

Demand Response Management in Smart Grids With Heterogeneous Consumer Preferences

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Abstract—Consumer demand profiles and fluctuating renewable power generation are two main sources of uncertainty in matching demand and supply. This paper proposes a model of the electricity market that captures the uncertainties on both the operator and user sides. The system operator (SO) implements a temporal linear pricing strategy that depends on real-time demand and renewable generation in the considered period combining real-time pricing with time-of-use pricing. The announced pricing strategy sets up a noncooperative game of incomplete information among the users with heterogeneous, but correlated consumption preferences. An explicit characterization of the optimal user behavior using the Bayesian Nash equilibrium solution concept is derived. This explicit characterization allows the SO to derive pricing policies that influence demand to serve practical objectives, such as minimizing peak-to-average ratio or attaining a desired rate of return. Numerical experiments show that the pricing policies yield close to optimal welfare values while improving these practical objectives.

Index Terms—Demand response management (DRM), game theory, renewable energy.

I. INTRODUCTION

MATCHING power production to power consumption is a complex problem in conventional energy grids, exacerbated by the introduction of renewable sources, which, by their very nature, exhibit significant output fluctuations. This problem can be mitigated with a system of smart meters that control the power consumption of users by managing the energy cycles of various devices while also enabling information exchange between users and the system operator (SO) [1], [2]. The flow of information between meters and the SO can be combined

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with sophisticated pricing strategies so as to encourage a better match between power production and consumption [3]–[7]. The effort of operators to guide the consumption of end users through suitable pricing policies is referred to as demand response management (DRM) [8].

To implement DRM, we consider pricing mechanisms that combine real-time pricing (RTP) with time-of-use (TOU) pricing. That is, the price depends on total consumption at each time slot (RTP) and, in addition, the SO divides the operation cycle into time slots (TOU). The use of TOU allows the SO to apply temporal policies based on its anticipation of consumption and renewable source generation in each time. The use of RTP transfers part of the risks and benefits to consumers and encourages their adaptation to power production. When producers use RTP, users agree to a pricing function but actual prices are unknown *a priori* because they depend on the realized aggregate demand. In this context, users must reason strategically about the consumption of others that will ultimately determine the realized price. Game-theoretic models of user behavior then arise naturally and various mechanisms and analyses have been proposed [3], [4], [8]–[11]—also see [12], [13] for more comprehensive expositions. A common feature of these schemes is that the SO and its users run an iterative algorithm to solve a distributed optimization problem prior to the start of an operating cycle. The outcome of this optimization results in individual power targets that the users agree to consume once the operating cycle starts.

This paper proposes an RTP mechanism for DRM in which users agree to a linear price function that depends on the total consumption and a parameter to incentivize the use of energy produced from renewable sources. Both total consumption and the amount of energy produced by renewable sources are unknown *a priori* and users must decide their consumption based on uncertain estimates made public by the SO. Instead of running an iterative optimization algorithm prior to the start of the operating cycle, we assume that this is all the information exchange that occurs between users and the SO (Section II). To determine their consumption levels, users only rely on this information to anticipate the behavior of others, be aware of their influence on price, and mind renewable resource generation forecasts. We provide an analysis of this pricing policy in which users' anticipatory behaviors are formally modeled as the actions of rational consumers with heterogeneous preferences repeatedly taking actions in a game with incomplete information (Section II-B). We define the Bayesian Nash equilibria (BNE) in these games as the

optimal user behavior, provide explicit characterizations of the BNE, and use the resulting characterizations to show desirable properties of the proposed RTP mechanism, e.g., the SO can shape uncertain demand based on its expected renewable generation, or other policy parameters (Sections III and IV). Given the price anticipating user behavior model, we propose two pricing policies that, respectively, aim to achieve a target rate of return and minimize consumption peak-to-average ratio (PAR) (Section V).

The proposed pricing schemes are compared to TOU and flat pricing schemes in which price-taking users respond to given price values at each time slot. In addition, we consider the complete information efficient competitive equilibrium benchmark where the SO maximizes welfare given all the information and users maximize selfish utilities [9], [14]. We show analytically that the proposed RTP is equivalent to TOU and efficient benchmark in expectation and the inefficiency in price-anticipating behavior diminishes with increasing number of users if the correlation among users also diminishes. Numerical analyses verify that the proposed RTP schemes improve user utility and reduce uncertainty in demand facilitating higher forecast accuracy. Finally, the proposed PAR-minimizing policy can indeed achieve its goal with marginal loss to welfare (Section V-C). We discuss the policy implications of these results in Section VI.

II. SMART GRID MODEL

An SO oversees a DRM model with N users denoted by the set $\mathcal{N} := \{1, \dots, N\}$, each equipped with a power consumption scheduler. User $i \in \mathcal{N}$ is characterized by the individual power consumption l_{ih} at time slot $h \in \mathcal{H} := \{1, \dots, H\}$. Accordingly, we represent the total consumption at time slot h with $L_h := \sum_{i \in \mathcal{N}} l_{ih}$.

A. System Operator Model

The total power consumption L_h results in the SO incurring a production cost of $C_h(L_h)$ units. Observe that the production cost function $C_h(L_h)$ depends on the time slot h and the total power produced L_h . When the generation cost per unit is constant, $C_h(L_h)$ is a linear function of L_h . More often, increasing the load L_h results in increasing unit costs as more expensive energy sources are dispatched to meet the load. This results in superlinear cost functions $C_h(L_h)$ with a customary model being the quadratic form¹

$$C_h(L_h) = \frac{1}{2} \frac{\kappa_h}{N} L_h^2 \quad (1)$$

for given constant $\kappa_h > 0$ that depends on the time slot h and that is normalized by the number of users N . The cost in (1) has been experimentally validated for thermal generators [15] and is otherwise widely accepted as a reasonable approximation [4], [8], [9].

The SO utilizes an adaptive pricing strategy whereby users are charged a slot-dependent price p_h that varies linearly with the average power consumption per capita $\bar{L}_h := L_h/N$ —see

¹It is possible to add linear and constant cost terms to $C_h(L_h)$ and have all the results in this paper still hold. We exclude these terms to simplify notation.

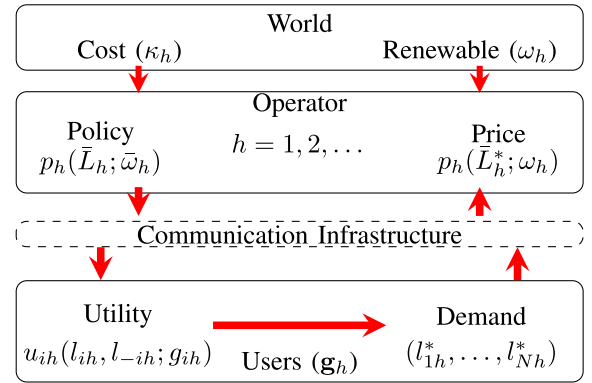


Fig. 1. Illustration of information flow between the power provider and consumers. The SO determines the pricing policy (2) and broadcasts it to the users along with its prediction of mean of the renewable term $\bar{\omega}_h$. Selfish users respond optimally to realize demand $L_h^* = \sum_{i \in \mathcal{N}} l_{ih}^*$. The realized demand \bar{L}_h^* and realized renewable generation term ω_h determine the price at time h .

operator box in Fig. 1. The SO dispatches power from renewable source plants such as wind farms and solar arrays, and incorporates renewable source generation into the pricing strategy by introducing a random variable $\omega_h \in \mathbb{R}$ that depends on the amount of renewable power produced at time h denoted by G_h —see [7] for models in which SO dispatches renewables. The per-unit power price at time slot $h \in \mathcal{H}$ is set as

$$p_h(\bar{L}_h; \omega_h) = \gamma_h(\bar{L}_h + \omega_h/N) \quad (2)$$

where $\gamma_h > 0$ is a policy parameter to be determined by the SO based on its objectives and the renewable source related random variable is normalized by the number of users. We present how the operator can pick its policy parameter $\gamma_h > 0$ to minimize PAR or achieve a desired rate of return in Section V after modeling and analyzing consumption behavior. The random variable ω_h is such that $\omega_h = 0$ when renewable sources operate at their nominal benchmark capacity \bar{G}_h^2 ; that is, the generation G_h at time h equals \bar{G}_h . If the realized production exceeds this benchmark, $G_h > \bar{G}_h$, the SO agrees to set $\omega_h < 0$ to discount the energy price and share the windfall brought about by favorable weather conditions. If the realized production is below the benchmark, i.e., $G_h < \bar{G}_h$, the SO sets $\omega_h > 0$ to reflect the additional charge on the users. The specific dependence of ω_h with the realized renewable energy production and the policy parameter γ_h are part of the supply contract between the SO and its users.

The operator's price function maps the amount of energy demanded to the market price. This is a standard model in pricing—see [16] for a similar formulation. A fundamental observation here is that the prices $p_h(\bar{L}_h; \omega_h)$ in (2) become known after the end of the time slot h . This is because prices depend on the average demand per user \bar{L}_h and the value of ω_h , which is determined by the amount of renewable power available in time h . Both of these quantities are unknown *a priori* as shown in Fig. 1.

²The nominal benchmark capacity at time slot h , \bar{G}_h refers to the amount of wind power expected to be available at time h in kWh. It can be determined with respect to the predicted wind power which then determines an expected generation capacity for the renewable generator [3].

