{k+1}$. We do not restrict $C(P(\hat{\gamma}), \gamma)$ to a specific form and allow for nonconvex and discontinuous rate functions. We only assume that $C(P(\hat{\gamma}), \gamma)$ is a nonnegative nondecreasing function of the product $P(\hat{\gamma})\gamma$ that takes finite values for finite arguments.

To achieve rate $C(P(\hat{\gamma}), \gamma)$ the transmitter has to select an appropriate code adapted to the received SNR $P(\hat{\gamma})\gamma/N_0$, e.g., the appropriate modulation and coding mode if AMC is used as in (2). This is not possible however, because the code selection depends on the unknown channel gain γ. A feasible alternative is to adapt the code to the estimated received SNR $P(\hat{\gamma})\hat{\gamma}/N_0$ and attempt transmission at a rate $C(P(\hat{\gamma}), \hat{\gamma})$. Observe that the transmitted rate does not coincide with the channel throughput due to the possibility of lost packets. Indeed, a channel outage is assumed to occur when the transmitted rate $C(P(\hat{\gamma}), \hat{\gamma})$ exceeds the maximum rate $C(P(\hat{\gamma}), \gamma)$ the channel can afford, i.e., when $C(P(\hat{\gamma}),\hat{\gamma}) > C(P(\hat{\gamma}),\gamma)$ or simply when $\hat{\gamma} > \gamma$. The instantaneous rate achieved in the channel is therefore given by

$$R(\gamma,\hat{\gamma}) = C(P(\hat{\gamma}),\hat{\gamma})\mathbb{I}[\hat{\gamma} \le \gamma], \tag{3}$$

which corrects for lost packets through the indicator $\mathbb{I}[\hat{\gamma} \le \gamma]$.

Selecting a code to attempt transmission at rate $C(P(\hat{\gamma}),\hat{\gamma})$ would likely result in a substantial number of dropped packets. For the sake of argument suppose that the conditional distribution $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ is symmetric around $\hat{\gamma}$. In such case about half of the packets are lost as the outage probability would be $\mathrm{P}\{\hat{\gamma} > \gamma\} = 0.5$. To alleviate the negative effect of outages, a channel backoff function $B = B(\hat{\gamma}) : \mathbf{R}_+ \to \mathbf{R}_+$ is used to determine a backed-off channel gain $B(\hat{\gamma})$. The code is then adapted to the received SNR $P(\hat{\gamma})B(\hat{\gamma})/N_0$—as opposed to $P(\hat{\gamma})\hat{\gamma}/N_0$- and communication proceeds at a rate $C(P(\hat{\gamma}),B(\hat{\gamma}))$. With codes adapted to $P(\hat{\gamma})B(\hat{\gamma})/N_0$, an outage occurs if $B(\hat{\gamma}) > \gamma$. Thus, the instantaneous transmission rate can be written as

$$R(\gamma,\hat{\gamma}) = C(P(\hat{\gamma}),B(\hat{\gamma}))\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}. \tag{4}$$

The idea is that making $B(\hat{\gamma}) < \hat{\gamma}$ reduces the chance of an outage thereby increasing the effective rate $R(\gamma,\hat{\gamma})$ even if the attempted transmission is more conservative [cf. (3) and (4)]. However, as we shall show later, making $B(\hat{\gamma}) > \hat{\gamma}$ is optimal in some cases.

A. Ergodic Rate Optimization

Our goal is to find the optimal power allocation function P and channel backoff function B such that the expected transmission rate is maximized subject to an average power constraint P_0,

$$\begin{aligned}\mathsf{P_s} = \max \mathbb{E}_{\gamma,\hat{\gamma}}&[C(P(\hat{\gamma}),B(\hat{\gamma}))\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}]\\ \text{s.t. } &\mathbb{E}_{\hat{\gamma}}[P(\hat{\gamma})] \le P_0.\end{aligned} \tag{5}$$

Solving (5) is challenging because: (I1) The objective includes an expectation over the random channel gain γ, whose realizations are not available at the transmitter and whose pdf is unknown. (I2) Variables in this optimization problem are functions P and B defined on $\mathbf{R}_+$, implying the dimensionality of the problem is infinite. (I3) The objective and the constraint involve expectations over channel estimates $\hat{\gamma}$, whose realizations are observed in each time slot but whose pdf is unknown. (I4) The channel capacity function $C(P(\hat{\gamma}),B(\hat{\gamma}))$ may be nonconvex or even discontinuous as in the case of AMC [cf. (2)].

To overcome issue (I1), we rewrite the expectation in the objective of (5) as a conditional expectation over γ with $\hat{\gamma}$ given, followed by an expectation over $\hat{\gamma}$, i.e.,

$$\begin{aligned}&\mathbb{E}_{\gamma,\hat{\gamma}}[C(P(\hat{\gamma}),B(\hat{\gamma}))\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}]\\ &\quad= \mathbb{E}_{\hat{\gamma}}\left[C(P(\hat{\gamma}),B(\hat{\gamma}))\mathbb{E}_{\gamma|\hat{\gamma}}[\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}]\right].\end{aligned} \tag{6}$$

Note that the inner expectation in (6) is just the probability $\Pr(B(\hat{\gamma}) \le \gamma|\hat{\gamma})$ of the backed off channel being smaller than the actual channel γ for a given estimate $\hat{\gamma}$. This probability can be written in terms of the complementary cumulative distribution function (ccdf) $M_{\gamma|\hat{\gamma}}(\,\cdot\,)$ of γ given $\hat{\gamma}$ as

$$M_{\gamma|\hat{\gamma}}(B(\hat{\gamma})) := \Pr(B(\hat{\gamma}) \le \gamma|\hat{\gamma}) = \mathbb{E}_{\gamma|\hat{\gamma}}[\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}]. \tag{7}$$

Since $m_{\gamma|\hat{\gamma}}(\,\cdot\,)$ is known—see Remark 1—the ccdf $M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))$ is available. This allows us to simplify (6) to

$$\begin{aligned}&\mathbb{E}_{\gamma,\hat{\gamma}}[C(P(\hat{\gamma}),B(\hat{\gamma}))\mathbb{I}\{B(\hat{\gamma}) \le \gamma\}]\\ &\quad= \mathbb{E}_{\hat{\gamma}}\left[C(P(\hat{\gamma}),B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))\right].\end{aligned} \tag{8}$$

Using (8) the objective in (5) can be written as a single expectation over $\hat{\gamma}$ yielding the equivalent formulation

$$\begin{aligned}\mathsf{P_s} = \max \mathbb{E}_{\hat{\gamma}}&\left[C(P(\hat{\gamma}),B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))\right]\\ \text{s.t. } &\mathbb{E}_{\hat{\gamma}}[P(\hat{\gamma})] \le P_0.\end{aligned} \tag{9}$$

Problems (5) and (9) are equivalent. Our goal is to find the optimal power allocation P^* with values $P^*(\hat{\gamma})$ and backoff function B^* with values $B^*(\hat{\gamma})$ that jointly solve problem (9). Since actual channel gains γ are not present in (9), issue (I1) has been resolved. Issues (I2)-(I4), however, still hold for problem (9). Sections II.B and II.C discuss a method to solve (9) that overcomes these issues. We pursue this after the following remark.

Remark 1: The probability distribution $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ depends on the channel estimation method. A typical way of estimating the channel is to send a training signal that is known to both the transmitter and the receiver and get feedback from the receiver on the measured channel gain. Due to estimation error and/or feedback delays, estimated channels $\hat{h}$ are different from actual channels h and are modeled as

$$\hat{h} = h + e, \tag{10}$$

where e is a complex Gaussian random noise $\mathcal{CN}(0,\sigma_e^2)$. For the model in (10) it holds that the pdf of γ given $\hat{\gamma}$ is a noncentral chi-square given by [21]

$$m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{1}{\sigma_e^2}\exp\left(-\frac{\gamma+\hat{\gamma}}{\sigma_e^2}\right)I_0\left(\frac{2\sqrt{\gamma\hat{\gamma}}}{\sigma_e^2}\right), \tag{11}$$

where $I_0(x) = \sum_{i=0}^{\infty}(x^2/4)^i/(i!)^2$ is the zeroth order modified Bessel function of the first kind. This particular form for the conditional pdf $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ is used to provide numerical results in Section V. The rest of the development in the paper holds independently of the particular form of this pdf. Note that we assume the conditional pdf $m_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ does not change over time.

Remark 2: The instantaneous rate (3) implicitly assumes that $\gamma < \hat{\gamma}$ implies $C(P(\hat{\gamma}),\gamma) < C(P(\hat{\gamma}),\hat{\gamma})$. This is not always true if the capacity function is not a strictly increasing function of channel condition, e.g., AMC (2). However, if $P(\hat{\gamma})$ takes value such that $\eta_k \le P(\hat{\gamma})\hat{\gamma}/N_0 \le \eta_{k+1}$, then the first inequality constraint must be satisfied with equality. This is because if $\eta_k < P(\hat{\gamma})\hat{\gamma}/N_0$ one can always reduce the power $P(\hat{\gamma})$ to a value such that $P(\hat{\gamma})\hat{\gamma}/N_0 = \eta_k$ to achieve the same capacity. As a result, $\gamma < \hat{\gamma}$ indeed implies $C(P(\hat{\gamma}),\gamma) < C(P(\hat{\gamma}),\hat{\gamma})$, even when the capacity function is not a strictly increasing function of the channel condition.

B. Optimal Power Allocation and Channel Backoff Functions

The optimization problem in (9) has only one constraint, implying that while the primal problem is infinite dimensional, the dual problem is one-dimensional. More importantly, it has been shown that problems like (9), where the non-convex functions appear inside expectations, have null duality gap as long as the pdf of the random variable with respect to which we take the expected value has no points of strictly positive probability (see Appendix A). As a result, working in the dual domain is equivalent. To introduce the dual function associate Lagrange multiplier λ with the power constraint and define the Lagrangian as

$$\begin{aligned}\mathcal{L}(P,B,\lambda) \\ &= \mathbb{E}_{\hat{\gamma}}\left[C(P(\hat{\gamma}),B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))\right] \\ &\quad + \lambda[P_0 - \mathbb{E}_{\hat{\gamma}}[P(\hat{\gamma})]] \\ &= \mathbb{E}_{\hat{\gamma}}\left[C(P(\hat{\gamma}),B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma})) - \lambda P(\hat{\gamma})\right] + \lambda P_0, \quad (12)\end{aligned}$$

where we rearranged terms to write the second equality. The dual function is then defined as the maximum of the Lagrangian over the sets of feasible functions P and B, i.e.,

$$g(\lambda) = \max_{P,B} \mathcal{L}(P,B,\lambda). \quad (13)$$

We now can write the dual problem as the minimum of $g(\lambda)$ over nonnegative λ, i.e.,

$$\mathsf{D_s} = \min_{\lambda\geq 0} g(\lambda) \quad (14)$$

Since the problem (9) and its dual (14) have been shown to have null gap we have that $\mathsf{P_s} = \mathsf{D_s}$. This property can be exploited to characterize the optimal power allocation and channel backoff functions as is done in the following theorem.

Theorem 1: The optimal power allocation function P^* with values $P^*(\hat{\gamma})$ and optimal backoff function B^* with values $B^*(\hat{\gamma})$ that solve problem (9) are determined by the optimal dual variable λ^* of the dual problem (14). In particular,

$$\{P^*(\hat{\gamma}), B^*(\hat{\gamma})\} \in \underset{b\in[0,\infty),p\in[0,P_{\max}]}{\operatorname{argmax}} \left\{C(p,b)M_{\gamma|\hat{\gamma}}(b) - \lambda^* p\right\}. \quad (15)$$

Proof: According to the definition of the dual function [cf. (13)], $g(\lambda^*)$ is the maximum of the Lagrangian $\mathcal{L}(P,B,\lambda^*)$ across all functions P and B. Since optimal functions P^* and B^* are possible arguments of the Lagrangian in this maximization it follows that $\mathcal{L}(P^*,B^*,\lambda^*)$ must be bounded above by $g(\lambda^*)$, i.e.,

$$\mathsf{D_s} = g(\lambda^*) = \max_{P,B} \mathcal{L}(P,B,\lambda^*) \geq \mathcal{L}(P^*,B^*,\lambda^*). \quad (16)$$

As per its definition in (12) the Lagrangian $\mathcal{L}(P^*,B^*,\lambda^*)$ can be written as

$$\begin{aligned}\mathcal{L}(P^*,B^*,\lambda^*) = \mathbb{E}_{\hat{\gamma}}\left[C(P^*(\hat{\gamma}),B^*(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B^*(\hat{\gamma}))\right] \\ + \lambda^*[P_0 - \mathbb{E}_{\hat{\gamma}}[P^*(\hat{\gamma})]].\end{aligned}$$

Since B^* and P^* are feasible for the primal problem, the average power constraint must be satisfied, i.e., $P_0 - \mathbb{E}_{\hat{\gamma}}[P^*(\hat{\gamma})] \geq 0$. Since we also know that $\lambda^* \geq 0$ we conclude that $\lambda^*[P_0 - \mathbb{E}_{\hat{\gamma}}[P^*(\hat{\gamma})]] \geq 0$ and as a result

$$\mathcal{L}(P^*,B^*,\lambda^*) \geq \mathbb{E}_{\hat{\gamma}}\left[C(P^*(\hat{\gamma}),B^*(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B^*(\hat{\gamma}))\right] = \mathsf{P_s}. \quad (17)$$

