Symbol Error Probabilities for General Cooperative Links*

Alejandro Ribeiro, Xiaodong Cai, and Georgios B. Giannakis,
Dept. of ECE, University of Minnesota
200 Union Street, Minneapolis, MN 55455

Abstract

Cooperative Diversity (CD) networks have been receiving a lot of attention recently as a distributed means of improving error performance and capacity. This paper derives the average Symbol Error Probability (SEP) for Amplify and Forward CD links. The resulting expressions are general as they hold for an arbitrary number of cooperating branches, arbitrary number of cooperating hops per branch, and many channel fading models. Their simplicity provides valuable insights to the performance of CD networks and allows their optimization. Besides revealing the diversity advantage, they clearly show from where this advantage comes from and prove that the diversity advantage holds independently of the channel fading model. Finally, they explain how diversity is improved in multi-hop CD networks.

Keywords: Fading, Performance analysis, Cooperative Diversity

1. Introduction

Cooperative diversity (CD) networks are attracting increasing attention as a new and promising diversity technique. Somewhat inspired by multi-antenna systems the technology exploits the fact that around a given terminal, there can be other single-antenna terminals which can be used to enhance diversity by forming a virtual (or distributed) multi-antenna system. As demonstrated in e.g., [1], [2], [3], CD networks can achieve a diversity order equal to the number of paths between the source and the destination, and in this sense, they offer similar advantages to any existing diversity technique.

Relying on high SNR approximations, this paper derives expressions for the average Symbol Error Probability (SEP). The results obtained are simple and general, which allows their application to complex CD scenarios. In particular, we show how to estimate the average SEP in networks with multiple cooperating branches each composed of multiple cooperating hops. They are valid for various fading models, provided that their probability density functions (*pdf*) are non-zero at zero instantaneous SNR.

* Work in this paper was prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon.

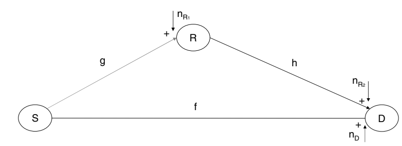

Figure 1. Single cooperating terminal.

2. SEP with a single cooperating terminal

2.1. System Model

Consider the simplest CD strategy shown in Fig. 1, where we have an information source, S, and a destination D, communicating over a complex channel f. A relay terminal, R, is willing to cooperate in this link providing D with a second copy of the original signal through the complex channels S-R and R-D with flat fading coefficients g and h, respectively. Without loss of generality, we assume that all the additive white Gaussian noise (AWGN) terms, n_{R_1}, n_{R_2}, and n_D have equal variance N_0. Similar to [3], [6], we suppose that the realizations of the random variables f, g, and h have been acquired at the receiver ends e.g., via training. Note that no particular assumptions are made on channel statistics.

We consider the Amplify and Forward (AF) model where relays simply amplify the signal received from the source [3]. Assuming that S and R transmit through orthogonal channels, the destination D receives two independent copies of the signal x:

$$\begin{aligned} y_D &= fx + n_D, \\ y_R &= hA(gx + n_{R_1}) + n_{R_2} = hAgx + n_R, \end{aligned} \tag{1}$$

where $n_R := hAn_{R_1} + n_{R_2}$, and A is the amplification factor which will be discussed later. The receiver collects these copies with a maximum ratio combiner (MRC) to form a decision variable z. For fixed f, g and h realizations, the variable z is Gaussian, and the SEP conditioned on the instantaneous SNR, γ_z, is given by $P_e = Q(\sqrt{k\gamma_z})$, where the constant k depends on the type of modulation (2 for PSK), and $Q(x) := (1/\sqrt{2\pi})\int_x^\infty e^{-u^2/2}\,du$. The SNR of z can be calculated readily as,

$$\gamma_z = |f|^2 \frac{P_x}{\sigma_D^2} + |Agh|^2 \frac{P_x}{\sigma_R^2} = \gamma_D + \gamma_R, \tag{2}$$

where P_x is the transmitted power at S, $\gamma_D := |f|^2 P_x/\sigma_D^2$, and $\gamma_R := |Agh|^2 Px/\sigma_R^2$. Being the sum of the SNR of the two independent paths, (2) will allow us to work with each of them independently and then analyze what happens when we sum up their contributions. Before that we choose the amplification factor A to maintain constant average power output, equal to the original transmitted power,