We assume that the SO uses a model on the renewable power generation—see [3] for the prediction of wind generation—to predict the value of ω_h at the beginning of the time h . The expectation of the renewable term denoted by $\bar{\omega}_h := E_{\omega_h}[\omega_h]$ is computed with respect to the predicted distribution P_{ω_h} and is made available to all users at the beginning of the time. By including a term that depends on renewable generation in the price function, the SO aims to use the flexibility of consumption behavior to compensate for the uncertainties in renewables in real time [3], [6], [7], [17].

A particular variable that is of interest is the net revenue of the SO at time h defined as the difference between its revenue $R_h(L_h) := L_h p_h(L_h; \omega_h)$ and its cost $C_h(L_h)$, that is, $\text{NR}_h = R_h(L_h) - C_h(L_h)$. The SOs net revenue over the horizon is the sum over its time slot net revenues, $\text{NR} := \sum_{h \in \mathcal{H}} \text{NR}_h$. Another metric that measures the well-being of the SO is the rate of return defined as the ratio of revenue to cost, $r_h := R_h(L_h)/C_h(L_h)$.

B. Power Consumer Model

The consumption preferences of users are determined by random variables $g_{ih} > 0$ that are possibly different across users and time. When user i consumes l_{ih} units of power at time slot h we assume that it receives a linear utility $g_{ih}l_{ih}$. The user has a diminishing marginal utility from consumption which is captured by the introduction of a quadratic penalty $\alpha_h l_{ih}^2$. This quadratic penalty implies that even when the price charged by the SO is zero, e.g., when $\gamma_h = 0$, it is not in the user's interest to consume infinite amounts of energy. Note that, the decay variable α_h may change across time but it is assumed to be the same for all the users. For each unit of power consumed, the SO charges the price $p_h(\bar{L}_h; \omega_h)$, which results in user i incurring the total cost $l_{ih}p_h(\bar{L}_h; \omega_h)$. The utility of user i is then given by the difference between the consumption return $g_{ih}l_{ih}$, the power cost $l_{ih}p_h(\bar{L}_h; \omega_h)$ and the overconsumption penalty $\alpha_h l_{ih}^2$

$$u_{ih}(l_{ih}, \bar{L}_h; g_{ih}, \omega_h) = -l_{ih}p_h(\bar{L}_h; \omega_h) + g_{ih}l_{ih} - \alpha_h l_{ih}^2. \quad (3)$$

Using the expressions for price in (2) and \bar{L}_h , we write (3) as

$$u_{ih}(l_{ih}, l_{-ih}; g_{ih}, \omega_h) = -l_{ih} \left[\frac{\gamma_h}{N} \left(\sum_{j \in \mathcal{N}} l_{jh} + \omega_h \right) \right] + g_{ih}l_{ih} - \alpha_h l_{ih}^2 \quad (4)$$

where we have also rewritten the utility of user i as $u_{ih}(l_{ih}, \bar{L}_h; g_{ih}, \omega_h) = u_{ih}(l_{ih}, l_{-ih}; g_{ih}, \omega_h)$ to emphasize the fact that it depends on the consumption $l_{-ih} := \{l_{jh}\}_{j \neq i}$ of other users. Note that, if the SOs policy parameter is set to $\gamma_h = 0$, the utility of user i is maximized by $l_{ih} = g_{ih}/2\alpha_h$. This quadratic utility form can also be used to capture target consumption of users at each time [5].

The utility of user i depends on the powers l_{-ih} that are consumed by other users in the current slot. These l_{-ih} power consumptions depend partly on their respective preferences, i.e., $g_{-ih} := \{g_{jh}\}_{j \neq i}$, which are, in general, unknown to user i . We assume, however, that there is a probability distribution $P_{\mathbf{g}_h}(g_h)$ on the vector of consumption preferences

$\mathbf{g}_h := [g_{1k}, \dots, g_{Nk}]^T$ from which these preferences are drawn from. We further assume that $P_{\mathbf{g}_h}$ is normal with mean $\bar{g}_h \mathbf{1}$ where $\bar{g}_h > 0$ and $\mathbf{1}$ is an $N \times 1$ vector with one in every element, and covariance matrix Σ_h

$$\mathbf{g}_h \sim N(\bar{g}_h \mathbf{1}, \Sigma_h). \quad (5)$$

We use the operator $E_{\mathbf{g}_h}$ to signify expectation with respect to the distribution $P_{\mathbf{g}_h}$ and $\sigma_{ij}^h := [[\Sigma_h]]_{ij}$ where the operator $[[\cdot]]_{ij}$ indicates the (i, j) th entry of its matrix argument. Having mean $\bar{g}_h \mathbf{1}$ implies that all users have equal average preferences in that $E_{\mathbf{g}_h}(g_{ih}) = \bar{g}_h$ for all i . If $\sigma_{ij}^h = 0$ for some pair $i \neq j$, it means that the preferences of these users are uncorrelated. In general, $\sigma_{ij}^h \neq 0$ to account for correlated preferences due to, e.g., weather. It is assumed that preferences \mathbf{g}_h and \mathbf{g}_l for different time slots $h \neq l$ are independent, e.g., the jump in consumption preference from off-peak to peak time is independent.

The probability distributions P_{ω_h} and $P_{\mathbf{g}_h}$ and the parameters α_h and γ_h are common knowledge among the SO and its users. That is, the probability distribution $P_{\mathbf{g}_h}$ in (5) is correctly predicted by the SO based on past data by assumption and is announced to the users—see [18] for a probabilistic model and online tracking of user preferences and also see [7] for a linear price responsive users. The pricing parameter γ_h and the operator's belief on the renewable energy parameter ω_h , P_{ω_h} are also announced. In addition, user i knows its private value of consumption preference g_{ih} .

A selfish user's goal is to maximize the utility $u_{ih}(l_{ih}, l_{-ih}; g_{ih}, \omega_h)$ in (4) given its partial knowledge on consumptions of others l_{-ih} . Given the selfish behavior of users, the aggregate utility of the population is defined as the sum of consumers' utility functions, $U_h(\{l_{jh}\}_{j \in \mathcal{N}}; \mathbf{g}_h, \omega_h) := \sum_{i \in \mathcal{N}} u_{ih}(l_{ih}, l_{-ih}; g_{ih}, \omega_h)$. The aggregate utility over the horizon is defined as $U := \sum_{h \in \mathcal{H}} U_h$. The welfare of the system at time h , W_h , considers the well-being of all the entities in the system and is defined as the sum of the net revenue at time h , NR_h , with the aggregate utility U_h

$$W_h := \text{NR}_h + U_h = -C_h(L_h) + \sum_{i \in \mathcal{N}} g_{ih}l_{ih} - \alpha_h l_{ih}^2 \quad (6)$$

where the second equality follows from the definition because the monetary costs to the users cancel out the revenue of the SO. The welfare over the horizon is defined as $W := \sum_{h \in \mathcal{H}} W_h$.

The dependence of each user's utility on the consumption of other users sets up a game among the users with players $i \in \mathcal{N}$ and payoffs given in (4). The load consumption that maximizes a player's payoff requires strategic reasoning, i.e., a model of behavior for other users that comes in the form of a BNE strategy that we introduce in the next section.

III. CONSUMERS' BAYESIAN GAME

User i 's load consumption at time h is determined by its belief q_{ih} and strategy s_{ih} . The belief of i is a conditional probability distribution on \mathbf{g}_h given g_{ih} , $q_{ih}(\cdot) := P_{\mathbf{g}_h}(\cdot | g_{ih})$. We use $E_{ih}[\cdot] := E_{\mathbf{g}_h}[\cdot | g_{ih}]$ to indicate conditional expectation with respect to belief q_{ih} of user i at time h . In order to second-guess

the consumption of other users, user i forms beliefs on their preferences given the common prior $P_{\mathbf{g}_h}$ and self-preferences up to time h $\{g_{it}\}_{t < h}$. Observe that self-preferences of previous times are not relevant to belief at time h as they are independent from the present preferences. Note further that if renewable generation is correlated with the user preferences, the user can refine its beliefs based on the prior P_{ω_h} . User i 's load consumption at time h is determined by its strategy which is a complete contingency plan that maps any possible local preference that it may have to its consumption, $s_{ih} : g_{ih} \mapsto \mathbb{R}$ for any g_{ih} , i.e., $l_{ih} = s_{ih}(g_{ih})$. In particular, for user i , its best response strategy is to maximize expected utility with respect to its belief q_{ih} given the strategies of other users $\mathbf{s}_{-ih} := \{s_{jh}\}_{j \neq i}$

$$\text{BR}(g_{ih}; \mathbf{s}_{-ih}) = \arg \max_{l_{ih}} E_{\omega_h} [E_{ih} [u_{ih}(l_{ih}, \mathbf{s}_{-ih}; g_{ih}, \omega_h)]] . \quad (7)$$

A BNE strategy profile $\mathbf{s}^* := \{s_{ih}\}_{i \in \mathcal{N}, h \in \mathcal{H}}$ is a strategy in which each user maximizes expected utility with respect to its own belief given that other users play with respect to their BNE strategy.