Substituting (17) into (16) gives us

$$\mathsf{D_s} = g(\lambda^*) \geq \mathcal{L}(P^*,B^*,\lambda^*) \geq \mathsf{P_s}. \quad (18)$$

Since the duality gap is null, i.e., $\mathsf{D_s} = \mathsf{P_s}$, the inequalities in (18) must be satisfied with equalities, i.e.,

$$\mathsf{D_s} = g(\lambda^*) = \mathcal{L}(P^*,B^*,\lambda^*) = \mathsf{P_s}. \quad (19)$$

The equality $g(\lambda^*) = \mathcal{L}(P^*,B^*,\lambda^*)$ in (19) implies that P^* and B^* are maximizers of the Lagrangian $\mathcal{L}(P,B,\lambda^*)$,

$$\{P^*, B^*\} \in \underset{P,B}{\operatorname{argmax}} \mathcal{L}(P,B,\lambda^*). \quad (20)$$

Note that in (20) we used set inclusion instead of equality because the maximizer may not be unique. Using the definition of the Lagrangian [cf. (12)], we can rewrite (20) as

$$\begin{aligned}\{P^*, B^*\} \in \underset{P,B}{\operatorname{argmax}} \mathbb{E}_{\hat{\gamma}}\left[C(P(\hat{\gamma}),B(\hat{\gamma}))\right. \\ \left.\times M_{\gamma|\hat{\gamma}}(B(\hat{\gamma})) - \lambda^* P(\hat{\gamma})\right], \quad (21)\end{aligned}$$

where we ignored the term $\lambda^* P_0$ since it does not depend on $P(\hat{\gamma})$ or $B(\hat{\gamma})$. Due to the linearity of the expectation operator, the maximization in (21) can be carried out inside the expectation. This yields separate maximizations for each channel state estimate $\hat{\gamma}$ as indicated in (15). ■

Provided that λ^* is available, Theorem 1 states that $P^*(\hat{\gamma})$ and $B^*(\hat{\gamma})$ can be obtained by solving the maximization in (15). Although the problem in (15) might be nonconvex, solving it is by no means a difficult task as it only involves two variables. This provides a great advantage because the problem dimensionality is reduced from infinity to 1. Also, we remark that Theorem 1 is true no matter what the capacity function is and how the underlying channel is distributed. Next, we shall develop online algorithms that find the optimal solutions for problem (9) using only instantaneous imperfect CSI.

C. *Online Learning Algorithms*

Unlike the nonconvex primal problem, the dual problem in (14) is always convex. This suggests that gradient descent algorithms in the dual domain are guaranteed to converge to the optimal multiplier λ^*. In particular, we use stochastic subgradient descent algorithms that iteratively compute primal and dual variables. Given dual variable $\lambda(t)$, the algorithm proceeds to a primal iteration in which it computes power allocation $p(t)$ and backoff function $b(t)$ as

$$\{p(t), b(t)\} \in \underset{p\in[0,P_{\max}],b\geq 0}{\operatorname{argmax}} \left\{C(p,b)M_{\gamma|\hat{\gamma}(t)}(b) - \lambda(t)p\right\}. \quad (22)$$

Multipliers $\lambda(t+1)$ are then updated based on $\lambda(t)$ and $p(t)$ as

$$\lambda(t+1) = [\lambda(t) - \epsilon(t)[P_0 - p(t)]]^+, \quad (23)$$

where $[x]^+ = \max\{0,x\}$ denotes projection on the nonnegative reals and $\epsilon(t) > 0$ is a possibly time dependent step size. The difference $P_0 - p(t)$ in (23) is a stochastic subgradient of the dual function as it can be shown that the expected value of $P_0 - p(t)$ is a (deterministic) subgradient of the dual function [22], [23]. This property implies that $P_0 - p(t)$ points to λ^* on an average sense and can be exploited to prove convergence in the dual