$$A^2 = \frac{P_x}{P_x|h|^2 + N_0}. \tag{3}$$

Substituting (3) into (2), we obtain,

$$\gamma_z = \frac{\gamma_g \gamma_h}{1 + \gamma_g + \gamma_h} + \gamma_f, \tag{4}$$

where γ_g and γ_h are the per-hop SNRs associated with the channels g and h, respectively, and are defined similar to γ_f; that is $\gamma_g := |g|^2 P_x/N_0$ and $\gamma_h := |h|^2 P_x/N_0$.

At high SNR, the 1 in the denominator of (4) is negligible; and thus (4) reduces to,

$$\gamma_z = \frac{\gamma_g \gamma_h}{\gamma_g + \gamma_h} + \gamma_f. \tag{5}$$

The SNR in (5) is analytically more tractable than that in (4), which will come handy when analyzing the SEP in Sections 2.2 and 3.2. For those justifiably skeptic about this approximation, [5] shows that for high SNR the average SEPs of (4) and (5) are indistinguishable.

Traditional approaches to obtaining the average SEP (see e.g. [7, chapter 8]) become cumbersome, and even if we were able to obtain the SEP in closed form, the resulting expression would provide limited insight. Instead, we will use a new tool developed in [8], which enables average SEP calculations for sufficiently large SNR by looking at the *pdf* of the SNR around zero. This approach will allow us to achieve insightful results with relatively simple computations.

2.2. SEP Analysis

The work in [8] shows that the average SEP for large SNR can be well approximated relaying on the Mc Laurin series of $p_{\hat{\gamma}}(\hat{\gamma})$. In particular, if the derivatives of $p_{\hat{\gamma}}(\hat{\gamma})$ up to order $t-1$ are null, we have the following asymptotic expression for the average SEP:

$$\bar{P}_e \to \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)k^{(t+1)}} \cdot \frac{1}{t!} \frac{\partial^t p_\gamma}{\partial \gamma^t}(0), \tag{6}$$

Based on (6), we have to study the behavior of $p_\gamma(\gamma)$ around zero, and this is precisely the aim of the following Proposition.

Proposition 1 *Consider three non-negative independent random variables X, Y, and Z with $pdfs$ $p_X(x)$, $p_Y(y)$ and $p_Z(z)$, respectively. These $pdfs$ are unknown except for their values at zero that are denoted as x_0, y_0, and z_0 and assumed to be nonzero. If the variable V is defined as,*

$$V := \frac{XY}{X+Y} + Z, \tag{7}$$

then $p_V(v) = 0$, and the first derivative of $p_V(v)$ evaluated at zero is given by,

$$\frac{\partial p_V}{\partial v}(0) = (x_0 + y_0) z_0. \tag{8}$$

The proof of Proposition 1 is given in [5]. If X, Y, and Z are the per-hop SNRs in (5), then V corresponds to γ_z, the SNR of the decision variable. What is more, the derivative in (8) can be plugged in (6) to yield an expression for the average SEP,

$$\bar{P}_e \to \frac{3}{4k^2}\left[P_{\gamma_g}(0) + P_{\gamma_h}(0)\right] P_{\gamma_f}(0). \tag{9}$$

In the special case of Rayleigh fading, the SNR is exponentially distributed with *pdf*: $p_\gamma(\gamma) = (1/\bar{\gamma}) e^{\frac{\gamma}{\bar{\gamma}}}$, and (9) reduces to,

$$\bar{P}_e \to \frac{3}{4k^2}\left(\frac{1}{\bar{\gamma}_g} + \frac{1}{\bar{\gamma}_h}\right)\frac{1}{\bar{\gamma}_f}. \tag{10}$$

This result is strikingly simple, and will allow us to draw some interesting conclusions in Section 2.3. Furthermore, the treatment in this section can be easily generalized to an arbitrary number of cooperative branches and hops per branch as we will see in Sections 3.1 and 3.2 respectively.

By now we have established that diversity of order two is possible for our simple CD network under various fading channel models. Furthermore, we have seen that the *diversity comes from the product of two independent SNRs* , that of the direct path and the one of the relay path.