Definition 1: A BNE strategy \mathbf{s}^* is such that for all $i \in \mathcal{N}$, $h \in \mathcal{H}$, and $\{q_{ih}\}_{i \in \mathcal{N}, h \in \mathcal{H}}$

$$E_{\omega_h} [E_{ih} [u_{ih}(s_{ih}^*, \mathbf{s}_{-ih}^*; g_{ih}, \omega_h)]] \geq E_{\omega_h} [E_{ih} [u_{ih}(s_{ih}, \mathbf{s}_{-ih}^*; g_{ih}, \omega_h)]] . \quad (8)$$

A BNE strategy (8) is computed using beliefs formed according to Bayes' rule. Note that, the BNE strategy profile is defined for all time slots, that is, no user at any given point of time has a profitable deviation to another strategy. Equivalently, a BNE strategy is one in which users play best response strategy given their individual beliefs as per (7) to best response strategies of other users—see [16], [19], [20] for a detailed explanation. As a result, a BNE strategy is defined with the following fixed point equations:

$$s_{ih}^*(g_{ih}) = \text{BR}(g_{ih}; \mathbf{s}_{-ih}^*) \quad (9)$$

for all $i \in \mathcal{N}$, $h \in \mathcal{H}$, and g_{ih} . Using the definition in (9), the following result characterizes the unique BNE strategy.

Proposition 1: Consider the game defined by the payoff in (4) at time $h \in \mathcal{H}$. Let the information given to user i be its preference g_{ih} , the common normal prior on preferences $P_{\mathbf{g}_h}$ as per (5) and the prior on renewable generation P_{ω_h} at each time h . Then, the unique BNE strategy of user i is linear in $\bar{\omega}_h$, \bar{g}_h , and g_{ih} for all $h \in \mathcal{H}$ such that

$$s_{ih}^*(g_{ih}) = a_{ih}(\bar{g}_h - \bar{\omega}_h \gamma_h / N) + b_{ih}(g_{ih} - \bar{g}_h) \quad (10)$$

where the constants a_{ih} and b_{ih} are entries of the vectors $\mathbf{a}_h = [a_{1h}, \dots, a_{Nh}]^T$ and $\mathbf{b}_h = [b_{1h}, \dots, b_{Nh}]^T$ which are given by

$$\mathbf{a}_h = ((N+1)\gamma_h/N + 2\alpha_h)^{-1} \mathbf{1}, \quad \mathbf{b}_h = \rho_h \mathbf{d}(\boldsymbol{\Sigma}_h) \quad (11)$$

with constant $\rho_h = (2(\gamma_h/N + \alpha_h))^{-1}$ and inference vector

$$\mathbf{d}(\boldsymbol{\Sigma}_h) = (\mathbf{I} + \rho_h \gamma_h \mathbf{S}(\boldsymbol{\Sigma}_h) / N)^{-1} \mathbf{1} \quad (12)$$

obtained from the pairwise inference matrix $\mathbf{S}(\boldsymbol{\Sigma}_h)$ defined as

$$[[\mathbf{S}(\boldsymbol{\Sigma}_h)]]_{ii} = 0, \quad [[\mathbf{S}(\boldsymbol{\Sigma}_h)]]_{ij} = \sigma_{ij}^h / \sigma_{ii}^h \quad \forall i \in \mathcal{N}, j \in \mathcal{N} \setminus i. \quad (13)$$

Proof: See the Appendix. \blacksquare

Proposition 1 shows that there exists a unique BNE strategy. Furthermore, the unique BNE strategy is linear in self-preference g_{ih} at each time slot. This is a direct consequence of the fact that the utility in (4) has quadratic form and the prior on preferences is normal (5). From the linear strategy in (10), we observe that increase in mean preference \bar{g}_h causes an increase in consumption when $a_{ih} > b_{ih}$. From the first set of strategy coefficients in (11), \mathbf{a}_h , we observe that the estimated effect of renewable power $\bar{\omega}_h$ has a decreasing effect on individual consumption. This is expected since increasing $\bar{\omega}_h$ implies an expected increase in the price which lowers the incentive to consume. We remark that the users only need the mean estimate $\bar{\omega}_h$ to respond optimally. Hence, the SO does not need to send the distribution of ω_h , P_{ω_h} to the users.

Observe that the strategy coefficients a_{ih} and b_{ih} do not depend on information specific to user i . A consequence of this observation is that the SO knows the strategy functions of all the rational users via the action coefficient equations in (11). On the other hand, the realized load consumption l_{ih} is a function of realized preference g_{ih} , i.e., $l_{ih}^* = s_{ih}^*(g_{ih})$, which is private. Hence, by knowing the strategy function the SO cannot predict the consumption level of the users with certainty. Nevertheless, the SO can use the BNE strategy to estimate the expected total consumption in order to achieve its policy design objectives as we discuss in Section V.

The strategy coefficients \mathbf{a}_h and \mathbf{b}_h in (11) depend on the inference vector $\mathbf{d}(\boldsymbol{\Sigma}_h)$ which is driven by the covariance matrix $\boldsymbol{\Sigma}_h$. In order to identify the effect of correlation among preferences on user behavior, we define the notion σ -correlated preferences.

Definition 2: The preferences of users are σ -correlated at time h if $\sigma_{ij}^h = \sigma$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$ and $\sigma_{ii}^h = 1$ for all $i \in \mathcal{N}$ where $0 \leq \sigma \leq 1$.

In σ -correlated preferences, the correlation among users varies according to the parameter σ . Hence, the definition does not allow heterogeneous correlation among pairs. When the parameter σ is varied, the preference correlation change is ubiquitous. The inference vector $\mathbf{d}(\boldsymbol{\Sigma}_h)$ is well-defined for σ -correlated preferences where $0 \leq \sigma \leq 1$. We interpret the effect of correlation on the users' BNE strategies in the next result.

Proposition 2: Denote the BNE strategy weights by \mathbf{a}_h^σ , \mathbf{b}_h^σ when preferences are σ -correlated. Then, when $\sigma'' > \sigma'$, we have the following relationship:

$$a_{ih}^{\sigma'} = a_{ih}^{\sigma''} \quad \text{and} \quad b_{ih}^{\sigma'} > b_{ih}^{\sigma''} \quad \forall i \in \mathcal{N}. \quad (14)$$

Proof: First note that \mathbf{a}_h in (11) does not depend on the covariance matrix, hence, it remains the same. When the preferences are σ -correlated, the off-diagonal elements of the inference matrix $\mathbf{S}(\boldsymbol{\Sigma}_h)$ in (13) are equal to σ . As a result, we

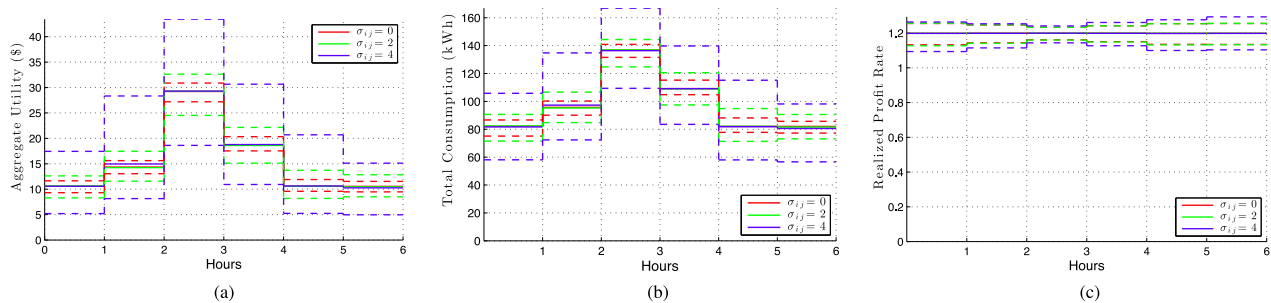


Fig. 2. Effect of preference distribution on performance metrics. (a) Aggregate utility U_h , (b) total consumption L_h , and (c) realized rate of return r_h . Each line represents the value of the performance metric with respect to three values of $\sigma_{ij} \in \{0, 2, 4\}$ as color coded in the legends. Solid lines represent the average value over 100 instantiations. Dashed lines indicate the maximum and minimum values of 100 instantiations. Changes in user preferences do not affect mean rate of return of the SO.

can write it as $\mathbf{S}(\boldsymbol{\Sigma}_h) = \sigma(\mathbf{1}\mathbf{1}^T - \mathbf{I})$ which allows us to express the inference vector as $\mathbf{d}(\boldsymbol{\Sigma}_h) = (\mathbf{I} + \rho_h \gamma_h \sigma (\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1} \mathbf{1}$. Use the relationship that $(\mathbf{I} + c(\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1} \mathbf{1} = ((N-1)c + 1)^{-1} \mathbf{1}$ for a constant c to obtain the following weights for \mathbf{a}_h^σ and \mathbf{b}_h^σ in (11):

$$\mathbf{b}_h^\sigma = \rho_h ((N-1)\gamma_h \rho_h \sigma / N + 1)^{-1} \mathbf{1}. \quad (15)$$

Equation (14) follows by comparing individual entries of (15). \blacksquare

Proposition 2 shows that user i 's strategy is to place less weight on self-preference g_{ih} when the correlation between the users increases. If user i 's preference is higher than the mean, $g_{ih} > \bar{g}_h$, then increasing correlation coefficient σ decreases consumption of user i . When $g_{ih} < \bar{g}_h$, user i 's consumption increases as σ is increased. The intuition is as follows. Consider the case where $g_{ih} > \bar{g}_h$. As the correlation coefficient increases, it is more likely that others' preferences are also above the mean. For instance, others' preferences are certainly above the mean when $\sigma = 1$, given $g_{ih} > \bar{g}_h$. This implies that consumption willingness of others is similar to i , which then means the price will be higher than what is expected when the population's preference is at the mean. As a result, user i decreases its consumption. An identical reasoning follows when $g_{ih} < \bar{g}_h$.