Algorithm 1: Optimal power control and channel backoff for point-to-point channels

```
Initialize Lagrangian multiplier λ(0);
for t = 0, 1, 2, ··· do
    Compute primal variables as per (22):
        {p(t), b(t)} ∈
            argmax_{p∈[0,P_max], b≥0} {C(p, b) M_{γ|γ̂(t)}(b) − λ(t)p};
    Transmit with power p(t) and rate C(p(t), b(t));
    Update dual variables as per (23):
        λ(t+1) = [λ(t) − ε(t) [P_0 − p(t)]]^+;
end
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domain. The computations in (22) and (23) are summarized in Algorithm 1.

Particular convergence properties depend on whether constant or time varying step sizes are used. We first consider diminishing step sizes. If $\epsilon(t)$ is nonsummable but square summable, i.e., $\sum_{t=0}^{\infty} \epsilon(t) = \infty$ and $\sum_{t=0}^{\infty} \epsilon^2(t) < \infty$, then using standard stochastic approximation techniques it can be shown that $\lambda(t)$ converges to λ^* almost surely [24]. As a consequence of Theorem 1, this indicates that the primal variables almost surely converge to the optimal values as time grows, i.e., $p(t) = P^*(\hat{\gamma}(t))$ and $b(t) = B^*(\hat{\gamma}(t))$ almost surely as t goes to infinity.

In addition to diminishing step size, constant step size can be used for the algorithm. However, with a constant step size the dual iterates $\lambda(t)$ no longer converge to the optimal value almost surely. Instead, they stay within a small distance of λ^* with probability close to 1 as t goes to infinity and convergence can be established in a time average sense only [22]. Specifying Theorem 1 of [22] to the stochastic subgradient descent algorithm in (22)–(23) yields the following property.

Property 1: If constant step sizes $\epsilon(t) = \epsilon > 0$ for all t are used in Algorithm 1, the average power constraint is almost surely satisfied

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} p(u) \le P_0 \quad \text{a.s.,} \tag{24}$$

and the ergodic limit of $C(p(t), b(t)) M_{\gamma(t)|\hat{\gamma}(t)}(b(t))$ almost surely converges to a value within $\kappa\epsilon/2$ of optimal,

$$\mathsf{P_s} - \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} C(p(u), b(u)) M_{\gamma(u)|\hat{\gamma}(u)}(b(u)) \le \kappa\epsilon/2 \quad \text{a.s.,} \tag{25}$$

where $\kappa \ge \mathbb{E}[(P_0 - p(t))^2 \mid \lambda(t)]$ is a constant bounding the second moment $\mathbb{E}[(P_0 - p(t))^2 \mid \lambda(t)]$ of the stochastic subgradient $P_0 - p(t)$.

Since $p(t)$ can only take values in $[0, P_{\mathsf{max}}]$, the constant κ in (25) is upper bounded by $\max\{P_0^2, (P_{\mathsf{max}} - P_0)^2\}$. It follows that the time average of $C(p(t), b(t)) M_{\gamma(t)|\hat{\gamma}(t)}(b(t))$ can be made arbitrarily close to optimal by reducing the step size ϵ. Notice however that the rate $C(p(t), b(t)) M_{\gamma(t)|\hat{\gamma}(t)}(b(t))$ is an average across possible channel realizations $\gamma(t)$ for given estimate $\hat{\gamma}(t)$, which is in general different from the instantaneous transmission rate $C(p(t), b(t)) \mathbb{I}\{b(t) \le \gamma(t)\}$ achieved by the algorithm. Despite this disparity in instantaneous values, their ergodic limits are almost surely equal. To see this just note that according to its definition in (7) it holds $M_{\gamma(t)|\hat{\gamma}(t)}(b(t)) = \mathbb{E}_{\gamma(t)|\hat{\gamma}(t)}[\mathbb{I}\{b(t) \le \gamma(t)\}]$. With estimates $\hat{\gamma}(\mathbb{N})$ given for all times t, the stochastic process $\gamma(\mathbb{N})$ is ergodic. It then follows that we must have

$$\begin{aligned} &\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} C(p(u), b(u)) \mathbb{I}\{b(u) \le \gamma(u)\} \\ &= \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} C(p(u), b(u)) \mathbb{E}_{\gamma(u)|\hat{\gamma}(u)} \\ &\quad \times [\mathbb{I}\{b(u) \le \gamma(u)\}] \quad \text{a.s.,} \end{aligned} \tag{26}$$

because with $\hat{\gamma}(\mathbb{N})$ given the term $C(p(u), b(u))$ is just a constant. Substituting the equality $M_{\gamma(t)|\hat{\gamma}(t)}(b(t)) = \mathbb{E}_{\gamma(t)|\hat{\gamma}(t)}[\mathbb{I}\{b(t) \le \gamma(t)\}]$ into (26) and the resulting expression into (25) gives

$$\mathsf{P_s} - \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} C(p(u), b(u)) \mathbb{I}\{b(u) \le \gamma(u)\} \le \kappa\epsilon/2 \quad \text{a.s.} \tag{27}$$

Equation (27) shows that although the algorithm with constant step sizes does not find $P^*(\hat{\gamma}(t))$ and $B^*(\hat{\gamma}(t))$ it generates sequences $p(t)$ and $b(t)$ whose time averages are almost surely near optimal. The near optimality gap can be made arbitrarily small by reducing the step size ϵ as we have already noted. The advantage of using a constant step size is that if the channel distributions change slowly the algorithm can adapt to that change.

III. Orthogonal Frequency Division Multiplexing

Consider now an OFDM channel where a common access point (AP) spends an average power budget P_0 to communicate with N terminals $\{T_n\}_{n=1}^{N}$ using a set of orthogonal frequencies $\mathcal{F}$. As in the point-to-point channel case of Section II, time is slotted and indexed by t. The time-varying channel gain between the AP and terminal T_n for all frequencies $f \in \mathcal{F}$ is modeled as block fading and denoted by $\gamma_n^f(t)$. In each time slot the AP observes channel gain estimates for all terminals and frequencies which we denote as a vector $\hat{\boldsymbol{\gamma}}(t) := \{\hat{\gamma}_n^f(t) : n \in \mathcal{N}, f \in \mathcal{F}\}$. Based on $\hat{\boldsymbol{\gamma}}(t)$, the AP decides on frequency allocation $a_n^f(t) := A_n^f(\hat{\boldsymbol{\gamma}}(t)) \in \{0, 1\}$ and power allocation $p_n^f(t) := P_n^f(\hat{\boldsymbol{\gamma}}(t)) \in [0, P_{\mathsf{max}}]$. If $a_n^f(t) = 1$, it transmits to T_n using frequency f. Since a given frequency cannot be used by more than one terminal in the same time slot, we require $\sum_{n=1}^{N} A_n^f(\hat{\boldsymbol{\gamma}}) \le 1$ for all $f \in \mathcal{F}$. Define the vector $\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) := [A_1^f(\hat{\boldsymbol{\gamma}}), \ldots, A_n^f(\hat{\boldsymbol{\gamma}})]^T$ grouping the schedules of all terminals for given frequency and channel realization. We can then express the frequency exclusion constraint as

$$\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A} := \{\mathbf{a} = [a_1, \ldots, a_N]^T : a_n \in \{0, 1\}, \mathbf{a}^T \mathbf{1} \le 1\}, \tag{28}$$

which simply states that at most one component of $\mathbf{A}^f(\hat{\boldsymbol{\gamma}})$ can be 1.

If frequency f is scheduled for communication to T_n the AP determines power allocations $P_n^f(\hat{\boldsymbol{\gamma}}(t))$ for the communication to terminal T_n in frequency f for joint channel estimates $\hat{\boldsymbol{\gamma}}(t)$ as well as a channel backoff value $B_n^f(\hat{\boldsymbol{\gamma}}(t))$ that we also let be a function of all channel estimates $\hat{\boldsymbol{\gamma}}(t)$. The intent of the backoff function $B_n^f(\hat{\boldsymbol{\gamma}}(t))$ is to reduce the likelihood of channel outages as in the point-to-point channel case discussed in Section II.

Therefore, channel coding is selected according to the value of the product $P_n^f(\hat{\boldsymbol{\gamma}}(t))B_n^f(\hat{\boldsymbol{\gamma}}(t))$ and communication is attempted at a rate $C(P_n^f(\hat{\boldsymbol{\gamma}}(t)), B_n^f(\hat{\boldsymbol{\gamma}}(t)))$. The instantaneous throughput for the link to terminal T_n has to discount for channel outages and to account for all frequencies $f \in \mathcal{F}$ scheduled for this transmission yielding the instantaneous rate

$$R_n(\boldsymbol{\gamma}, \hat{\boldsymbol{\gamma}}) = \sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}(t)) C\left(P_n^f(\hat{\boldsymbol{\gamma}}(t)), B_n^f(\hat{\boldsymbol{\gamma}}(t))\right) \times \mathbb{I}\left\{B_n^f(\hat{\boldsymbol{\gamma}}(t)) \le \gamma_n^f(t)\right\}. \quad (29)$$

The term $A_n^f(\hat{\boldsymbol{\gamma}}(t))$ in (29) is a binary indicator of wether T_n is scheduled in frequency f for channel realization $\hat{\boldsymbol{\gamma}}(t)$, the factor $C(P_n^f(\hat{\boldsymbol{\gamma}}(t)), B_n^f(\hat{\boldsymbol{\gamma}}(t)))$ is the attempted transmission rate in such case, and the indicator $\mathbb{I}\{B_n^f(\hat{\boldsymbol{\gamma}}(t)) \le \hat{\gamma}_n^f(t)\}$ accounts for dropped packets.

Since we are interested in ergodic rates, we define the average rate $r_n := \mathbb{E}_{\boldsymbol{\gamma},\hat{\boldsymbol{\gamma}}}[R_n(\boldsymbol{\gamma}, \hat{\boldsymbol{\gamma}})]$. Upon defining $M_{\gamma_n^f|\hat{\gamma}_n^f}(\cdot)$ as the ccdf of γ_n^f given $\hat{\gamma}_n^f$, we can express the ergodic rate from the AP to T_n as [cf. (8)]

$$\begin{aligned} r_n &= \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}(t)) C\left(P_n^f(\hat{\boldsymbol{\gamma}}(t)), B_n^f(\hat{\boldsymbol{\gamma}}(t))\right) \times M_{\gamma_n^f|\hat{\gamma}_n^f}\left(B_n^f(\hat{\boldsymbol{\gamma}}(t))\right)\right] \\ &:= \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}(t)) R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}(t)), B_n^f(\hat{\boldsymbol{\gamma}}(t)); \hat{\gamma}_n^f(t)\right)\right], \end{aligned} \quad (30)$$

where in the second equality we defined $R_n^f(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) := C(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}))M_{\gamma_n^f|\hat{\gamma}_n^f}(B_n^f(\hat{\boldsymbol{\gamma}}))$ as the expected throughput of terminal n on frequency f. The expected throughput is the rate at which the AP expects to convey information to terminal T_n on frequency f when the channel estimate is $\hat{\boldsymbol{\gamma}}$. By expected throughput here we refer to the conditional expectation with respect to $\boldsymbol{\gamma}$ given $\hat{\boldsymbol{\gamma}}$.

To evaluate the performance of the system, introduce utility functions $U_n(r_n)$ to measure the value of ergodic rate r_n for terminal n. The AP's goal is to find optimal subcarrier assignment, power allocation and channel backoff functions such that the sum utility $\sum_{n=1}^N U_n(r_n)$ is maximized. Recalling the expression for r_n in (30) and introducing an average sum power constraint, the optimal operating point is obtained as the solution of the optimization problem

$$\begin{aligned} \mathsf{P}_\mathsf{b} = \max & \sum_{n=1}^N U_n(r_n) \\ \text{s.t. } & r_n \le \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right)\right], \\ & P_0 \ge \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\sum_{n=1}^N \sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) P_n^f(\hat{\boldsymbol{\gamma}})\right], \\ & \mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A}, \quad P_n^f(\hat{\boldsymbol{\gamma}}) \in [0, P_{\mathsf{max}}], \\ & B_n^f(\hat{\boldsymbol{\gamma}}) \ge 0, \quad r_n \in [0, r_{\mathsf{max}}], \end{aligned} \quad (31)$$

where r_{max} is a given upper bound on the rates r_n of each user. The relaxation of the rate equality constraint in (30) to the corresponding inequality constraint in (31) is without loss of optimality. The average sum power constraint is enforced by the inequality $P_0 \ge \mathbb{E}_{\hat{\boldsymbol{\gamma}}}[\sum_{n=1}^N \sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}})P_n^f(\hat{\boldsymbol{\gamma}})]$ in (31). The factor $A_n^f(\hat{\boldsymbol{\gamma}})P_n^f(\hat{\boldsymbol{\gamma}})$ is the power used for communication to T_n on frequency f for channel estimate $\hat{\boldsymbol{\gamma}}$. This term is not null only if $A_n^f(\hat{\boldsymbol{\gamma}}) = 1$ which means that terminal T_n is scheduled on frequency f. Individual power consumptions $A_n^f(\hat{\boldsymbol{\gamma}})P_n^f(\hat{\boldsymbol{\gamma}})$ are summed for all terminals $n = 1, \dots, n$ and all frequencies $f \in \mathcal{F}$ to determine the total power consumption for gain estimate $\hat{\boldsymbol{\gamma}}$. These sums of instantaneous power consumptions are averaged over the distribution of $\hat{\boldsymbol{\gamma}}$ to determine the average power expenditure that cannot exceed the budget P_0.