2.3. Relay Selection

We consider here the relay selection problem which is easily tractable with our approach. We focus on Rayleigh fading but the results can be readily generalized to other models. Suppose that several terminals are available to cooperate with the source, and the source has to decide which one will be the best possible cooperator. The optimization problem under consideration is selecting the relay that minimizes the average SEP. Looking at (10), we can see that minimizing the average SEP is equivalent to minimizing the function:

$$\Omega = \frac{1}{|\bar{g}|^2} + \frac{1}{|\bar{h}|^2}, \tag{11}$$

where $|\bar{g}|^2 := E(|\bar{g}|^2)$, and $|\bar{h}|^2 := E(|\bar{h}|^2)$. Note that Ω is twice the inverse of the harmonic mean of $|\bar{g}|^2$ and $|\bar{h}|^2$. So, the solution to the relay selection problem is selecting the pair which maximizes the harmonic mean of the fading coefficients average power; i.e.,

$$R_{opt} = R_i : \max\{\mu_{\mathrm{H}}(|\bar{\mathrm{g}}_{\mathrm{i}}|^2, |\bar{\mathrm{h}}_{\mathrm{i}}|^2)\}, \tag{12}$$

where μ_H denotes the harmonic mean function. This is potentially applicable to routing problems in CD networks.

A slightly different problem arises when $|\bar{g}|^2$ and $|\bar{h}|^2$ cannot vary independently due to physical limitations arising from e.g., the path-loss between two terminals. Letting d_{SR} denote the distance S-R and d_{SD} the distance S-D, we define the relative distance from source to relay as $\rho := d_{SR}/d_{SD}$, from where we can easily infer that $|\bar{g}|^2$ and $|\bar{h}|^2$ are given by:

$$|\bar{g}|^2 = \frac{|\bar{f}|^2}{\rho^\alpha} \qquad |\bar{h}|^2 = \frac{|\bar{f}|^2}{(1-\rho)^\alpha}, \tag{13}$$

where $|\bar{f}|^2 := E(|\bar{f}|^2)$, and α is the pathloss slope [4, p.104]. Given this physical model, (11) takes the form

$$\Omega = \frac{1}{|\bar{f}|^2}\left[\rho^\alpha + (1-\rho)^\alpha\right], \tag{14}$$

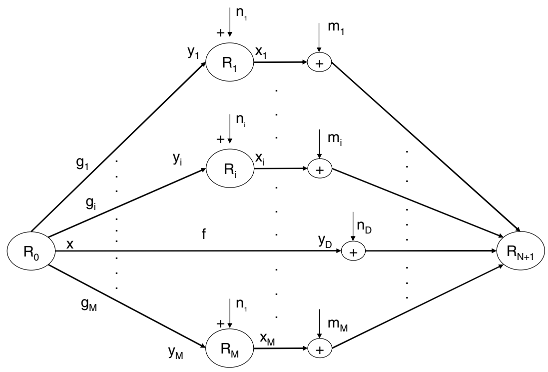

Figure 2. Multi-branch cooperation. M terminals cooperate with S to attain diversity of order $M+1$.

which has a maximum for $\rho = 1/2$, for any $\alpha > 1$. This implies the intuitively appealing result that the relay should be placed just in the middle between source and destination. Moreover, this result does not depend on the detailed path-loss model. It is clear from (14) that for exponents $\alpha > 3$, usually encountered in practice, the minimum SEP has also null second derivative. Thus, this minimum is relatively robust to relay displacements from the optimum position, which is always desirable for optimal designs.

The optimal placement obtained sheds light to more general cooperation scenarios of interest. In this sense, *we should focus on CD schemes that work well when h and g have balanced (ideally equal) power profiles,* because placing the relay either close to the source or close to the destination offer suboptimal solutions. This result also speaks for the flexibility optimally placed CD systems have in improving the average SEP relative to non-CD multi-antenna systems of diversity order two.

3. General Cooperative Links

3.1. Multi-branch Cooperative Diversity

The ideas of Section 2 can be generalized to multi-branch CD networks such as those depicted in Fig. 2. In addition to the direct path with fading coefficient f, we consider M cooperating terminals (relays) $\{R_1, \ldots, R_M\}$. The channel coefficient between S and relay R_i is denoted as g_i, while that between R_i and D as h_i. We assume that the relays transmit over mutually orthogonal channels. At each relay, a noise term n_i is present and a second noise term m_i is introduced at reception. At each cooperating terminal, the amplification factor is that defined in (3).