The increase in correlation coefficient enhances the ability of individuals to predict others' preferences. Alternatively, this increase in prediction ability can be achieved via communication among individuals, e.g., sharing of preferences or consumption levels. Hence, Proposition 2 states that if communication is such that the predictive abilities of the individuals increase, then users decrease their weight on self-preferences and relatively increase the weight on the mean estimate \bar{g}_h . In [20], a similar result is shown to hold for the beauty contest game where in contrast to the game considered here, individuals have the incentive to increase their action when others increase theirs.

We note that the strategy coefficients of all users are the same when the preferences are σ -correlated; that is, $a_{ih}^\sigma = a_{jh}^\sigma$ and $b_{ih}^\sigma = b_{jh}^\sigma$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$. Furthermore, the effect of γ_h on strategy coefficients is readily identified from (15). BNE strategy coefficients \mathbf{a}_h^σ and \mathbf{b}_h^σ decrease with respect to increasing γ_h . The downward trend on consumption is conceivable since increasing γ_h means increasing the elasticity of price with respect to total consumption.

We remark that similar analysis as in Proposition 2 follows when σ_{ii} is equal to some constant $c > \sigma$ for all $i \in \mathcal{N}$, that is, it suffices that the diagonals of $\boldsymbol{\Sigma}_h$ are equal.

IV. NUMERICAL EXAMPLES

We numerically explore the effects of the preference distribution $P_{\mathbf{g}_h}$ (Section IV-A), policy parameter γ_h (Section IV-B), and prediction errors of renewable power term ω_h (Section IV-C) on aggregate utility U_h , total consumption L_h , price p_h in (2), and the SOs realized rate of return r_h defined in Section II-A.

In the numerical setup, there are $H = 6$ h and $N = 10$ users. The mean preference profile for the horizon is given as $\bar{\mathbf{g}} := [\bar{g}_1, \dots, \bar{g}_H] = [30, 35, 50, 40, 30, 30]$. We choose the preference covariance matrix $\boldsymbol{\Sigma}_h$ to be identical for all times, i.e., $\boldsymbol{\Sigma}_h = \boldsymbol{\Sigma}$ for $h \in \mathcal{H}$. Furthermore, we consider σ -correlated preferences with diagonal elements $\sigma_{ii} = 4$ and the correlation is set to $\sigma_{ij} = 2$ for all users unless otherwise stated. Note that, we consider σ -correlated preferences but use σ_{ij} to refer to off-diagonal elements of $\boldsymbol{\Sigma}$. Users are selfish with utility in (4) and the decay parameter chosen as $\alpha_h = 1.5$. The cost function parameter value is $\kappa_h = 1$ for $h \in \mathcal{H}$. For the baseline setup, the policy parameter is set to $\gamma_h = 0.6$ for $h \in \mathcal{H}$. Unless stated otherwise, the renewable power term ω_h comes from normal distribution with mean $\bar{\omega}_h = 0$ and variance $\sigma_{\omega_h} = 2$ for $h \in \mathcal{H}$.

A. Effect of Consumption Preference Distribution

In Figs. 2(a)–(c), we plot aggregate utility U_h , total consumption L_h , and realized rate of return r_h with respect to time, respectively. Each solid line is the mean value for the corresponding metric over 100 realizations of the random variables (\mathbf{g}_h, ω_h) for each correlation value $\sigma_{ij} \in \{0, 2, 4\}$. Each dashed plot refers to the maximum and minimum values among the scenarios considered.

Mean preference \bar{g}_h has a significant effect on all of the performance metrics except the realized profit. We observe that as \bar{g}_h increases, e.g., from $h = 1$ to $h = 2$ or from $h = 2$ to $h = 3$, aggregate utility and total consumption increases in Fig. 2(a) and (b), respectively. The price also increases in peak hours with a jump in total consumption as per (2). The increase in price does not automatically translate to an increase in realized profit ratio in Fig. 2(c) since both revenue

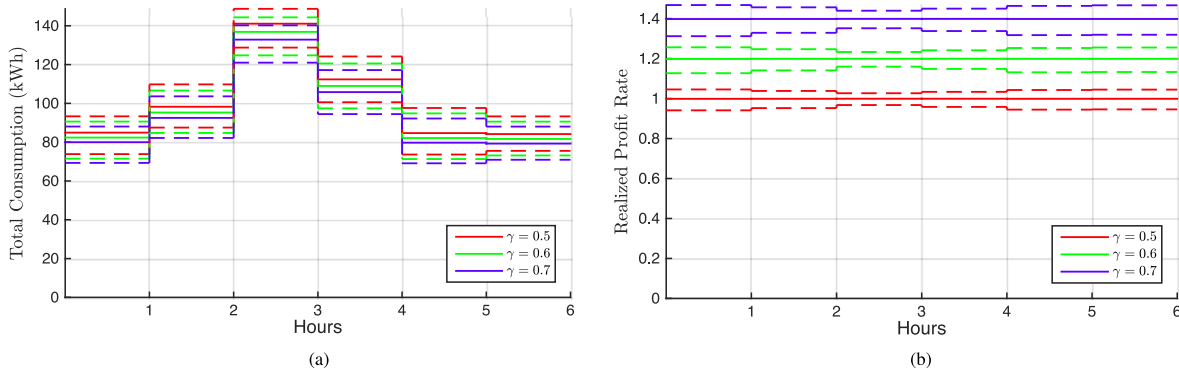


Fig. 3. Effect of policy parameter on performance metrics. (a) Total consumption L_h and (b) realized rate of return r_h . Each solid line represents the average value (over 100 realizations) of the performance metric with respect to three values of $\gamma \in \{0.5, 0.6, 0.7\}$ where $\gamma_h = \gamma$ for $h \in \mathcal{H}$ color coded in each figure. Dashed lines mark minimum and maximum values over all scenarios. Total consumption decreases with increasing γ .

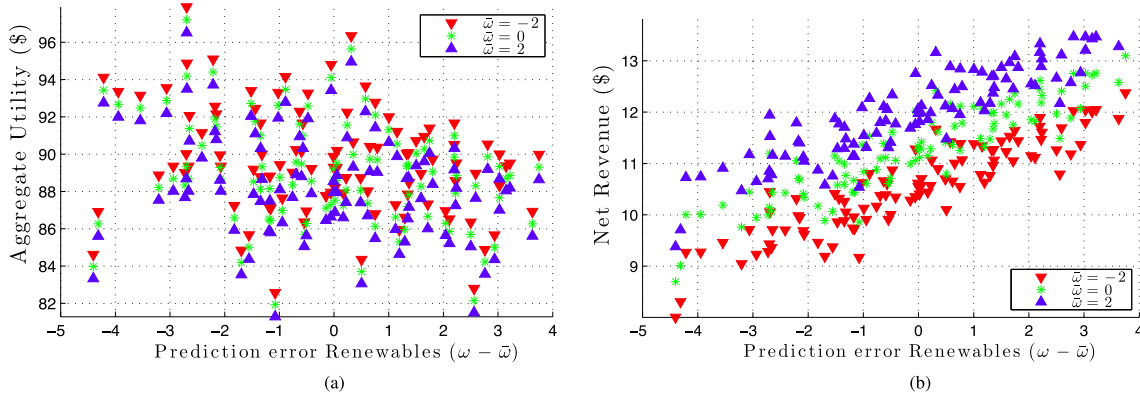


Fig. 4. Effect of prediction error of renewable power uncertainty ω_h on performance metrics. (a) Aggregate utility $\sum_{h \in \mathcal{H}} U_h$ and (b) net revenue NR. In both figures, the horizontal axis shows the prediction error for the renewable term in price, that is, $\omega_h = \omega$ and $\bar{\omega}_h = \bar{\omega}$ for $h \in \mathcal{H}$ and it shows $\omega - \bar{\omega}$. Each point in the plots corresponds to the value of the metric at a single initialization. When the realized renewable term ω is larger than predicted $\bar{\omega}$, net revenue increases.

and cost grow quadratically with total consumption. The correlation value σ_{ij} affects the minimum–maximum band that total consumption moves in as shown in Fig. 2(b). Specifically, the uncertainty in consumption is higher when user preferences have higher correlation. This is reasonable since higher correlation means that if one user’s realization of the preference is higher than the mean preference \bar{g}_h , others’ preferences are also likely to be higher, whereas in low correlation others are likely to balance the high consumption preference of a given user. This indicates that the SO can estimate consumption behavior with higher accuracy and it requires less reserve energy when the preferences are less correlated. We further observe that the mean welfare over the horizon is not affected by the correlation coefficient and is equal to \$100, \$99.8, and \$100.5 for σ_{ij} values equal to 0, 2, and 4, respectively. Finally, we observe that the difference between maximum and minimum values of the rate of return decreases as \bar{g}_h increases in Fig. 2(c).

B. Effect of Policy Parameter

Fig. 3(a) and (b) illustrates the effect of policy parameter γ_h on total consumption L_h and realized rate of return r_h , respectively. We fix the policy parameter across time, that is, $\gamma_h = \gamma \in \{0.5, 0.6, 0.7\}$ for all $h \in \mathcal{H}$. As before, solid

lines indicate average value over 100 instantiations (\mathbf{g}_h, ω_h) and dashed lines indicate minimum and maximum values over these 100 runs.