Solving problem (31) bears the same challenges as solving (9). The problem is infinite dimensional due to the optimization variables being functions of the channel estimates $\hat{\boldsymbol{\gamma}}$ and the expectations are with respect to the random variable $\hat{\boldsymbol{\gamma}}$ whose pdf is unknown. The problem is also not convex because the function $R_n^f(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f)$ is not concave on the variables $P_n^f(\hat{\boldsymbol{\gamma}})$ and $B_n^f(\hat{\boldsymbol{\gamma}})$. In this case we also have the requirement of variables $A_n^f(\hat{\boldsymbol{\gamma}})$ being binary as represented by the nonconvex set constraint $\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A}$. As we show in the next section, and as in the case of point-to-point channels, these issues are resolved by working on the dual domain.

A. Optimal Solution

The optimization problem in (31) also has the structure of the problems shown to have null duality gap in [19], [25]. Therefore, we can work on its dual problem which is finite dimensional and convex without loss of optimality. To do so, introduce multipliers λ_n associated with the ergodic rate constraint of user T_n and μ associated with the average power constraint [cf. (31)]. Further define the vector $\boldsymbol{\Lambda} := \{\lambda_1, \dots, \lambda_N, \mu\}$ grouping all dual variables and vectors $\mathbf{P}(\hat{\boldsymbol{\gamma}}) := \{A_n^f(\hat{\boldsymbol{\gamma}}), P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}) : n \in \{1, \cdots, N\}, f \in \mathcal{F}\}$ and $\mathbf{r} := \{r_1, \dots, r_N\}$ respectively grouping resource allocation and ergodic rates. Further let $\mathbf{P}$ stand for the resource allocation function with values $\mathbf{P}(\hat{\boldsymbol{\gamma}})$. The Lagrangian of the optimization problem in (31) can then be written as

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}) &= \sum_{n=1}^N U_n(r_n) \\ &+ \sum_{n=1}^N \lambda_n \left[\mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) \times R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right)\right] - r_n\right] \\ &+ \mu\left[\mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[P_0 - \sum_{n=1}^N \sum_{f\in\mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) P_n^f(\hat{\boldsymbol{\gamma}})\right]\right]. \end{aligned} \quad (32)$$

The dual function and the dual problem are then given by

$$\mathsf{D}_\mathsf{b} = \min_{\lambda_n \ge 0, \mu \ge 0} g(\boldsymbol{\Lambda}) = \min_{\lambda_n \ge 0, \mu \ge 0} \max_{\mathbf{P}, \mathbf{r}} \mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}). \quad (33)$$

Note that the Lagrangian in (32) exhibits a separable structure because all summands involve a single primal variable. To explain this observation consider all summands of (32) that involve transmission rate r_n associated with terminal n and define the Lagrangian component associated with $\mathbf{r}$ as

$$\mathcal{L}^{(1)}(\mathbf{r}, \boldsymbol{\Lambda}) := \sum_{n=1}^{N} [U_n(r_n) - \lambda_n r_n]. \quad (34)$$

Define also the per channel Lagrangian components $\mathcal{L}^{(2)}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \hat{\boldsymbol{\gamma}}, \boldsymbol{\Lambda})$ grouping all summands of (32) that involve resource allocation $\mathbf{P}(\hat{\boldsymbol{\gamma}})$ and a given channel estimate $\hat{\boldsymbol{\gamma}}$, i.e.,

$$\mathcal{L}^{(2)}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \hat{\boldsymbol{\gamma}}, \boldsymbol{\Lambda}) := \sum_{n=1}^{N} \sum_{f \in \mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) \times \left[\lambda_n R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu P_n^f(\hat{\boldsymbol{\gamma}})\right]. \quad (35)$$

It is easy to see by reordering summands in (32) that we can rewrite the Lagrangian as a sum of the component $\mathcal{L}^{(1)}(\mathbf{r}, \boldsymbol{\Lambda})$ and an expectation of the per channel components $\mathcal{L}^{(2)}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \hat{\boldsymbol{\gamma}}, \boldsymbol{\Lambda})$,

$$\mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}) = \mathcal{L}^{(1)}(\mathbf{r}, \boldsymbol{\Lambda}) + \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[\mathcal{L}^{(2)}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \hat{\boldsymbol{\gamma}}, \boldsymbol{\Lambda})\right] + \mu P_0. \quad (36)$$

By leveraging the null duality gap, i.e., the equivalence $\mathsf{P}_\mathsf{b} = \mathsf{D}_\mathsf{b}$, and the Lagrangian separability in (36) we can characterize the optimal solution of the primal problem using the optimal solution of the dual problem, as shown in the following theorem.

Theorem 2: The optimal subcarrier assignment function A_n^{f*} with values $A_n^{f*}(\hat{\boldsymbol{\gamma}})$, channel backoff function B_n^{f*} with values $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ and power allocation function P_n^{f*} with values $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ for solving problem (31) are determined by the optimal variables λ_n^* and μ^* of the dual problem (33). In particular, for a given frequency $f \in \mathcal{F}$ and channel estimate $\hat{\boldsymbol{\gamma}}$ values $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ and $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ of the optimal power and backoff functions are given by

$$\left\{P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}})\right\} \in \underset{p \in [0, P_{\max}], b \geq 0}{\operatorname{argmax}} \lambda_n^* R_n^f\left(p, b; \hat{\gamma}_n^f\right) - \mu^* p. \quad (37)$$

To determine optimal frequency allocation values $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ compute discriminants $\lambda_n^* R_n^f(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}})$ for all n and f. Determine the index of the terminal with maximum discriminant,

$$n^f = \underset{n}{\operatorname{argmax}}\, \lambda_n^* R_n^f\left(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}}), \quad (38)$$

and set $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 0$ for all $n \neq n^f$. For $n = n^f$ set $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 1$ if $\lambda_n^* R_n^f(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}}) > 0$.

Proof: As we have shown in the proof of Theorem 1, the fact that $\mathsf{P}_\mathsf{b} = \mathsf{D}_\mathsf{b}$ implies that optimal functions $\mathbf{P}^*$ and variables $\mathbf{r}^*$ are maximizers of the Lagrangian $\mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}^*)$, i.e.,

$$\{\mathbf{P}^*, \mathbf{r}^*\} \in \underset{\mathbf{P}, \mathbf{r}}{\operatorname{argmax}}\, \mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}^*). \quad (39)$$

Since $\mathbf{P}$ and $\mathbf{r}$ appear in different summands in $\mathcal{L}(\mathbf{P}, \mathbf{r}, \boldsymbol{\Lambda}^*)$ [cf. (34)–(36)], we can separate the maximizations for $\mathbf{P}(\hat{\boldsymbol{\gamma}})$ and $\mathbf{r}$ and write $\mathbf{P}^*$ as

$$\mathbf{P}^* \in \underset{\mathbf{P}}{\operatorname{argmax}} \sum_{n=1}^{N} \sum_{f \in \mathcal{F}} \mathbb{E}_{\hat{\boldsymbol{\gamma}}}\left[A_n^f(\hat{\boldsymbol{\gamma}})\left[\lambda_n^* R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu^* P_n^f(\hat{\boldsymbol{\gamma}})\right]\right]. \quad (40)$$

Due to the linearity of expectation, the maximization in (40) can be carried out inside the expectation. Using the definition of $\mathbf{P}^*(\hat{\boldsymbol{\gamma}})$ we have the following relationship

$$\left\{A_n^{f*}(\hat{\boldsymbol{\gamma}}), P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}})\right\} \in \underset{\mathbf{P}}{\operatorname{argmax}} \sum_{f \in \mathcal{F}} A_n^f(\hat{\boldsymbol{\gamma}}) \times \left[\lambda_n^* R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu^* P_n^f(\hat{\boldsymbol{\gamma}})\right]. \quad (41)$$

Since for a fixed $f \in \mathcal{F}$ at most one $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ can be 1 [cf. (28)], the computation of $A_n^{f*}(\hat{\boldsymbol{\gamma}})$, $P_n^{f*}(\hat{\boldsymbol{\gamma}})$, and $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ as per (41) can be further separated into the computations in (37) and (38). Indeed, if $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 1$ the best possible values for $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ and $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ are the ones that maximize the factor $\lambda_n^* R_n^f(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) - \mu^* P_n^f(\hat{\boldsymbol{\gamma}})$. If $A_n^{f*}(\hat{\boldsymbol{\gamma}}) = 0$ any value of $P_n^{f*}(\hat{\boldsymbol{\gamma}})$ and $B_n^{f*}(\hat{\boldsymbol{\gamma}})$ is optimal, in particular the one that maximizes this factor. Therefore

$$\left\{P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}})\right\} \in \underset{P_n^f(\hat{\boldsymbol{\gamma}}) \in [0, P_{\max}], B_n^f(\hat{\boldsymbol{\gamma}}) \geq 0}{\operatorname{argmax}} \lambda_n^* R_n^f\left(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu^* P_n^f(\hat{\boldsymbol{\gamma}}). \quad (42)$$

Upon the change of variables $p = P_n^f(\hat{\boldsymbol{\gamma}})$ and $b = B_n^f(\hat{\boldsymbol{\gamma}})$ (37) follows.

To decide on indicator variables $A_n^f(\hat{\boldsymbol{\gamma}})$ substitute the optimal power and backoff values in (42) into the sum maximization in (41) to obtain

$$\left\{A_n^{f*}(\hat{\boldsymbol{\gamma}})\right\} \in \underset{\mathbf{A}^f(\hat{\boldsymbol{\gamma}}) \in \mathcal{A}}{\operatorname{argmax}} A_n^f(\hat{\boldsymbol{\gamma}})\left[\lambda_n^* R_n^f\left(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f\right) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}})\right]. \quad (43)$$

If all discriminants $\lambda_n^* R_n^f(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}})$ are negative, the maximum in (43) is attained by making $A_n^f(\hat{\boldsymbol{\gamma}}) = 0$ for all n implying that frequency f is not used by any terminal during the time slot. Otherwise, the largest objective in (43) is obtained by making $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ for the terminal with the largest discriminant. These computations coincides with (38). ■

With optimal multipliers given, optimal power allocation and channel backoff can be computed using (37). Optimal frequency allocations are determined by comparing the discriminants in (38) and assigning frequency f to the terminal with the largest discriminant if this discriminant is positive. Notice that the maximization required in (37) is of a nonconvex objective, but this involves just two variables and is analogous to the maximand in (22) for the case of point-to-point channels. We can interpret (22) as establishing a decomposition on per terminal, per frequency and per fading state subproblems. Further observe that Theorem 2 indicates that the optimal solution is opportunistic. Frequency f is used only when at least one terminal observes a good channel on this frequency so as to have a positive discriminant $\lambda_n^* R_n^f(P_n^{f*}(\hat{\boldsymbol{\gamma}}), B_n^{f*}(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f) - \mu^* P_n^{f*}(\hat{\boldsymbol{\gamma}})$ [cf. (38)].

B. Online Learning Algorithms

Similar to the case of point-to-point channels we can solve the optimization problem in (31) using stochastic subgradient descent in the dual domain. To determine stochastic subgradients of the dual function start with given dual variable $\boldsymbol{\Lambda}(t)$

and channel realization $\hat{\boldsymbol{\gamma}}(t)$ and proceed to determine the Lagrangian maximizers