Total consumption decreases as γ increases in Fig. 3(a) as noted in the discussion following Proposition 2. Furthermore, PAR in total consumption is not altered when γ is fixed over the time horizon in Fig. 3(a) where PAR of the average total consumption over all runs is 1.4 for each $\gamma \in \{0.5, 0.6, 0.7\}$. As a policy to reduce PAR, the SO might choose to increase γ when \bar{g}_h is high and lower γ when \bar{g}_h is low—see Section V. In Fig. 3(b), we observe that the mean realized profit ratio is in proportion with the policy parameter γ . This is expected since both revenue and cost grow with the square of the total consumption multiplied by constants γ and $\kappa_h/2$, respectively. Hence, the rate of return is expected to be equal to $2\gamma/\kappa_h$ which gives the mean rate of return in Fig. 3(b) for each γ value. We further observe that the mean realized welfare over the horizon W is not affected by the changes in γ , that is, for $\gamma \in \{0.5, 0.6, 0.7\}$ mean welfare is equal to \$99, \$99.8, and \$100.2, respectively. At the same time, mean user aggregate utility U decreases, that is, for $\gamma \in \{0.5, 0.6, 0.7\}$ it is equal to \$99, \$93.8, and \$89, respectively. Hence, the loss in aggregate utility is compensated by the increase in SOs net revenue.

C. Effect of Uncertainty in Renewable Power

From the BNE strategy in (10), we observe that an increase in the expectation $\bar{\omega}_h$ reduces the load of the users linearly. Hence, the SO can use the response of its users to mitigate the effects of fluctuations in renewable source generation. However, the contract between the operator and users is such that the latter are charged based on the realization of the random variable ω_h . We analyze the effect of prediction errors of the renewable term, $\omega - \bar{\omega}$, on the aggregate utility U and NR in Fig. 4(a) and (b), respectively. Each point in the plots corresponds to the value of the metric at a single initialization given $\bar{\omega} \in \{-2, 0, 2\}$. There are 100 initializations for each $\bar{\omega}$ value.

Fig. 4(a) shows that aggregate utility is higher when the predicted renewable variable discounts price, i.e., $\bar{\omega} = -2$. This is regardless of the prediction error. We observe that there is a small decrease in mean aggregate utility on average with increasing $\bar{\omega}$, i.e., average aggregate utility across all runs is equal to \$89.6, \$89, and \$88.3, respectively, for $\bar{\omega} \in \{-2, 0, 2\}$. We do not observe any correlation with the prediction error of renewables and aggregate utility in Fig. 4(a). Fig. 4(b) shows that NR is likely to be larger when the realized value of ω is larger than $\bar{\omega}$. This is reasonable since users respond to $\bar{\omega}$, however, when the realized ω is larger than predicted $\bar{\omega}$, users pay more than what they expected. Furthermore, given a fixed amount of prediction error $\omega - \bar{\omega}$, observe that an increase in the announced estimate $\bar{\omega}$ is beneficial to the NR in Fig. 4(b). Finally, the mean welfare is not affected by the announced estimate $\bar{\omega}$, that is, mean welfare across all runs is equal to \$100.2 for each $\bar{\omega} \in \{-2, 0, 2\}$.

V. PRICING POLICY MECHANISMS

We propose desired rate of return and PAR minimization as the two objectives according to which the SO determines its pricing policy parameters $\{\gamma_h\}_{h \in \mathcal{H}}$ given price anticipating users. Below we first explain these two pricing schemes and then compare them with flat and TOU pricing schemes.

1) *Desired Rate of Return RTP*: The SO can pick its policy parameter γ_h to target an expected rate of return $r_h^* = E[R_h(L_h(\gamma_h))/C_h(L_h(\gamma_h))]$ at time h . Given its uncertainty on user preferences \mathbf{g}_h , the SO can rely on the user behavior determined by the BNE (10) to reason about total load $L_h(\gamma_h)$. The term $L_h(\gamma_h)$ makes the SOs influence on demand through the parameter γ_h explicit. In a budget balancing scheme, the SO would set desired rate of return to $r_h^* = 1$. Otherwise, it is customary that $r_h^* > 1$ —see [8], [10] for similar pricing policies. Solving for the policy parameter γ_h to attain an expected rate of return of r_h^* yields that $\gamma_h = r_h^* \kappa_h / 2$ when the renewable generation term is $\omega_h = 0$ as per the discussion in Section IV-B.

2) *PAR Minimizing Price*: The PAR of load profile $\{L_h\}_{h \in \mathcal{H}}$ is defined as the ratio of the maximum load over the operation cycle to the average load profile. The SO can pick the policy parameter $\{\gamma_h\}_{h \in \mathcal{H}}$ to minimize the expected PAR of consumption behavior which is formulated as follows:

$$\min_{\{\gamma_h\}_{h \in \mathcal{H}}} E \left[\frac{H \max_{h=1, \dots, H} L_h(\gamma_h)}{\sum_{h=1}^H L_h(\gamma_h)} \right]. \quad (16)$$

In computing its expected PAR, the SO relies on the model of rational user behavior in (10). From the perspective of the SO, the total consumption at equilibrium $L_h = \sum_{i \in \mathcal{N}} s_{ih}^*(g_{ih})$ is a normal random variable with mean $N a_h (\bar{g}_h - \bar{\omega}_h \gamma_h / N)$ and variance $b_h^2 (N + N(N-1)\sigma)$ when the preferences are σ -correlated. We use $L_h(\gamma_h)$ above to indicate that the distribution of this normal random variable is parametrized by γ_h . Similarly, the average consumption over the horizon is also a normal random variable. Hence, the PAR expression inside the expectation in (16) is a random variable that is the maximum of jointly normal random variables divided by the sum of these random variables both of which are parametrized by $\{\gamma_h\}_{h \in \mathcal{H}}$. To the best of our knowledge, an exact expression for neither the density function nor the mean of this random variable exists. Hence, we cannot hope to find a closed form solution to the minimization in (16). Therefore, we use the evolutionary algorithm presented in [21] to determine the minimizing policy profile $\{\gamma_h^*\}_{h \in \mathcal{H}}$. The evolutionary algorithm starts with a candidate set of policy profiles and iteratively evolves the set based on the expected PAR achieved for each profile in the set. For each policy profile $\{\gamma_h\}_{h \in \mathcal{H}}$ considered in this set, we evaluate the expected PAR using Monte Carlo sampling.

In both of the pricing schemes above the users are assumed to be price anticipating and strategic as price value is not known at time of their decision making as per the discussion in Section II-A. Next, we present two pricing schemes, flat and TOU pricing, in which the SO determines the price value in advance.

3) *Flat Price*: Users are charged with a flat price (FLAT) p during the horizon. Users respond by optimizing their utility in (3) with price replaced by FLAT p , that is, they are price takers. The optimal user response is obtained by solving the first order conditions $l_{ih}^* = (g_{ih} - p) / 2\alpha_h$ assuming a non-negative solution, that is, $g_{ih} > p$ for all i . The SO wants to pick the price that maximizes the expected welfare over the horizon $E[W]$, $p^* = \arg \max_p E[W]$. Note that, W depends on price-taker user response which is random to the SO. However, the SO can use the form of the price-taker user behavior to solve explicitly for the p^* based on $P_{\mathbf{g}_h}$. When $\alpha_h = \alpha$ and $\kappa_h = \kappa$ for all $h \in \mathcal{H}$, we obtain the following expression for the FLAT:

$$p^* = \frac{\kappa \sum_{h \in \mathcal{H}} \bar{g}_h}{H(\kappa + 2\alpha)}. \quad (17)$$

Note that, the price scales linearly with the time average of the mean preferences $\{\bar{g}_h\}_{h \in \mathcal{H}}$ over the horizon. In order for the FLAT to maximize welfare, the non-negative consumption requirement, $g_{ih} > p^*$, needs to be satisfied for all $i \in \mathcal{N}$ and $h \in \mathcal{H}$. Given the optimal FLAT (17), this condition is equivalent to $\bar{g} - \sum_h \bar{g}_h / H \geq -2\alpha \bar{g} / \kappa$ where $\bar{g} := \min_{i \in \mathcal{N}, h \in \mathcal{H}} \{g_{ih}\}$. Since the preferences have normal distribution the probability that the condition will be violated is positive. In particular, this probability is small when κ is small or α is large, or when the minimum mean value over the whole horizon is away from zero, $\min_{h \in \mathcal{H}} \bar{g}_h \gg 0$, and time average of the mean preferences is relatively close to the minimum mean preference.

4) *TOU Price*: Users are charged with hourly prices p_h that are determined by maximizing hourly expected welfare (6), that is, $p_h^* = \arg\max_p E[W_h]$, given that users optimally respond to announced hourly prices by selecting $l_{ih}^* = (g_{ih} - p_h^*)/2\alpha_h$. Note that, the W_h in (6) does not explicitly depend on price. Its dependence on p_h comes from the price-taker user response. Given the optimal behavior of users, the price that maximizes expected welfare is explicitly expressed as

$$p_h^* = \frac{\kappa_h \bar{g}_h}{\kappa_h + 2\alpha_h}. \quad (18)$$

The price above scales linearly with the mean preference of the time slot \bar{g}_h . The condition for non-negativity of consumption $l_{ih} \geq 0$ reduces to $\underline{g}_h - \bar{g}_h \geq -2\alpha_h \underline{g}_h / \kappa_h$ for all $h \in \mathcal{H}$ where $\underline{g}_h := \min_{i \in \mathcal{N}} g_{ih}$. The probability of violating this requirement is small when α_h is large or κ_h is small. Furthermore, it is small when the smallest mean preference is large $\min_{h \in \mathcal{H}} \bar{g}_h \gg 0$ or when σ_{ii} is relatively small. When the probability of violation is high, the SO has to consider the probability that some users might choose not to consume any deferrable loads for a given time slot. In this case, the user behavior distribution will not be normal from the perspective of the SO and the TOU will not have the form in (18).

Next, we present the efficient competitive equilibrium price with complete information (CCE) as a benchmark to compare the aforementioned pricing schemes against.

A. Efficient Competitive Equilibrium

A competitive equilibrium is a tuple of prices $\mathbf{p}^W := [p_1^W, \dots, p_H^W]$ and consumption profiles $\{l_{ih}\}_{i \in \mathcal{N}, h \in \mathcal{H}}$ such that each user picks the consumption to maximize its selfish utility given the price, $l_{ih}^W = (g_{ih} - p_h^W)/2\alpha_h$ and the market clears, that is, total consumption demand is met by the SO [9], [10]. Note that, in a competitive equilibrium, users respond to announced price value of the SO. A competitive equilibrium is efficient when it maximizes welfare W . In order to compare the aforementioned pricing schemes, we provide an explicit characterization of the unique efficient competitive equilibrium under certain conditions on the values of the preferences $\{g_h\}_{h \in \mathcal{H}}$.

Proposition 3: Consider the welfare W with user utility functions in (4) and SOs cost function in (1). There exists a unique competitive equilibrium price $\mathbf{p}^W := [p_1^W, \dots, p_H^W]$ such that when price takers respond optimally by maximizing their selfish utility, $l_{ih}^W = (g_{ih} - p_h^W)/2\alpha_h$, the welfare W is maximized. Furthermore, if the minimum preference value $\underline{g}_h := \min_{i \in \mathcal{N}} \{g_{ih}\}$ satisfies the following condition:

$$\underline{g}_h - \sum_{j \in \mathcal{N}} g_{jh} / N \geq -2\alpha_h \underline{g}_h / \kappa_h \quad \forall h \in \mathcal{H} \quad (19)$$

then the competitive equilibrium price \mathbf{p}^W is characterized by

$$p_h^W = \frac{\kappa_h \sum_{i \in \mathcal{N}} g_{ih}}{N(\kappa_h + 2\alpha_h)} \quad \forall h \in \mathcal{H}. \quad (20)$$

Proof: At the efficient competitive equilibrium, welfare W is maximized and market clears, that is, demand equals supply, $\sum_{i \in \mathcal{N}} l_{ih} = Q_h$ where Q_h is defined as the SOs

supply variable. We can translate this definition to the following optimization problem:

$$\begin{aligned} \max_{\{\{l_{ih}\}_{i \in \mathcal{N}}, Q_h\}_{h \in \mathcal{H}}} & \sum_{h \in \mathcal{H}} \left(-C_h(Q_h) + \sum_{i \in \mathcal{N}} g_{ih} l_{ih} - \alpha_h l_{ih}^2 \right) \\ \text{s.t.} & \sum_{i \in \mathcal{N}} l_{ih} = Q_h \quad h \in \mathcal{H} \\ & l_{ih} \geq 0 \quad i \in \mathcal{N}, h \in \mathcal{H}. \end{aligned} \quad (21)$$

Consider the Lagrangian of the above optimization problem obtained by relaxing the market clearance constraint with the corresponding price variables $\mathbf{p} := [p_1, \dots, p_H]$

$$\begin{aligned} \mathcal{L}(\{\{l_{ih}\}_{i \in \mathcal{N}}, Q_h\}_{h \in \mathcal{H}}, \mathbf{p}) &= \sum_{h \in \mathcal{H}} -C_h(Q_h) + \sum_{i \in \mathcal{N}} g_{ih} l_{ih} \\ &\quad - \alpha_h l_{ih}^2 + \sum_{h \in \mathcal{H}} p_h \left(Q_h - \sum_{i \in \mathcal{N}} l_{ih} \right). \end{aligned} \quad (22)$$

When the Lagrangian (22) is maximized with respect to the primal variables l_{ih} and Q_h given $C_h(Q_h)$ in (1), we respectively obtain the following conditions:

$$g_{ih} - 2\alpha l_{ih} - p_h = 0 \quad \forall i \in \mathcal{N}, h \in \mathcal{H} \quad (23)$$

$$-\kappa_h Q_h / N + p_h = 0 \quad h \in \mathcal{H}. \quad (24)$$

Note that, (23) enforces that users are price takers, $l_{ih} = (g_{ih} - p_h)/2\alpha_h$ and (24) indicates that the optimal price is linear in Q_h , $p_h = \kappa_h Q_h / N$. By the Karush–Kuhn–Tucker (KKT) optimality conditions, the feasibility conditions stated in (21) has to be satisfied. From the power balance constraint, we get that the optimal price is a linear function of total consumption, that is, $p_h = \kappa_h \sum_{i \in \mathcal{N}} l_{ih} / N$ for all $h \in \mathcal{H}$. Now using the price taker behavior and assuming $l_{ih} \geq 0$, we obtain

$$p_h = \frac{\kappa_h}{N} \sum_{i \in \mathcal{N}} (g_{ih} - p_h) / 2\alpha_h. \quad (25)$$

Solving the above equation for p_h , we get the competitive equilibrium price in (20). When we plug in the price p_h^W in (20) into the price taker consumption $l_{ih}^W = (g_{ih} - p_h^W)/2\alpha_h$, the consumption non-negativity assumption is satisfied given the condition $\kappa_h (g_{ih} - \sum_{j \in \mathcal{N}} g_{jh} / N) + 2\alpha g_{ih} \geq 0$ for all $i \in \mathcal{N}$. Since the inequality in the condition has to be satisfied by all the user preferences, this condition reduces to the condition in (19).

The solution to (21) is a competitive equilibrium because each user responds optimally l_{ih}^W with respect to their selfish utility by the KKT conditions (23) and the market clears at the equilibrium price p_h^W . Furthermore, the equilibrium is efficient because W is maximized. Finally, the solution is unique as the optimization in (21) is strictly concave with feasible linear constraints. ■

The proposition provides a characterization of CCE \mathbf{p}_H^W in (23) given the condition in (19) holds. The proof relies on expressing the efficient competitive equilibrium as a welfare maximization problem with the constraints that demand matches supply and the consumption of users is non-negative. The optimality conditions yield that the user consumption that

maximizes welfare is equivalent to users maximizing their selfish utility (4) given p_h^W . This shows that the feasible optimal consumption to the maximization problem is an efficient competitive equilibrium.

The condition in (19) is required due to the non-negativity constraint on user consumption. The condition implies that the minimum realized preference \underline{g}_h in the population cannot be too small with respect to the realized mean preference $\sum_{i \in \mathcal{N}} g_{ih}/N$. The condition is akin to the condition for TOU pricing except that here we replace \bar{g}_h with the mean of realized preferences $\sum_{i \in \mathcal{N}} g_{ih}/N$. As a result the discussion for TOU pricing on parameters that make the probability of violation small applies to (19) verbatim. For increasing α_h , \bar{g}_h and decreasing κ_h , the violation probability is small. If the correlation σ among users increases, the probability of violating the condition decreases. We expect to have high correlation among user preferences that have means larger than zero—e.g., the electric vehicle charging demand profiles in [18].

The CCE in (20) gives us a benchmark to compare the proposed pricing schemes that operate under incomplete information of the preferences. In [14], the authors proposed a decentralized algorithm that converges to an efficient competitive equilibrium when the SO does not know the preferences of its users. In [9], a taxing scheme, which incentivizes users to truthfully reveal their preferences and aligns Nash equilibrium behavior with the competitive equilibrium is proposed. In this paper, we consider unilateral information feeding from the SO to the users, hence, the SO only has estimates of the preferences of the users. Next, we compare the presented pricing schemes.

B. Analytical Comparison Among Pricing Policies

We expand the RTP price by substituting in the BNE strategy in (10) given σ -correlated preferences for the term \bar{L}_h in (2)

$$p_h(L_h; \omega_h) = \frac{\gamma_h(N\bar{g}_h + \omega_h(\gamma_h/N + 2\alpha_h))}{N((N+1)\gamma_h/N + 2\alpha_h)} + \frac{\gamma_h b_h^\sigma}{N} \sum_{i \in \mathcal{N}} g_{ih} - \bar{g}_h \quad (26)$$

where the coefficient b_h^σ is a single element of the vector \mathbf{b}_h^σ in (15). The first term above is obtained by grouping and simplifying all the terms that relate to the first term in user behavior (10) and the term ω_h/N in (2). When we take the expectation of the price above, the second term is nulled and we replace ω_h with $\bar{\omega}_h$ in the first term. As expected, increasing the expectation of ω_h means an expected increase in price. Furthermore, when $\bar{\omega}_h = 0$, we observe that increasing γ_h increases the price by decreasing the relative weight of the $2\alpha_h$ term in the denominator. When $\bar{\omega}_h = 0$ and $\gamma_h = \kappa_h$, the expected price in (26) is equal to $\kappa_h \bar{g}_h / ((N+1)\kappa_h/N + 2\alpha_h)$ which is smaller than the TOU price in (18) since it has a larger denominator. That is, we expect the TOU price be larger than RTP when $\gamma_h = \kappa_h$. However, the SO can solve for γ_h that equates the expected price of RTP with the TOU price. Moreover, the expectation of competitive price in (20), that is, expectation prior to the realization of the preferences, is equal

to the TOU price in (18). Consequently, the RTP price can be made in expectation equal to the expectation of the CCE price by selecting γ_h . Since in both TOU and CCE pricing schemes, users respond optimally to the given price, we expect that users in TOU will behave on average the same as the users in CCE. The same argument cannot be made between the RTP pricing scheme and CCE pricing as user behavior differs in the two schemes. Finally, note that FLAT is equal to the time average of TOU. That is, FLAT is not equal to the CCE price unless all preferences are distributed according to the same mean [22].