$$\mathbf{r}(t) \in \operatorname*{argmax}_{\mathbf{r}} \mathcal{L}^{(1)}(\mathbf{r}, \boldsymbol{\Lambda}(t)), \tag{44}$$

$$\mathbf{p}(t) \in \operatorname*{argmax}_{\mathbf{p}} \mathcal{L}^{(2)}(\mathbf{p}, \hat{\boldsymbol{\gamma}}(t), \boldsymbol{\Lambda}(t)), \tag{45}$$

Notice that in (45) we determine a power allocation that corresponds to the current channel estimate $\hat{\boldsymbol{\gamma}}(t)$. Using the definition of the Lagrangian components $\mathcal{L}^{(1)}(\mathbf{r}, \boldsymbol{\Lambda})$, $\mathcal{L}^{(2)}(\mathbf{P}(\hat{\boldsymbol{\gamma}}), \hat{\boldsymbol{\gamma}}, \boldsymbol{\Lambda})$ [cf. (34) and (35)] and attempted transmission rate $R_n^f(P_n^f(\hat{\boldsymbol{\gamma}}), B_n^f(\hat{\boldsymbol{\gamma}}); \hat{\gamma}_n^f)$, the primal iterates in (44) and (45) can be computed as

$$r_n(t) = \operatorname*{argmax}_{r_n \in [0, r_{\max}]} U_n(r_n) - \lambda_n(t) r_n, \tag{46}$$

$$\begin{aligned}\{a_n^f(t), p_n^f(t), b_n^f(t)\} &= \operatorname*{argmax}_{\mathbf{a}\in\mathcal{A},\, p\in[0,P_{\max}], b>0} \sum_{n,f} a_n^f \\ &\quad \times \left[\lambda_n(t) R_n^f(b, p; \hat{\gamma}_n^f(t)) - \mu(t) p\right]. \end{aligned} \tag{47}$$

Following the logic used for deriving (37)–(38) the maximization in (47) can be further simplified to the computation of power allocation and backoff function

$$\{p_n^f(t), b_n^f(t)\} = \operatorname*{argmax}_{p\in[0,P_{\max}], b\geq 0} \lambda_n(t) R_n^f(b, p; \hat{\gamma}_n^f(t)) - \mu(t) p, \tag{48}$$

followed by the determination of terminal indices

$$n^f(t) = \operatorname*{argmax}_{n} \lambda_n(t) R_n^f\left(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)\right) - \mu(t) p_n^f(t). \tag{49}$$

We then set $a_n^f(t) = 0$ for all $n \neq n^f(t)$ and $a_n^f(t) = 1$ for $n = n^f(t)$ if $\lambda_n(t) R_n^f(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)) - \mu(t) p_n^f(t) > 0$.

A subgradient of the dual function $g(\boldsymbol{\Lambda}(t))$ can be obtained by evaluating the instantaneous constraint slacks associated with the Lagrangian maximizers. Denoting as $s_{\lambda_n}(t)$ the subgradient components along the λ_n direction and $s_\mu(t)$ the component along the μ direction we have

$$\begin{aligned} s_{\lambda_n}(t) &= \sum_{f\in\mathcal{F}} a_n^f(t) R_n^f\left(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)\right) - r_n(t), \\ s_\mu(t) &= P_0 - \sum_{n=1}^{N} \sum_{f\in\mathcal{F}} a_n^f(t) p_n^f(t). \end{aligned} \tag{50}$$

The algorithm is completed with an update of the dual variables along the stochastic subgradient direction moderated by a possibly time varying step size $\epsilon(t)$,

$$\begin{aligned} \lambda_n(t+1) &= \left[\lambda_n(t) - \epsilon(t) \left[\sum_{f\in\mathcal{F}} a_n^f(t) \right.\right. \\ &\quad \left.\left. \times\ R_n^f\left(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)\right) - r_n(t)\right]\right]^+, \end{aligned} \tag{51}$$

$$\mu(t+1) = \left[\mu(t) - \epsilon(t) \left[P_0 - \sum_{n=1}^{N} \sum_{f\in\mathcal{F}} a_n^f(t) p_n^f(t)\right]\right]^+, \tag{52}$$

As in the case of point-to-point channels, particular convergence properties depend on whether constant or time varying step sizes are used. With diminishing step size, $\lambda_n(t)$ and $\mu(t)$ converge to optimal dual variables λ_n^* and μ^* almost surely. With constant step size convergence is established in an ergodic sense by applying Theorem 1 of [22] to the optimization problem in (31) and the stochastic dual descent algorithm in (46) and (48)–(52). The resulting property is specified in the following.

Property 2: If constant step sizes $\epsilon(t) = \epsilon > 0$ for all t are used in Algorithm 2, rate and power constraints are almost surely satisfied on average

$$\begin{aligned} \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} r_n(u) &\leq \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \left[\sum_{f\in\mathcal{F}} a_n^f(u) \right. \\ &\quad \left. \times\ R_n^f\left(p_n^f(u), b_n^f(u); \hat{\gamma}_n^f(u)\right)\right] \quad \text{a.s.}, \end{aligned} \tag{53}$$

$$\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \left[\sum_{n=1}^{N} \sum_{f\in\mathcal{F}} a_n^f(u) p_n^f(u)\right] \leq P_0 \quad \text{a.s.} \tag{54}$$

The sum utility of the ergodic limit of $r_n^f(t)$ almost surely converges to a value within $\kappa\epsilon/2$ of optimal,

$$\mathsf{P_b} - \sum_{n=1}^{N} U_n\left(\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \sum_{f\in\mathcal{F}} r_n^f(u)\right) \leq \kappa\epsilon/2 \quad \text{a.s.}, \tag{55}$$

where $\kappa \geq \mathbb{E}[\sum_n s_{\lambda_n}^2(t) + s_\mu^2(t) \mid \lambda(t)]$ is a constant bounding the second moment of the norm of the stochastic subgradient with components given as in (50).

Property 2 establishes optimality of the sequences of primal variables generated by (46) and (48)–(52). In particular, ergodic limits of these sequences almost surely satisfy problem constraints and are within a small factor of optimal. As in the case of point to point channels the rate $a_n^f(u) C(p_n^f(u), b_n^f(u)) M_{\gamma_n^f(u)|\hat{\gamma}_n^f(u)}(b_n^f(u))$ in the ergodic limit in (53) is different from the instantaneous transmission rate $a_n^f(u) C(p_n^f(u), b_n^f(u)) \mathbb{I}\{b_n^f(u) \leq \gamma_n^f(u)\}$ achieved by the algorithm. However, their ergodic limits are equivalent since the stochastic process $\boldsymbol{\gamma}(\mathbb{N})$ is ergodic given estimates $\hat{\boldsymbol{\gamma}}(\mathbb{N})$, i.e.,

$$\begin{aligned} &\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} a_n^f(u) C\left(p_n^f(u), b_n^f(u)\right) M_{\gamma_n^f(u)|\hat{\gamma}_n^f(u)}\left(b_n^f(u)\right) \\ &\quad = \lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} a_n^f(u) C\left(p_n^f(u), b_n^f(u)\right) \\ &\quad\quad \times \mathbb{I}\left\{b_n^f(u) \leq \gamma_n^f(u)\right\} \quad \text{a.s.}. \end{aligned} \tag{56}$$

Substituting (56) into (53) and the resulting expression into (55) we have

$$\begin{aligned} \mathsf{P_b} - \sum_{n=1}^{N} U_n &\left(\lim_{t\to\infty} \frac{1}{t} \sum_{u=1}^{t} \sum_{f\in\mathcal{F}} a_n^f(u) C\left(p_n^f(u), b_n^f(u)\right)\right. \\ &\quad \left. \times \mathbb{I}\left\{b_n^f(u) \leq \gamma_n^f(u)\right\}\right) \leq \kappa\epsilon/2 \quad \text{a.s.}. \end{aligned} \tag{57}$$

Algorithm 2: Optimal subcarrier assignment, power control and channel backoff for OFDM

1 Initialize Lagrangian multipliers $\lambda_n(0), \mu(0)$;
2 **for** $t = 0, 1, 2, \cdots$ **do**
3 Compute primal variables as per (46) for all n
4 $r_n(t) = \underset{r_n \in [0, r_{\max}]}{\operatorname{argmax}} \; U_n(r_n) - \lambda_n(t) r_n$;
5 Accept $r_n(t)$ packets to T_n's queue for transmission;
6 **for** *each frequency* $f \in \mathcal{F}$ **do**
7 For all n, compute primal variables $p_n^f(t), b_n^f(t)$ as per (48):
8 $\{p_n^f(t), b_n^f(t)\} =$
9 $\underset{p \in [0, P_{\max}], b \geq 0}{\operatorname{argmax}} \; \lambda_n(t) R_n^f(p, b; \hat{\gamma}_n^f(t)) - \mu(t) p$;
10 Find terminal with maximum discriminant as per (49):
11 $n^f(t) =$
12 $\underset{n}{\operatorname{argmax}} \, \lambda_n(t) R_n^f(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)) - \mu(t) p_n^f(t)$;
13 **if** $n = n^f(t)$ and $\lambda_n(t) R_n^f(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)) - \mu(t) p_n^f(t) > 0$ **then**
14 Set $a_n^f(t) = 1$, transmit to T_n on frequency f with power $p_n^f(t)$ and rate $C(p_n^f(t), b_n^f(t))$;
15 **end**
16 **end**
17 Update dual variables as per (51) and (52):
18 $\lambda_n(t+1) = \Big[\lambda_n(t) -$
19 $-\epsilon(t) \Big[\sum_{f \in \mathcal{F}} a_n^f(t) R_n^f(p_n^f(t), b_n^f(t); \hat{\gamma}_n^f(t)) - r_n(t) \Big]\Big]^+$;
20 $\mu(t+1) = \Big[\mu(t) - \epsilon(t) \Big[P_0 - \sum_{n=1}^{N} \sum_{f \in \mathcal{F}} a_n^f(t) p_n^f(t) \Big]\Big]^+$;
21 **end**

The inequality in (57) establishes that the utility of the ergodic limits of the transmission rates achieved by the algorithm is within $\kappa\epsilon/2$ of the optimal value P_b. Since κ is a constant, the optimality gap can be made arbitrarily small by reducing the step size ϵ.

The procedure is summarized in Algorithm 2. Multipliers $\lambda_n(0)$ and $\mu(0)$ are initialized at time slot 0. Primal and dual variables are computed iteratively in subsequent time slots. In particular, for each time slot t the algorithm first computes variable $r_n(t)$ for all users as per (46) which decides the number of packets to be accepted into T_n's queue awaiting for transmission (line 3). The algorithm then iterates over frequencies and calculates power allocation $p_n^f(t)$, channel backoff $b_n^f(t)$ for all n by solving the two-variable maximization as per (48) (line 7). The subcarrier assignments $a_n^f(t)$ are then determined by setting $a_n^f(t) = 1$ for n such that $\lambda_n(t) R_n^f(t) - \mu(t) p_n^f(t)$ the largest positive discriminant among all n as per (49) while setting $a_n^f(t) = 0$ for all the rest users (line 11). Note that for a given frequency there is at most one $a_n^f(t) = 1$. If $a_n^f(t) = 1$, the AP transmit to T_n over frequency f using power $p_n^f(t)$ and rate $C(p_n^f(t), b_n^f(t))$. The algorithm then proceeds to update multipliers for the next time slot based on multipliers and primal variables of the current time slot according to (51) and (52) (lines 15–16).

IV. Random Access

Consider now a multiple access channel in which N terminals contend for communication to a common AP using random access. The channel between terminals and the AP is modeled as block fading and denoted as $\gamma_n(t)$. Assume each terminal only observes an imperfect version of its local channel $\hat{\gamma}_n(t)$. Based on its local channel, terminals decide channel access $a_n(t) := A_n(\hat{\gamma}_n) \in \{0, 1\}$, power allocations $p_n(t) := P_n(\hat{\gamma}_n) \in [0, P_{\mathsf{max}}]$ and channel backoffs $b_n(t) := B_n(\hat{\gamma}_n) \geq 0$. We remark that A_n, P_n and B_n are functions of local channels only as opposed to functions of all channel realizations as in the case of OFDM considered in Section III. Since terminals contend for channel access, a transmission from terminal T_n in time slot t is successful if and only if $a_n(t) = 1$ and $a_m(t) = 0$ for all $m \neq n$. If the transmission of T_n is successful, its transmission rate is determined by $C(p_n(t), b_n(t))$. As a consequence, the instantaneous transmission rate for T_n in time slot t is

$$r_n(t) = C(p_n(t), b_n(t)) \mathbb{I}\{b_n(t) \leq \gamma_n(t)\} a_n(t) \times \prod_{m=1, m \neq n}^{N} [1 - a_m(t)]. \quad (58)$$

The ergodic rate is given by a time average of the instantaneous rates in (58) which due to ergodicity can be equivalently written as

$$\begin{aligned} r_n &= \mathbb{E}_{\hat{\gamma}} \Bigg[C(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n)) M_{\gamma_n | \hat{\gamma}_n(t)}(B_n(\hat{\gamma}_n)) A_n(\hat{\gamma}_n) \\ &\quad \times \prod_{m=1, m \neq n}^{N} [1 - A_m(\hat{\gamma}_m)] \Bigg] \\ &= \mathbb{E}_{\hat{\gamma}} \Bigg[R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) A_n(\hat{\gamma}_n) \\ &\quad \times \prod_{m=1, m \neq n}^{N} [1 - A_m(\hat{\gamma}_m)] \Bigg], \end{aligned} \quad (59)$$

where in the second equality we defined the average attempted transmission rate for terminal n as $R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) = C(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n)) M_{\gamma_n | \hat{\gamma}_n}(B_n(\hat{\gamma}_n))$. An important observation here is that since terminals are required to make channel access and power control decisions independently of each other, $A_n(\hat{\gamma}_n), P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n)$ are independent of $A_m(\hat{\gamma}_m), P_m(\hat{\gamma}_m), B_m(\hat{\gamma}_m)$ for all $n \neq m$. This allows us to rewrite r_n as

$$r_n = \mathbb{E}_{\hat{\gamma}_n} [R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) A_n(\hat{\gamma}_n)] \times \prod_{m=1, m \neq n}^{N} [1 - \mathbb{E}_{\hat{\gamma}_m}[A_m(\hat{\gamma}_m)]]. \quad (60)$$

The objective is to maximize the proportional fair utility of the ergodic rates r_n, i.e.,

$$U(\mathbf{r}) = \sum_{n=1}^{N} \log(r_n), \quad (61)$$

where $\mathbf{r} := \{r_n : n \in \{1, \ldots, N\}\}$. In a network where channel pdfs vary among users, maximizing sum log utility $U(\mathbf{r})$ yields solutions that are fair since it prevents users from having very low transmission rates. The optimal random access with imperfect CSI is formulated as the following optimization problem

$$\begin{aligned}&\mathsf{P}_{\mathsf{r}} = \max\ U(\mathbf{r})\\&\text{s.t. } r_n = \mathbb{E}_{\hat{\gamma}_n}[R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) A_n(\hat{\gamma}_n)]\\&\qquad\times \prod_{m=1, m\neq n}^{N} [1 - \mathbb{E}_{\hat{\gamma}_m}[A_m(\hat{\gamma}_m)]],\\&\mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) P_n(\hat{\gamma}_n)] \leq P_{0n},\\&A_n(\hat{\gamma}_n) \in \{0,1\}, P_n(\hat{\gamma}_n) \in [0, P_{\mathsf{max}}], B_n(\hat{\gamma}_n) \geq 0, \end{aligned} \tag{62}$$

where the second inequality indicates each terminal has an average power budget of P_{0n}. Since we require A_n, P_n and B_n to be functions of local channel estimates only, we need a distributed solution for problem (62). However, its formulation is not amenable for distributed implementations because the rate constraint involves actions of all terminals. Thus, we need to separate problem (62) into per terminal subproblems. To do so, we substitute r_n into $U(\mathbf{r})$ and express the logarithm of a product as a sum of logarithms so as to write

$$\begin{aligned}U(\mathbf{r}) &= \sum_{n=1}^{N} \Bigg[\log \mathbb{E}_{\hat{\gamma}_n}[R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) A_n(\hat{\gamma}_n)]\\&\qquad + \sum_{m=1, m\neq n}^{N} \log[1 - \mathbb{E}_{\hat{\gamma}_m}[A_m(\hat{\gamma}_m)]]\Bigg] \qquad (63)\\&= \sum_{n=1}^{N} [\log \mathbb{E}_{\hat{\gamma}_n}[R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) A_n(\hat{\gamma}_n)]\\&\qquad + (N-1) \log[1 - \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n)]]]. \qquad (64)\end{aligned}$$

where in (64) we grouped terms related to T_n. To maximize $U(\mathbf{r})$ for the whole system it suffices to separately maximize corresponding summand for each terminal n. Upon introducing auxiliary variables $x_n = \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n)]$ and $y_n = \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n)]$, it follows that (62) is equivalent to the following per terminal subproblems

$$\begin{aligned}&\mathsf{P}_{\mathsf{r,n}} = \max \sum_{f\in\mathcal{F}} \log x_n + (N-1)\log(1-y_n)\\&\quad\text{s.t. } x_n \leq \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n)],\\&\qquad y_n \geq \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n)],\\&\qquad P_{0n} \geq \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) P_n(\hat{\gamma}_n)],\\&\qquad x_n \geq 0, 0 \leq y_n \leq 1,\ A_n(\hat{\gamma}_n) \in \{0,1\},\\&\qquad P_n(\hat{\gamma}_n) \in [0, P_{\mathsf{max}}],\ B_n(\hat{\gamma}_n) \geq 0.\end{aligned} \tag{65}$$

In particular, we have $\mathsf{P}_{\mathsf{r}} = \sum_{n=1}^{N} \mathsf{P}_{\mathsf{r,n}}$. Therefore, to solve problem (62) we only need to solve problem (65) for all terminals in a distributed manner.

A. Optimal Solution

Similar to the case of point-to-point channel, problem (65) has null duality gap which allows us to work on its dual domain without loss of optimality. To define the problem's Lagrangian, associate multipliers α_n with the constraint involving x_n in (65), β_n with the constraint involving y_n, and ν_n with the average power constraint. Further define $\mathbf{\Lambda}_n := \{\alpha_n, \beta_n, \nu_n\}$, $\mathbf{P}_n(\hat{\gamma}_n) := \{A_n(\hat{\gamma}_n), P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n)\}$, $\mathbf{x}_n := \{x_n, y_n\}$ grouping all multipliers, resource allocation variables and auxiliary variables, respectively. The Lagrangian is then given by

$$\begin{aligned}&\mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n)\\&\quad = \log x_n + (N-1)\log(1-y_n)\\&\qquad + \alpha_n [\mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n)] - x_n]\\&\qquad + \beta_n [y_n - \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n)]]\\&\qquad + \nu_n [P_{0n} - \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n) P_n(\hat{\gamma}_n)]].\end{aligned} \tag{66}$$

The dual problem can then be written as

$$\begin{aligned}\mathsf{D}_{\mathsf{r,n}} &= \min_{\alpha_n\geq 0, \beta_n \geq 0, \nu_n \geq 0} g_n(\mathbf{\Lambda}_n)\\&= \min_{\alpha_n\geq 0, \beta_n \geq 0, \nu_n \geq 0} \max_{\mathbf{P}_n, \mathbf{x}_n} \mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n).\end{aligned} \tag{67}$$

Observe that primal variables $\mathbf{P}_n$ and $\mathbf{x}_n$ appear in different summands in (66). This allows us to regroup terms involving $\mathbf{P}_n$ and $\mathbf{x}_n$ and decompose the Lagrangian. To do so, define $\mathcal{L}_n^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n)$ as the per terminal local Lagrangian component involving auxiliary variable $\mathbf{x}_n$

$$\begin{aligned}&\mathcal{L}_n^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n)\\&\quad = [\log x_n - \alpha_n x_n] + [(N-1)\log(1-y_n) + \beta_n y_n],\end{aligned} \tag{68}$$

and $\mathcal{L}_n^{(2)}(\mathbf{P}_n(\hat{\gamma}_n), \hat{\gamma}_n, \mathbf{\Lambda}_n)$ as the per terminal per fading state Lagrangian component involving resource allocation variable $\mathbf{P}_n(\hat{\gamma}_n)$

$$\begin{aligned}&\mathcal{L}_n^{(2)}(\mathbf{P}_n(\hat{\gamma}_n), \hat{\gamma}_n, \mathbf{\Lambda}_n) = A_n(\hat{\gamma}_n)\\&\quad \times [\alpha_n R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) - \beta_n - \nu_n P_n(\hat{\gamma}_n)].\end{aligned} \tag{69}$$

As a result, the Lagrangian in (66) can be rewritten as

$$\begin{aligned}&\mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n) = \mathcal{L}_n^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n)\\&\qquad + \mathbb{E}_{\hat{\gamma}_n}\left[\mathcal{L}_n^{(2)}(\mathbf{P}_n(\hat{\gamma}_n), \hat{\gamma}_n, \mathbf{\Lambda}_n)\right] + \nu_n P_{0n},\end{aligned} \tag{70}$$

By leveraging the property of null duality gap, i.e., $\mathsf{P}_{\mathsf{r,n}} = \mathsf{D}_{\mathsf{r,n}}$, we can characterize the optimal solution of the primal problem using the optimal solution of the dual problem, as shown in the following theorem.

Theorem 3: The optimal subcarrier assignment function A_n^* with values $A_n^*(\hat{\gamma}_n)$, channel backoff function B_n^* with values $B_n^*(\hat{\gamma}_n)$ and power allocation function P_n^* with values $P_n^*(\hat{\gamma}_n)$ for solving problem (65) are uniquely determined by the optimal variables α_n^*, β_n^* and ν_n^* of the dual problem (67). In particular, for given terminal n we have

$$\begin{aligned}&\{P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n)\}\\&\quad \in \operatorname*{argmax}_{p\in[0,P_{\mathsf{max}}], b\geq 0} \alpha_n^* R_n(p, b; \hat{\gamma}_n) - \beta_n^* - \nu_n^* p, \qquad (71)\\&A_n^*(\hat{\gamma}_n)\\&\quad = \mathbb{I}\{\alpha_n^* R_n(P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n); \hat{\gamma}_n) - \beta_n^*\\&\qquad - \nu_n^* P_n^*(\hat{\gamma}_n)) > 0\}. \qquad (72)\end{aligned}$$

Proof: As we did in the proof of Theorem 1, by exploiting the null duality gap we can show that the optimal functions $\mathbf{P}_n^*$ and variables $\mathbf{x}_n$ are maximizers of the Lagrangian $\mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n^*)$, i.e.,

$$\{\mathbf{P}_n^*, \mathbf{x}_n^*\} \in \operatorname*{argmax}_{\mathbf{P}_n, \mathbf{x}_n} \mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n^*). \tag{73}$$

Note that since $\mathbf{P}_n^*$ and $\mathbf{x}_n^*$ appear in different summands in $\mathcal{L}_n(\mathbf{P}_n, \mathbf{x}_n, \mathbf{\Lambda}_n^*)$ [cf. (68)–(70)], we can write $\mathbf{P}_n^*$ as the maximizer of the corresponding summand, i.e.,

$$\mathbf{P}_n^* \in \operatorname*{argmax}_{\mathbf{P}_n} \mathbb{E}_{\hat{\gamma}_n}[A_n(\hat{\gamma}_n)[\alpha_n^* R_n(P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n); \hat{\gamma}_n) - \beta_n^* - \nu_n^* P_n(\hat{\gamma}_n)]]. \tag{74}$$

Due to the linearity of expectation, the maximization in (74) can be carried out inside the expectation. We therefore have

$$\{A_n^*(\hat{\gamma}_n), P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n)\} \in \operatorname*{argmax}_{a\in\{0,1\}, p\in[0,P_{\max}], b>0} a[\alpha_n^* R_n(p, b; \hat{\gamma}_n) - \beta_n^* - \nu_n^* p]. \tag{75}$$

where we have used the definition of the aggregate variable $\mathbf{P}_n(\hat{\gamma}_n) := \{A_n(\hat{\gamma}_n), P_n(\hat{\gamma}_n), B_n(\hat{\gamma}_n)\}$.

Since the variable a in (75) can only take values in $\{0,1\}$, the objective in (74) can only be 0 or $\alpha_n^* R_n(p, b; \hat{\gamma}_n) - \beta_n^* - \nu_n^* p$. Thus, to solve (75) we just need to find the optimal $P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n)$ when $a = 1$ and set $A_n^*(\hat{\gamma}_n) = 1$ if the resulting objective is strictly positive. This procedure is what (71) and (72) state. ■

Given the optimal dual variable $\mathbf{\Lambda}_n^*$, optimal functions for power allocation $P_n^*(\hat{\gamma}_n)$, channel backoff $B_n^*(\hat{\gamma}_n)$, and channel access $A_n^*(\hat{\gamma}_n)$ can be determined in a distributed manner through (71) and (72) using local information only. This satisfies the design requirement that terminals have to operate independently of each other. It is also worth remarking that the resulting transmission policy is opportunistic with respect to channel estimates $\hat{\gamma}_n$ because terminal n transmits only when $\alpha_n^* R_n(P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n); \hat{\gamma}_n) - \beta_n^* - \nu_n^* P_n^*(\hat{\gamma}_n) > 0$. For this inequality to be true we need to have a sufficiently large rate $R_n(P_n^*(\hat{\gamma}_n), B_n^*(\hat{\gamma}_n); \hat{\gamma}_n)$, which in turn requires large channel estimates $\hat{\gamma}_n$. In fact it is not difficult to see that (72) implies a threshold policy in which terminals transmit if and only if the channel $\hat{\gamma}_n$ exceeds a threshold that can be computed in terms of the optimal multiplier values. This is consistent with similar observations in the case of perfect CSI [23], [26].

The computation of the optimal power allocation and optimal channel backoff in random access [cf. (71)] is similar to that of OFDM [cf. (37)] in the sense that they both solve a two-variable nonconvex optimization problem. However, determination of their corresponding optimal subcarrier assignments is different. For a given frequency, the optimal subcarrier assignment $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ in OFDM is determined jointly for all n and at most one $A_n^{f*}(\hat{\boldsymbol{\gamma}})$ can be 1. The optimal $A_n^*(\hat{\gamma}_n)$ in the case of random access is computed locally and there might be more than one $A_n^*(\hat{\gamma}_n)$ set to 1 in for different n. This is because in the case of random access all terminals act independently of each other and there is no coordination among them while in the case of OFDM the AP plays the role of a central decision maker.

B. Online Learning Algorithms

To solve problem (65) without knowledge of the channel pdf we implement the stochastic subgradient descent algorithm in the dual domain as we did in the case of point-to-point and OFDM channels. To find stochastic subgradients we compute maximizers of the local Lagrangian components $\mathcal{L}^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n)$ and $\mathcal{L}^{(2)}(\mathbf{p}_n, \hat{\gamma}_n, \mathbf{\Lambda}_n)$ for given channel estimate $\hat{\gamma}_n(t)$ and Lagrangian multiplier $\mathbf{\Lambda}_n(t)$, i.e.,

$$\mathbf{x}_n(t) = \operatorname*{argmax}_{\mathbf{x}_n} \mathcal{L}_n^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n(t)), \tag{76}$$