Next, we consider the effect of population size N on RTP (26), FLAT (17), TOU price (18), and CCE (20). First note that flat and TOU prices are not affected by the number of users. As N grows, the expectation of RTP price, i.e., the first term of (26), converges to $\gamma_h \bar{g}_h / (\gamma_h + 2\alpha_h)$ which is identical to TOU price when $\gamma_h = \kappa_h$. Furthermore, when the covariance matrix Σ_h is diagonal, that is, $\sigma = 0$, the RTP price in (26) converges to TOU price almost surely by the strong law of large numbers. The convergence still holds when the correlation coefficient σ is positive but decays with N [23]. The same set of conditions can be used to show that the CCE price in (20) converges to TOU price almost surely. Since the users in TOU pricing scheme are price takers and by definition TOU price maximizes expected welfare, it is not surprising that TOU price becomes closer to the competitive equilibrium as N grows. On the other hand, the same argument is not that straightforward for price anticipating users. Yet, observe that as N grows RTP price approaches a value that depends on mean prior preference. As a result, price anticipating users become price takers as a single user's influence on price diminishes. As N grows, RTP schemes approach the competitive equilibrium given diminishing correlation among preferences. This discussion relates to inefficiency in markets [24], [25] and to the competitive limit theorems for Cournot markets [16], [26].

C. Numerical Comparison Among Pricing Policies

In Fig. 5(a)–(c), we numerically compare the pricing schemes with respect to their influence on welfare W , PAR of total consumption, and total consumption over the horizon $\sum_{h \in \mathcal{H}} L_h$, respectively. We use the same setup described in Section IV unless otherwise stated. We choose the desired rate of return in RTP to be $r_h^* = 1.5$ which yields $\gamma_h = r_h^* \kappa_h / 2 = 0.75$. For PAR pricing, we let $\gamma_h \in [0.5, 1.5]$. The optimal policy parameters are found to be $[\gamma_1^*, \dots, \gamma_6^*] = [0.55, 0.5, 1.5, 0.78, 0.54, 0.6]$. The PAR minimizing choices of high policy parameter in the peak time $h = 3$ when $\bar{g}_3 = 50$ and lower γ_h other times supports the intuition developed from Fig. 3(b). The FLAT determined according to (17) is equal to $p^* = \$0.09/\text{kWh}$. The TOU and CCE prices are determined according to (18) and (20), respectively. The TOU price profile is equal to $\mathbf{p}^* = [0.075, 0.088, 0.125, 0.1, 0.075, 0.075] \text{ \$/kWh}$. Each point in Fig. 5(a)–(c) corresponds to the value attained in the performance metric in that scenario out of the 100 instantiations for a given pricing model. We indicate the mean value over the 100 scenarios with a colored dashed line, each color corresponding to a pricing model

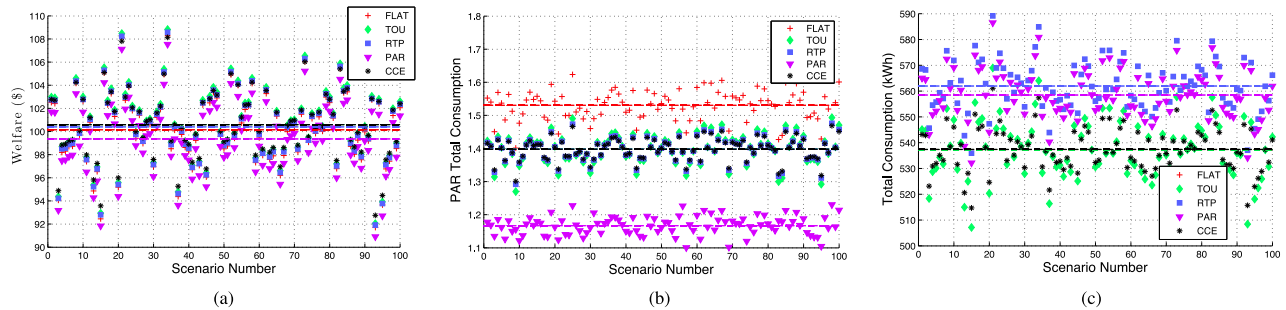


Fig. 5. Comparison of different pricing schemes with respect to (a) welfare W , (b) PAR of total consumption, and (c) total consumption $\sum_{h \in \mathcal{H}} L_h$. In (a)–(c), each point corresponds to the value of the metric for that scenario and dashed lines correspond to the average value of these points over all scenarios with colors associating the point with the pricing scheme in the legend. The PAR-minimizing policy performs better than others in minimizing PAR of consumption while at the same time being comparable to CCE in welfare.

indicated in the legend. We remark that the non-negativity conditions of FLAT, TOU, and CCE are satisfied in all of the instantiations.

As per the discussion above, the average CCE price across the scenarios is equal to the TOU price. The mean RTP price over the 100 scenarios trends below the TOU price and is equal to $\bar{p}^{\text{RTP}} = [0.06, 0.07, 0.1, 0.08, 0.06, 0.06]$ \$/kWh. In comparison, PAR pricing achieves a lower mean price for low preference hours and higher price in the high preference hours, $\bar{p}^{\text{PAR}} = [0.05, 0.05, 0.16, 0.08, 0.05, 0.05]$ \$/kWh. In addition, the variance of the RTP and PAR prices are low with the standard deviation less than \$0.003. We note that some of the variation observed in metrics for RTP and PAR are due to the uncertainty introduced by the renewable energy term ω_h .

In Fig. 5(a), we observe that PAR attains the lowest mean welfare \$99.4, and CCE and TOU have the highest mean welfare \$100.6. The RTP pricing scheme is close to the CCE welfare with mean welfare \$100.4. The FLAT pricing has a mean welfare \$100.1 that is in between PAR and RTP mean welfares. In addition, the breakdown of welfare to aggregate utility and net revenue changes depending on price-anticipating or price-taking behavior model, e.g., for PAR, mean aggregate utility U is equal to \$83.5 whereas for CCE it is \$75.5. This means the SOs net revenue is higher in price-taking models.

In Fig. 5(b), we see that PAR pricing achieves the lowest mean PAR of consumption value 1.17 with small deviation from the mean across runs 0.03. CCE, RTP, and TOU attain mean PAR consumption values close to each other around 1.4 but TOU pricing has a higher standard deviation 0.05. As can be expected FLAT has the largest mean PAR of consumption value 1.53 and high deviation 0.05 across runs as it does not adjust to varying consumption preferences of the users.

When we compare the total consumption over the whole horizon in Fig. 5(c), we observe that RTP and PAR pricing have means 561 and 558 kWh, respectively, that are close to each other. This is due to the fact that the average of PAR pricing policy parameters is $\sum_h \gamma_h / H = 0.75$ which is equal to policy parameter of RTP. CCE, TOU, and FLAT attain a lower consumption mean \$537. In addition, the deviation of total consumption across runs is smaller for RTP and PAR models with deviation equal to 9.8 kWh compared to the standard

deviation of total consumption in TOU and FLAT that is equal to 11.4 kWh. This indicates that the forecast certainty of the SO is higher when users anticipate price.

In sum, the proposed PAR minimizing pricing achieves a low PAR by incentivizing users to shift their consumption to off-peak times. This shift does not hurt the welfare of the system compared to other pricing schemes and is beneficial to the aggregate utility of the users compared to CCE and other price-taking schemes. Furthermore, users are facing similar prices. Hence, the increase in aggregate utility is due to the increased total consumption in RTP and PAR, that is, users consume more but pay similar amounts of money. In both RTP and PAR, the price anticipation of the users helps to reduce total consumption variance, increasing the demand predictability. Finally, PAR and RTP by design admit renewable integration via the term ω_h in (2) as discussed in Section IV-C.

VI. CONCLUSION AND POLICY IMPLICATIONS

We considered a DRM model where users with unknown and heterogeneous marginal utilities respond to real time prices announced by the SO ahead of each time slot. The pricing mechanism is such that the SO announces a pricing function that linearly increases with total consumption per capita and decreases with increasing renewable energy generation in that time slot. The pricing provides the SO with the versatility to charge hourly prices that incentivizes users to behave according to its goals. However, the users' consumption preferences are random to the SO and it may be that the users behave in a manner that trumps the SOs intentions in order to achieve their selfish goals. Our analysis shows that this would not happen if agents, selfish as they may be, act rationally.