$$\mathbf{p}_n(t) = \operatorname*{argmax}_{\mathbf{p}_n} \mathcal{L}_n^{(2)}(\mathbf{p}_n, \hat{\gamma}_n(t), \mathbf{\Lambda}_n(t)), \tag{77}$$

Recall that x_n and y_n appear in different summands in $\mathcal{L}_n^{(1)}(\mathbf{x}_n, \mathbf{\Lambda}_n(t))$ [cf. (68)]. As a result, we can separate the maximizations for $x_n(t)$ and $y_n(t)$, i.e.,

$$x_n(t) = \operatorname*{argmax}_{x\geq 0} \log x - \alpha_n(t)x = \frac{1}{\alpha_n(t)}, \tag{78}$$

$$y_n(t) = \operatorname*{argmax}_{0\leq y\leq 1} (N-1)\log(1-y) + \beta_n(t)y = \left[1 - \frac{N-1}{\beta_n(t)}\right]^+. \tag{79}$$

Furthermore, using the definition of $\mathbf{p}_n(t)$ and $\mathcal{L}_n^{(2)}(\mathbf{p}_n, \hat{\gamma}_n(t), \mathbf{\Lambda}_n(t))$, the resource allocation computation in (77) can be rewritten as

$$\begin{aligned}\{a_n(t), p_n(t), b_n(t)\} &= \operatorname*{argmax}_{a\in\{0,1\}, p\in[0,P_{\max}], b\geq 0} a[\alpha_n(t) R_n(p_n(t), b_n(t); \hat{\gamma}_n(t)) \\ &\quad - \beta_n(t) - \nu_n(t)p].\end{aligned} \tag{80}$$

Optimizations for $x_n(t)$ and $y_n(t)$ are relatively easy because their objectives are both convex functions with a single variable [cf. (78) and (79)]. Determination of $a_n(t), b_n(t)$ and $p_n(t)$ as per (80) is more complicated but it can be simplified since $a_n(t)$ can only take values in $\{0,1\}$. Using this fact as we did in the proof of Theorem 3 we conclude that (80) is equivalent to

$$\begin{aligned}\{b_n(t), p_n(t)\} &= \operatorname*{argmax}_{p\in[0,P_{\max}], b\geq 0} \alpha_n(t) R_n(p_n(t), b_n(t); \hat{\gamma}_n(t)) \\ &\quad - \beta_n(t) - \nu_n(t)p,\end{aligned} \tag{81}$$

$$\begin{aligned}a_n(t) &= \mathbb{I}(\alpha_n(t) R_n(p_n(t), b_n(t); \hat{\gamma}_n(t)) - \beta_n(t) \\ &\quad - \nu_n(t) p_n(t)).\end{aligned} \tag{82}$$

The stochastic subgradients of the dual function are obtained by evaluating the instantaneous constraint violations using $\mathbf{p}_n(t)$ and $\mathbf{x}_n(t)$. The dual variables are then updated using the stochastic subgradient as

$$\begin{aligned}\alpha_n(t+1) &= [\alpha_n(t) - \epsilon(t)[a_n(t) R_n(p_n(t), b_n(t); \hat{\gamma}_n(t)) \\ &\quad - x_n(t)]]^+,\end{aligned} \tag{83}$$

$$\beta_n(t+1) = [\beta_n(t) - \epsilon(t)[y_n(t) - a_n(t)]]^+, \tag{84}$$

$$\nu_n(t+1) = [\nu_n(t) - \epsilon(t)[P_n - a_n(t)p_n(t)]]^+. \tag{85}$$

As in algorithms 1 and 2 use of diminishing step sizes results in almost sure convergence whereas use of constant step sizes results in an ergodic mode of convergence that we summarize in the following property.

Property 3: If constant step size $\epsilon(t)=\epsilon>0$ is used in Algorithm 3, it follows from ([23], Theorem 1) that primal variables generated by the algorithm are almost surely feasible and almost surely near optimal in an ergodic sense for problem (65). In particular, the average power constraint in (65) is almost surely satisfied, i.e.,

$$\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}a_n(u)p_n(u)\le P_{0,n}\quad \text{a.s.,} \tag{86}$$

and the utility of the ergodic limit of the transmission rates almost surely converges to a value within $\kappa\epsilon$ of optimal,

$$\mathsf{P}_\mathsf{r}-\sum_{n=1}^{N}\log\left(\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}R_n(p_n(u),b_n(u);\hat{\gamma}_n(u))a_n(u)\times\prod_{m=1,m\neq n}^{N}[1-a_m(u)]\right)\le\kappa\epsilon\quad\text{a.s.,} \tag{87}$$

where κ is a constant upper bounding the second moment of the norm of the stochastic subgradient.

Note that the term $R_n(p_n(u),b_n(u);\hat{\gamma}_n(u))a_n(u)\prod_{m=1,\neq n}^{N}[1-a_m(u)]$ in (87) is different from the instantaneous transmission rate $r_n(u)$ in (58) achieved by the policy. To establish optimality results for the ergodic limits of the instantaneous transmission rate, we write the following relationship by using the definition of $R_n(p_n(u),b_n(u);\hat{\gamma}_n(u))$ and the ergodicity property of $\gamma_n(\mathbb{N})$ given $\hat{\gamma}_n(\mathbb{N})$

$$\begin{aligned}&\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}R_n(p_n(u),b_n(u);\hat{\gamma}_n(u))a_n(u)\\&\quad\times\prod_{m=1,m\neq n}^{N}[1-a_m(u)]\\&=\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}C(p_n(u),b_n(u))\mathbb{I}\{b_n(u)<\hat{\gamma}_n(u)\}a_n(u)\\&\quad\times\prod_{m=1,m\neq n}^{N}[1-a_m(u)]\\&=\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}r_n(u)\quad\text{a.s..}\end{aligned} \tag{88}$$

Substituting (88) into (87) we can show the sum logarithm of the ergodic limits of the instantaneous transmission rate $r_n(t)$ is within $\kappa\epsilon$ of the optimal value P_r, i.e.,

$$\mathsf{P}_\mathsf{r}-\sum_{n=1}^{N}\log\left(\lim_{t\to\infty}\frac{1}{t}\sum_{u=1}^{t}r_n(u)\right)\le\kappa\epsilon\quad\text{a.s.,} \tag{89}$$

The procedure is summarized in Algorithm 3. The algorithm initializes multipliers $\alpha_n(0),\beta_n(0)$ and $\nu_n(0)$ at time 0. In each time slot t, it iteratively computes primal variables $x_n(t),y_n(t),p_n(t),b_n(t),a_n(t)$ by (78)–(82) (lines 4–7). If $\alpha_n(t)R_n(p_n(t),b_n(t);\hat{\gamma}_n(t))-\beta_n(t)-\nu_n(t)p_n(t)$ is greater than 0, $a_n(t)$ is set to 1 and terminal n transmits on frequency f using power $p_n(t)$ and rate $C(p_n(t),b_n(t))$. Dual variables $\alpha_n(t),\beta_n(t),\nu_n(t)$ are then computed according to (83)–(85) (lines 12–14). Note that while the algorithm for OFDM (see Algorithm 2) is applied to the system with n users the presented algorithm for RA is for each individual terminal. In RA channels, each terminal distributedly operates based on Algorithm 3. Since each terminal makes channel access decisions based only on its local imperfect CSI and channels for different terminals are assumed to be independent, terminals' actions are independent of each other.

Algorithm 3: Optimal channel access, power control and channel backoff for T_n in random access

1 Initialize Lagrangian multipliers $\alpha_n(0),\beta_n(0)$ and $\nu_n(0)$;
2 **for** $t=0,1,2,\cdots$ **do**
3 Compute primal variables as per (78) - (82):
4 $x_n(t)=\underset{x\ge0}{\operatorname{argmax}}\log x-\alpha_n(t)x=\dfrac{1}{\alpha_n(t)}$;
5 $y_n(t)=\underset{0\le y\le1}{\operatorname{argmax}}(N-1)\log(1-y)+\beta_n(t)y$
6 $=\left[1-\dfrac{N-1}{\beta_n(t)}\right]^+$;
7 $\{b_n(t),p_n(t)\}=$
8 $\underset{p\in[0,P_{\max}],b\ge0}{\operatorname{argmax}}\ \alpha_n(t)R_n(p,b;\hat{\gamma}_n(t))-\beta_n(t)-\nu_n(t)p$;
9 $a_n(t)=$
10 $\mathbb{I}\{(\alpha_n(t)R_n(p_n(t),b_n(t);\hat{\gamma}_n(t))-\beta_n(t)-\nu_n(t)p_n(t))\}$;
11 **if** $a_n(t)=1$ **then**
12 Transmit using power $p_n(t)$ and rate $C(p_n(t),b_n(t))$;
13 **end**
14 Update dual variables as per (83) - (85):
15 $\alpha_n(t+1)=$
16 $[\alpha_n(t)-\epsilon(t)\,[a_n(t)R_n(p_n(t),b_n(t);\hat{\gamma}_n(t))-x_n(t)]]^+$;
17 $\beta_n(t+1)=[\beta_n(t)-\epsilon(t)\,[y_n(t)-a_n(t)]]^+$;
18 $\nu_n(t+1)=\left[\nu_n(t)-\epsilon(t)\left[P_n-a_n(t)p_n(t)\right]\right]^+$;
19 **end**

V. Numerical Results

The performance of the proposed algorithms is further evaluated through numerical tests. We consider point-to-point channels in Section V.A, OFDM channels in Section V.B, and random access channels in Section V.C.

A. Point-to-Point Channel

We assume the real channel coefficient h follows a complex Gaussian distribution $\mathcal{CN}(0,2)$ and the channel estimation error is modeled by (10). The average power budget is $P_0=1$ and the channel capacity function takes the form of (1). Without loss of generality, we assume N_0 is normalized to 1.

In the first set of tests, two channel estimation error variances $\sigma_e^2=0.1$ and $\sigma_e^2=0.7$, corresponding to small and large channel errors, are simulated. We apply diminishing step size $\epsilon(t)=1/\sqrt{t}$ to obtain the optimal dual variable λ^* for both cases and then find the optimal power allocation function $P^*(\hat{\gamma})$ and the optimal channel backoff function $B^*(\hat{\gamma})$ according to Theorem 1, as shown in Fig. 1. For comparison purposes, $P^*(\hat{\gamma})$ and $B^*(\hat{\gamma})$ for $\sigma_e^2=0$ are also depicted. For both small and large channel errors, the optimal power allocation functions are given by water-filling as in the case of perfect CSI. However, as the error in channel estimates increases, power is allocated more conservatively when channel gain estimates are small. The difference between the channel backoff functions for small and large channel errors are more significant. When $\sigma_e^2=0.1$, the

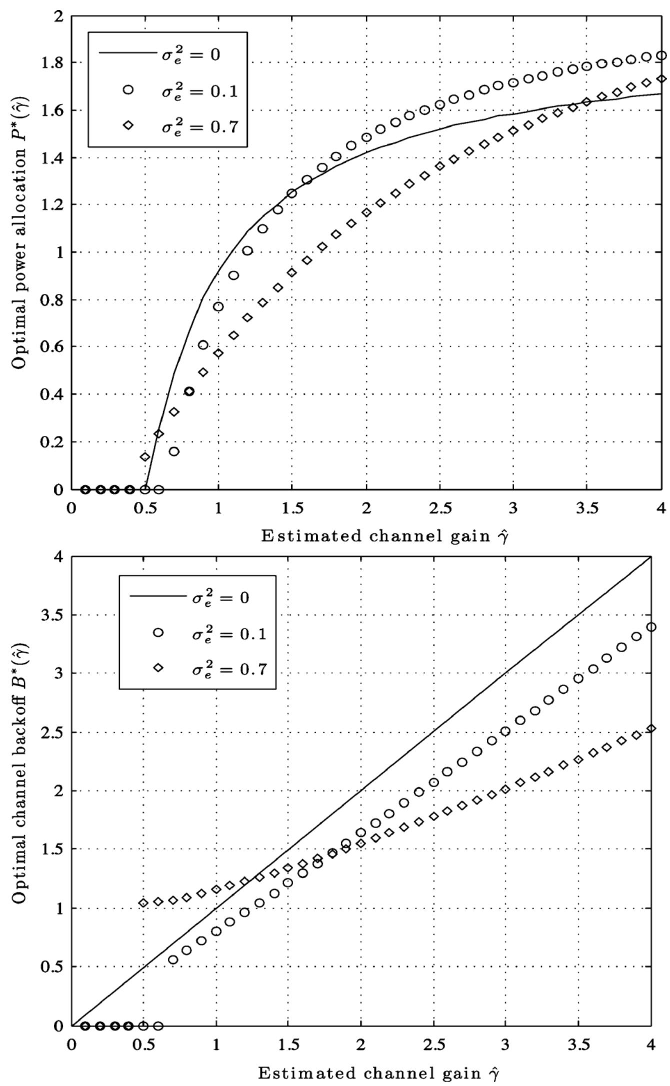

Fig. 1. Optimal power allocation function $P^*(\hat{\gamma})$ (top) and channel backoff function $B^*(\hat{\gamma})$ (bottom) for single user point-to-point channel. Curves shown for channel state information (CSI) variance $\sigma_e^2 = 0.1$, $\sigma_e^2 = 0.1$, and $\sigma_e^2 = 0$, corresponding to perfect CSI. As CSI variance increases power allocation is more conservative for small channel values. When the CSI variance is large, the backoff function selects codes of a higher rate than what is dictated by the channel estimate. Channel coefficient follows a complex Gaussian distribution $\mathcal{CN}(0, 2)$, average power budget $P_0 = 1$, and channel conditional pdf $m_{\gamma|\hat{\gamma}}$ as in (11).

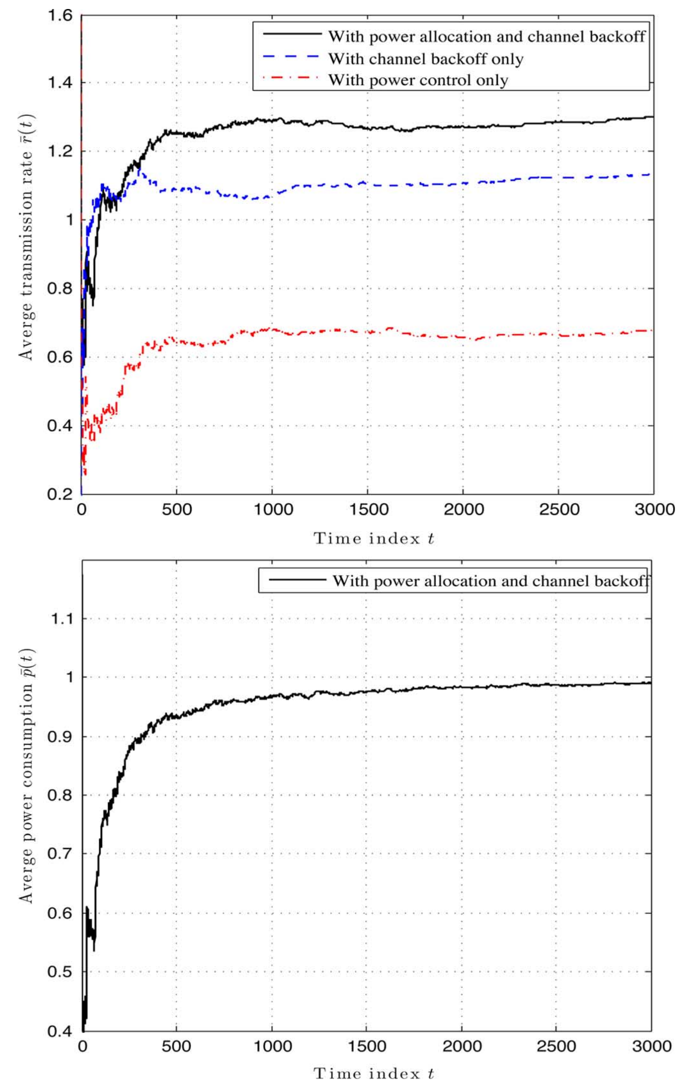

Fig. 2. Convergence of average transmission rate (top) and average power consumption (bottom) for Algorithm 1. Average transmission rate as a function of time is shown for Algorithm 1 and cases in which only the backoff function is optimized—meaning $p(t) = P_0$—or only the power allocation function is optimized—implying $b(t) = \hat{\gamma}(t)$. Joint optimization yields substantial increase of average communication rate. Average power budget $P_0 = 1$, constant step size $\epsilon = 0.01$, and channel estimation error $\sigma_e^2 = 0.1$.

channel backoff is almost linear and $B^*(\hat{\gamma}) < \hat{\gamma}$ for all $\hat{\gamma}$, i.e., making $B^*(\hat{\gamma})$ smaller is always beneficial. When $\sigma_e^2 = 0.7$ the channel backoff function is farther away from linear. It is interesting to note that $B^*(\hat{\gamma}) > \hat{\gamma}$ for small channel estimates $0.5 \leq \hat{\gamma} \leq 1.2$. In that sense the use of the term backoff is a misnomer as it is actually beneficial to select a transmission mode more aggressive than what the channel estimate indicates. The intuition here is that when σ_e^2 is comparable to $\hat{\gamma}$, it is likely that γ is greater than $\hat{\gamma}$ because we must have $\gamma \geq 0$. Therefore, making $B(\hat{\gamma})$ a little bigger than γ is not likely to result in an outage.

In our next simulation, we test the algorithm with constant step size $\epsilon = 0.01$ and assuming channel error $\sigma_e^2 = 0.1$. Other parameters remain the same as before. We define the average transmission rate $\bar{r}(t) = (1/t)\sum_{u=1}^{t} C(p(u), b(u)) \cdot \mathbb{I}\{b(u) \leq \gamma(u)\}$ and average power consumption $\bar{p}(t) = (1/t)\sum_{u=1}^{t} p(u)$. We compare average rates achieved by: 1) with both power allocation and channel backoff; 2) with channel backoff only (i.e., $p(t) = P_0$); 3) with power allocation only (i.e., $b(t) = \hat{\gamma}(t)$). Fig. 2(top) shows average rates achieved by these algorithms. There is a considerable improvement in average transmission rate when power allocation and channel backoff are jointly optimized. Furthermore, Fig. 2(bottom) shows that the average power constraint is always satisfied, coinciding with the almost sure feasibility result in (24).

B. Downlink OFDM Channel

To test Algorithm 2 for downlink OFDM channels, we assume that the number of users is $N = 8$ and that there are $|\mathcal{F}| = 4$ frequency tones available. As in the case of single user point-to-point channel, we model the complex channel

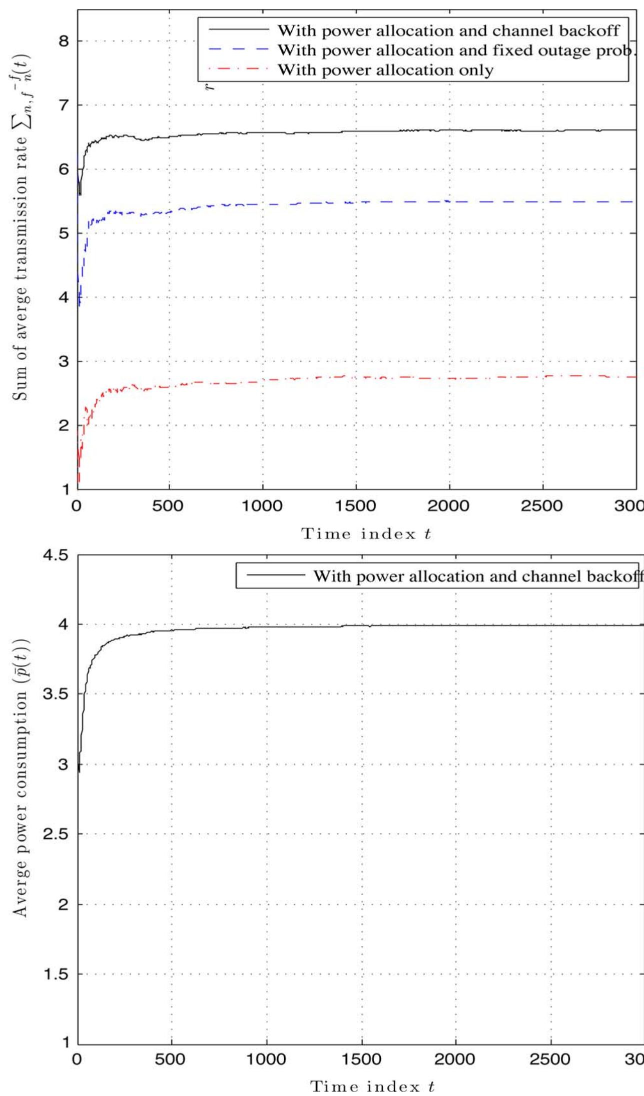

Fig. 3. Rate (top) and power (bottom) convergence of Algorithm 2. Sum of average transmission rates is shown for Algorithm 2 and two suboptimal solutions. One case uses a backoff function with fixed outage probability 0.01—meaning $M_{\gamma_n^f(t)|\hat{\gamma}_n^f(t)}(b_n^f(\hat{\gamma}(t))) = 0.99$—and the other case optimizes power allocation only—implying $b_n^f(t) = \hat{\gamma}_n^f(t)$. Joint optimization yields substantial increase of average communication rate. Average power budget $P_0 = 4$, constant step size $\epsilon = 0.01$, and channel estimation error $\sigma_e^2 = 0.1$.

coefficients h_n^f as random variables with complex Gaussian distributions $\mathcal{CN}(0,2)$ and the channel estimation error as having a complex Gaussian distribution $\mathcal{CN}(0,\sigma_e^2)$ with $\sigma_e^2 = 0.1$ modeled by (10). The total average power budget is $P_0 = 4$ and the channel capacity function takes the form of (1). Without loss of generality, we assume noise power is normalized to $N_0 = 1$. Sum utility $U_n(r_n) = r_n$ is used. We define the average utility as the sum of average transmission rates $\bar{U}(t) = \sum_{n=1}^N \bar{r}_n(t) = (1/t)\sum_{n=1}^N \sum_{f\in\mathcal{F}} \sum_{u=1}^t C(p_n^f(u), b_n^f(u)) \cdot \mathbb{I}\{b_n^f(u) \leq \gamma_n^f(u)\}$ and the average total power consumption as the sum of average power allocated to each terminal $\bar{p}(t) = (1/t)\sum_{n=1}^N \sum_{f\in\mathcal{F}} \sum_{u=1}^t p_n^f(u)$. Average sum utility $\bar{U}(t)$ is shown in Fig. 3(top) and average power $\bar{p}(t)$ is shown in Fig. 3(bottom).

In addition to Algorithm 2, two alternative solutions are also implemented. For the first method, the value of the channel backoff function is chosen such that a fixed outage probability 0.01 is achieved [12], i.e., $b_n^f(t)$ is calculated such that $M_{\gamma_n^f(t)|\hat{\gamma}_n^f(t)}(b_n^f(t)) = 1 - 0.01 = 0.99$ for each observed $\hat{\gamma}_n^f(t)$, and power is then allocated such that the average total power constraint is satisfied. For the second one, we do not perform any channel backoff, i.e., $b_n^f(t) = \hat{\gamma}_n^f(t)$. We remark that both are suboptimal solutions since power allocation and channel backoff functions are not jointly optimized.

We run Algorithm 2 and these two suboptimal alternatives with constant step size $\epsilon(t) = 0.01$ and compare their performance in terms of average utility $\bar{U}(t)$. Fig. 3(top) shows that the average utilities over 3000 time slots achieved by the proposed algorithm, the algorithm with fixed outage probability and the algorithm without channel backoff are 6.6, 5.5 and 2.8, respectively. By introducing channel backoff functions, there is a significant increase in average utility (6.6 vs. 2.8). This implies that channel backoff is indeed very important when dealing with imperfect CSI. Moreover, jointly optimizing power allocation and channel backoff results in 20% performance improvement (6.6 vs. 5.5). Fig. 3(bottom) shows the total average power used by the proposed Algorithm 2. We see that the average power budget $P_0 = 4$ is satisfied.

C. Uplink RA Channel

We run a set of simulations to test algorithms for the random access channel with imperfect CSI. Assume similar parameters as in the case of OFDM: $N = 8$, $N_0 = 1$, channel coefficient and channel estimation error modeled by complex Gaussian distributions $\mathcal{CN}(0,2)$ and $\mathcal{CN}(0,0.1)$, respectively. The power constraint for each terminal is set to $P_{0,n} = 1$.

The proposed Algorithm 3 is implemented in which channel access, power allocation and channel backoff functions are jointly optimized. Two other suboptimal solutions are also simulated: an algorithm without power control—$p_n(t) = P_{0,n}$ is always constant—and an algorithm without channel backoff—$b_n(t) = \hat{\gamma}_n(t)$ always equal to the real estimated channel gain. To compare their performance, define the average proportional fair utility as the sum of the logarithms of the average transmission rates, i.e., $\bar{U}(t) = \sum_{n=1}^N \log \bar{r}_n(t) = \sum_{n=1}^N \log(1/t)\sum_{u=1}^t r_n(u)$. Further define the average power consumption of each terminal as $\bar{p}_n(t) = (1/t)\sum_{u=1}^t p_n(u)$. Fig. 4(top) compares the average proportional fair utility $\bar{U}(t)$ achieved by the three algorithms. The utility over 3000 time slots achieved by the proposed algorithm, the algorithm with fixed outage probability and the algorithm without channel backoff are $-13.8, -15.3$ and -20.7, respectively. Again, we observe that by jointly optimizing the channel access, power allocation and channel backoff the proposed algorithm achieves the highest utility. Moreover, Fig. 4(bottom) shows that the average power budget for terminal 1 is satisfied. Note that the convergence rate of the algorithm for random access [cf. Fig. 4(top)] is slower than the rate of OFDM [cf. Fig. 3(top)]. This is because it takes longer to average out randomness in the case of RA since in OFDM the central decision maker has access to the channel of all users whereas in the case of RA each terminal only knows its own channel.

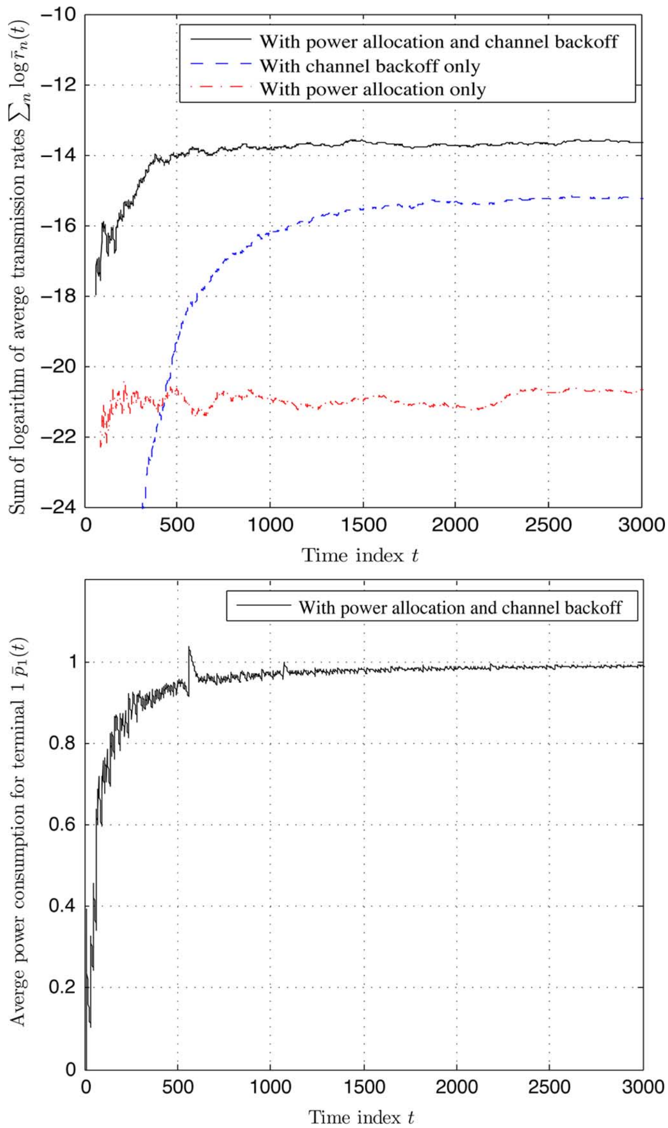

Fig. 4. Rate (top) and power (bottom) convergence of Algorithm 3. Proportional fair utility of average transmission rates is shown for Algorithm 3 and two suboptimal solutions in which only the backoff function—meaning $p_n(t) = P_{0,n}$—or only the power allocation function—implying $b_n(t) = \hat{\gamma}_n(t)$—are optimized. Joint optimization yields substantial increase of average communication rate. Average power budget $P_{0,n} = 1$ (bottom), constant step size $\epsilon = 0.01$, and channel estimation error $\sigma_e^2 = 0.1$.

VI. Conclusion

We considered optimal transmission over single user point-to-point channels, downlink OFDM channels and uplink RA channels with imperfect CSI in order to maximize expected transmission rates subject to average power constraints. For all cases we showed that the optimal solutions are determined by parameters in the form of the optimal multipliers of the Lagrange dual problem. We further developed stochastic subgradient descent algorithms on the dual domain that operate without knowledge of the channels' probability distributions. For vanishing step sizes these dual stochastic descent algorithms converge to the optimal multipliers. With constant step sizes optimal multipliers are not found but a policy that is optimal in an ergodic sense is determined. Numerical results showed significant performance gains of the proposed algorithms.

Appendix A
Proof of Null Duality Gap of Problem (9)

To prove problem (9) has null duality gap, we introduce variable $c = \mathbb{E}_{\hat{\gamma}}[C(P(\hat{\gamma}), B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))]$ and rewrite problem (9) as

$$\begin{aligned} \mathsf{P}_{\mathsf{s}} = \max \quad & c \\ \text{s.t. } & \mathbb{E}_{\hat{\gamma}}[C(P(\hat{\gamma}), B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma}))] \geq c, \\ & \mathbb{E}_{\hat{\gamma}}[-P(\hat{\gamma})] \geq -P_0, \end{aligned} \tag{90}$$

where we relaxed the first equality to inequality without loss of optimality. Further define $\mathbf{P}(\hat{\gamma}) = [P(\hat{\gamma}), B(\hat{\gamma})]^T, \mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma})) = [C(P(\hat{\gamma}), B(\hat{\gamma}))M_{\gamma|\hat{\gamma}}(B(\hat{\gamma})), -P(\hat{\gamma})]^T, \mathbf{x} = [c, -P_0]^T$ and $f_0(\mathbf{x}) = c$ and write (90) as

$$\begin{aligned} \mathsf{P}_{\mathsf{s}} = & \max f_0(\mathbf{x}) \\ & \text{s.t.} \mathbb{E}_{\hat{\gamma}}\left[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))\right] - \mathbf{x} \geq 0. \end{aligned} \tag{91}$$

Note that problem (91) and (9) are equivalent. To establish zero duality gap, consider a perturbed version of (91)

$$\begin{aligned} \mathsf{P}_{\mathsf{s}}(\boldsymbol{\delta}) = & \max f_0(\mathbf{x}) \\ & \text{s.t.} \mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))] - \mathbf{x} \geq \boldsymbol{\delta}, \end{aligned} \tag{92}$$

where we allow the constraint to be violated by $\boldsymbol{\delta}$. To prove that the duality gap for problem (91) is zero, it suffices to show that $\mathsf{P}_{\mathsf{s}}(\boldsymbol{\delta})$ is a concave function of $\boldsymbol{\delta}$; see, e.g., ([27], Sec. 6.2). Let $\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$ be a pair of perturbations, and $(\mathbf{P}, \mathbf{x})$, $(\mathbf{P}', \mathbf{x}')$ be optimal solutions corresponding to the perturbations. Define $\boldsymbol{\delta}_\alpha = \alpha\boldsymbol{\delta} + (1-\alpha)\boldsymbol{\delta}'$ where $\alpha \in [0, 1]$. We are interested in showing

$$\begin{aligned} \mathsf{P}_{\mathsf{s}}(\boldsymbol{\delta}_\alpha) &= \mathsf{P}_{\mathsf{s}}(\alpha\boldsymbol{\delta} + (1-\alpha)\boldsymbol{\delta}') \\ &\geq \alpha\mathsf{P}_{\mathsf{s}}(\boldsymbol{\delta}) + (1-\alpha)\mathsf{P}_{\mathsf{s}}(\boldsymbol{\delta}'). \end{aligned} \tag{93}$$

To establish concativity of the perturbation function, we study properties of the expectation $\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))]$. Define $\mathcal{Y}$ as a set that contains all possible values that $\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))]$ can take, i.e., $\mathcal{Y} := \{\mathbf{y} : \exists \mathbf{P} \text{ for which } \mathbf{y} = \mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))]\}$. If channel pdf has no points of positive probability, then $\mathcal{Y}$ is convex ([25], Theorem 3). Therefore, there must exist $\mathbf{P}_\alpha(\hat{\gamma})$ such that

$$\begin{aligned} \mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}_\alpha(\hat{\gamma}))] = & \alpha\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))] \\ & + (1-\alpha)\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}'(\hat{\gamma}))]. \end{aligned} \tag{94}$$

Since $\mathbf{P}(\hat{\gamma})$ and $\mathbf{P}'(\hat{\gamma})$ are feasible to problem (92), it follows that

$$\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}(\hat{\gamma}))] \geq \mathbf{x} + \boldsymbol{\delta}. \tag{95}$$

$$\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}'(\hat{\gamma}))] \geq \mathbf{x}' + \boldsymbol{\delta}'. \tag{96}$$

Substituting (95) and (96) into (94) yields

$$\begin{aligned} \mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}_\alpha(\hat{\gamma}))] &\geq \alpha(\mathbf{x} + \boldsymbol{\delta}) + (1-\alpha)(\mathbf{x}' + \boldsymbol{\delta}') \\ &= \alpha\mathbf{x} + (1-\alpha)\mathbf{x}' + \boldsymbol{\delta}_\alpha. \end{aligned} \tag{97}$$

Define $\mathbf{x}_\alpha = \alpha\mathbf{x}+(1-\alpha)\mathbf{x}'$, then we have $\mathbb{E}_{\hat{\gamma}}[\mathbf{f}_1(\hat{\gamma}, \mathbf{P}_\alpha(\hat{\gamma}))] \geq \mathbf{x}_\alpha + \boldsymbol{\delta}_\alpha$ implying that $\mathbf{P}_\alpha(\hat{\gamma})$ and $\mathbf{x}_\alpha$ are feasible for problem (92) with perturbation $\boldsymbol{\delta}_\alpha$. In addition, since $f_0(\mathbf{x})$ is a linear function of $\mathbf{x}$ we have

$$f_0(\mathbf{x}_\alpha) = f_0(\alpha\mathbf{x} + (1-\alpha)\mathbf{x}') = \alpha f_0(\mathbf{x}) + (1-\alpha) f_0(\mathbf{x}'). \tag{98}$$

Since $(\mathbf{P}, \mathbf{x})$ is optimal for perturbation $\boldsymbol{\delta}$, we have $\mathsf{P}_\mathsf{s}(\boldsymbol{\delta}) = f_0(\mathbf{x})$, and likewise, $\mathsf{P}_\mathsf{s}(\boldsymbol{\delta}') = f_0(\mathbf{x}')$. Further note that the optimal solution $\mathsf{P}_\mathsf{s}(\boldsymbol{\delta}_\alpha)$ for perturbation $\boldsymbol{\delta}_\alpha$ must exceeds $f_0(\mathbf{x}_\alpha)$, we conclude that

$$\mathsf{P}_\mathsf{s}(\boldsymbol{\delta}_\alpha) \geq \alpha \mathsf{P}_\mathsf{s}(\boldsymbol{\delta}) + (1-\alpha)\mathsf{P}_\mathsf{s}(\boldsymbol{\delta}'). \tag{99}$$

Equation (99) coincides with (93). This completes the proof since (99) holds for any $\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$, and all $\alpha \in [0, 1]$.

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