In particular, from the perspective of the SO, the PAR of demand is reduced when the SO implements a PAR minimizing price, i.e., users shift their consumption to time slots in which it is cheaper for SO to produce. The variance in demand caused by randomness in user preferences at each time slot reduces, increasing the demand forecast accuracy of the SO. From the perspective of a regulator invested in the well-being of the system, the proposed tariff by the SO is fair to the users [27] and the welfare is expected to be close to the efficient welfare. Furthermore, the renewable penetration is likely to increase given accurate forecasts of renewable generation

due to deferrable loads serving as a buffer that absorbs the fluctuations of renewable generation. From the perspective of the users, the proposed tariff is expected to increase user utility, i.e., users can consume the same amounts but at a cheaper price.

It has to be observed that the aforementioned implications depend on specific modeling choices, namely, the assumption of rational user behavior, the consideration of perfect knowledge of the preference distribution \mathbf{g}_h , and the use of a quadratic form for the SOs cost. These choices may be simplistic, but the results outlined here still provide meaningful guidelines if these restrictions are lifted. Consider, e.g., the case in which users are sub-rational and recall that we considered two models of rational behavior: price taking and price anticipating. If the users respond to announced price values, they would be price takers and the price is in expectation equal to the competitive equilibrium price. If the users are selfish and anticipate their contribution to price function, then the price is shown to approach the competitive price as N grows under certain conditions, and otherwise numerical results indicate that welfare reduction is tolerable. These models of behavior capture the two extremes of user behavior, and therefore, sub-rational behavior is likely to exhibit a behavior that falls in between these two extremes.

Regarding the assumption of perfect knowledge of the user preference distribution $P_{\mathbf{g}_h}$ it is likely that the SO will have some uncertain estimates, and that the difference between the two is a random noise term. When the SO utilizes such noisy predictions of the mean preference \bar{g}_h , the rational users will discount the weight on the public information based on the uncertainty of the SO in their responses. While the overall performance of the system will degrade, the generalization will not affect the overall implications of the analysis. As for the use of quadratic energy costs, it is better to consider a model in which the cost for each device can be modeled as a linear function of the power dispatched from each device. In this case, the cost model is an increasing piecewise linear function of total consumption as power is dispatched from more costly generators with increasing total consumption [7]. The quadratic cost function is a tractable approximation for the piecewise linear cost function and captures the fundamental property that higher energy production requires dispatching from more costly sources. The quantitative specifics may change for piecewise linear functions but the qualitative conclusions will be similar.

APPENDIX

A. Proof of Proposition 1

Our plan is to propose a linear strategy as in (10) and use the fixed point equations of BNE in (9) to solve for the linear strategy coefficients. First, we obtain the best response function. In order to compute the best response in (7) we take the derivative of conditional expected utility with respect to l_{ih} , equate the resultant to zero and solve for l_{ih}

$$\text{BR}(g_{ih}; \mathbf{s}_{-ih}) = \frac{g_{ih} - \gamma_h \left(\bar{\omega}_h + \sum_{j \neq i} E_{\omega_h} [E_{ih} [s_{jh}]] \right) / N}{2(\gamma_h / N + \alpha_h)}. \quad (27)$$

Next, we use the best response expression in (27) in the BNE (9) and substitute the proposed linear strategy in (10) for the corresponding terms to get the following fixed point equation:

$$\begin{aligned} & a_{ih}(\bar{g}_h - \bar{\omega}_h \gamma_h) + b_{ih}(g_{ih} - \bar{g}_h) \\ &= \rho_h \left(g_{ih} - \gamma_h \left(\bar{\omega}_h + \sum_{j \neq i} E_{ih} [a_{jh}(\bar{g}_h - \bar{\omega}_h) \right. \right. \\ & \quad \left. \left. + b_{jh}(g_{jh} - \bar{g}_h)] \right) \right) \end{aligned} \quad (28)$$

for all $i \in \mathcal{N}$ where we used the definition of ρ_h . Given the normal prior on \mathbf{g}_h , we have $E_{ih}[g_{jh}] = (1 - \sigma_{ij}^h / \sigma_{ii}^h) \bar{g}_h + (\sigma_{ij}^h / \sigma_{ii}^h) g_{ih}$. Substituting the term for the expectation

$$\begin{aligned} & a_{ih}(\bar{g}_h - \bar{\omega}_h \gamma_h) + b_{ih}(g_{ih} - \bar{g}_h) \\ &= \rho_h g_{ih} - \rho_h \gamma_h \left(\bar{\omega}_h + \sum_{j \neq i} a_{jh}(\bar{g}_h - \bar{\omega}_h) \right. \\ & \quad \left. + b_{jh} \left(\left(1 - \frac{\sigma_{ij}^h}{\sigma_{ii}^h} \right) \bar{g}_h + \frac{\sigma_{ij}^h}{\sigma_{ii}^h} g_{ih} - \bar{g}_h \right) \right). \end{aligned} \quad (29)$$

Next, we add and subtract $\rho_h \bar{g}_h$ to the right-hand side and group the terms that multiply $g_{ih} - \bar{g}_h$ and $\bar{g}_h - \bar{\omega}_h \gamma_h$

$$\begin{aligned} & a_{ih}(\bar{g}_h - \bar{\omega}_h \gamma_h) + b_{ih}(g_{ih} - \bar{g}_h) \\ &= \rho_h \left(1 - \gamma_h \sum_{j \neq i} a_{jh} \right) (\bar{g}_h - \bar{\omega}_h \gamma_h) \\ & \quad + \rho_h \left(1 - \gamma_h \sum_{j \neq i} \frac{\sigma_{ij}^h}{\sigma_{ii}^h} b_{jh} \right) (g_{ih} - \bar{g}_h). \end{aligned} \quad (30)$$

Equating the terms that multiply $(\bar{g}_h - \bar{\omega}_h \gamma_h)$ and $(g_{ih} - \bar{g}_h)$ for all $i \in \mathcal{N}$, we get the following equations for \mathbf{a}_h and \mathbf{b}_h :

$$a_{ih} = \rho_h \left(1 - \gamma_h \sum_{j \neq i} a_{jh} \right), \quad b_{ih} = \rho_h \left(1 - \gamma_h \sum_{j \neq i} \frac{\sigma_{ij}^h}{\sigma_{ii}^h} b_{jh} \right) \quad (31)$$

for all $i \in \mathcal{N}$. We write the equations above in vector form

$$(\mathbf{I} + \rho_h \gamma_h (\mathbf{1}\mathbf{1}^T - \mathbf{I})) \mathbf{a}_h = \rho_h \mathbf{1} \quad (32)$$

$$(\mathbf{I} + \rho_h \gamma_h \mathbf{S}(\boldsymbol{\Sigma}_h)) \mathbf{b}_h = \rho_h \mathbf{1} \quad (33)$$

where in (33) we used the definition of the inference matrix (13). Action coefficient a_{ih} is obtained from (32) by multiplying both sides by $(\mathbf{I} + \rho_h \gamma_h (\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1}$ and using the following identity:

$$(\mathbf{I} + \rho_h \gamma_h (\mathbf{1}\mathbf{1}^T - \mathbf{I}))^{-1} \mathbf{1} = ((N - 1) \rho_h \gamma_h + 1)^{-1} \mathbf{1}. \quad (34)$$

The action coefficient b_{ih} is obtained from (33) by multiplying both sides of the equation by $(\mathbf{I} + \rho_h \gamma_h \mathbf{S}(\boldsymbol{\Sigma}_h))^{-1}$. Hence, we showed that there exists a BNE strategy that is linear as in (10).

To prove uniqueness, we first show that the game defined by payoff in (4) is a Bayesian potential game with potential

function $v(\{l_{ih}\}_{i \in \mathcal{N}}; \mathbf{g}_h, \omega_h)$ —see [28] for a definition—and then argue that the function is strictly concave which implies unique solution for the original game. Define the symmetric matrix $\mathbf{Q}_h \in \mathbb{R}^{N \times N}$ where $[[\mathbf{Q}_h]]_{ii} = 1$ for all $i = 1, \dots, N$ and $[[\mathbf{Q}_h]]_{ij} = \rho_h$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$. Let $\mathbf{l}_h := [l_{1h}, \dots, l_{Nh}]$. For the stage game $h \in \mathcal{H}$ with payoffs u_i in (4), there exists a potential function given by

$$v(\mathbf{l}_h; \mathbf{g}_h, \omega_h) = -(\gamma_h + \alpha_h) \mathbf{l}_h^T \mathbf{Q}_h \mathbf{l}_h + \mathbf{l}_h^T (\mathbf{g}_h - \gamma_h \omega_h \mathbf{1}^T). \quad (35)$$

Note that, $\partial v(\mathbf{l}_h) / \partial l_{ih} = \partial u_{ih}(\mathbf{l}_h) / \partial l_{ih}$. Hence, the stage game user payoffs (4) and information on \mathbf{g}_h is a Bayesian potential game with potential function in (35) in [28, Lemma 6].

This result implies that the equilibrium of the Bayesian potential game with function $v(\mathbf{l}_h; \mathbf{g}_h, \omega_h)$ is the same as the equilibrium of the stage game at time h with payoffs u_{ih} in (4). It can be shown that \mathbf{Q}_h is positive definite for all $h \in \mathcal{H}$ by Sylvester's criterion which states that if each m by m upper left sub-matrix of a symmetric matrix has a positive determinant for all $m = 1, \dots, N$ then the matrix is positive definite. This implies that Bayesian potential function is strictly concave with a unique maximizer. Hence, the equilibrium is unique. ■

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