The generalization of the results of section Section 2 is stated in the following proposition (for the proof see [5]).

Proposition 2 *Consider a set of non-negative random variables $\{X\} = \{X_0, X_1, ..., X_M\}$ whose pdfs $p_0, p_1, ..., p_M$ have non zero values at zero, and denote these values as $p_0(0), p_1(0), ..., p_M(0)$. If the random variable V_M is,*

$$V_M := \sum_{i=0}^{M} X_i, \tag{15}$$

then all the derivatives of p_{v_M} evaluated at zero up to order $(M-1)$ are zero, and the M^{th} order derivative is given by

$$\frac{\partial^M p_{V_M}}{\partial v^M}(0) = \prod_{i=0}^{M} p_i(0). \tag{16}$$

It is worth mentioning that this result is applicable to various conceivable diversity strategies, even outside the scope of CD networks. This generality will be exploited to analyze the multi-hop scenario in Section 3.3, but for now we will use the limit in Proposition 2 as an expression for the SNR of the multi-branch CD network of Fig. 2. The decision variable z at the MRC output is given by,

$$z = \left(\frac{f}{\sigma_D}\right)^* \tilde{y}_D + \sum_{i=1}^{M} \left(\frac{h_i A_i g_i}{\sigma_{R_i}}\right)^* \tilde{y}_{R_i}, \tag{17}$$

where we defined the variables $\tilde{y}_D := y_D/\sigma_D$ and $\tilde{y}_{R_i} := y_{R_i}/\sigma_{R_i}$. It follows easily from (17) that the SNR of the decision variable is approximately given by,

$$\gamma_z = \gamma_f + \sum_{i=1}^{M} \frac{\gamma_{g_i}\gamma_{h_i}}{\gamma_{g_i} + \gamma_{h_i}}, \tag{18}$$

where again we eliminated the one in the denominator of the relay path SNRs, which is equivalent to considering $A_i = 1/g_i$, and has no impact on the asymptotic SEP. We proceed by analogy to Section 2.2 applying Proposition 2 to the variable defined by (17),

$$\frac{\partial^M p_{\gamma_z}(\gamma_z)}{\partial \gamma_z^M} = p_{\gamma_f}(0) \prod_{i=1}^{M} [p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0)]. \tag{19}$$

Substituting (19) into (6), we obtain the asymptotic expression for the average SEP of the multi-branch CD system,

$$\bar{P}_e \approx C(M)\ \ p_{\gamma_f}(0) \prod_{i=1}^{M} \Big(p_{\gamma_{g_i}}(0) + p_{\gamma_{h_i}}(0)\Big), \tag{20}$$

where $C(M) = \frac{\prod_{k=1}^{M+1}(2k-1)}{2(M+1)!k^{(M+1)}}$ is a constant that depends on the number of cooperating branches M. It is interesting to note that $C(M)$ increases with M, which slightly affects the diversity advantage.

Equation (20) is quite general as it holds under any SNR distribution provided that the underlying pdf at the origin is nonzero. For the case of Rayleigh fading, it takes the form,

$$\bar{P}_e \approx \frac{C(M)}{k^{M+1}} \ \cdot\ \frac{1}{\gamma_f} \prod_{i=1}^{M} \left(\frac{1}{\gamma_{g_i}} + \frac{1}{\gamma_{h_i}}\right). \tag{21}$$

3.2. Multi-hop Cooperative Diversity

A second point of interest is what happens when we add multiple hops to each of the diversity branches. In principle, one

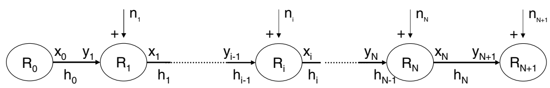

Figure 3. Multi-hop system with N intermediate relays ($N+1$ hops).

expects that as the results were generalizable to M branches, they should be generalizable to N hops. And indeed this is the case. We consider a set of N cooperating relays R_i, as depicted in Fig. 3.

Without loss of generality the relays will be considered to be an ordered set $\{R\} = \{R_1, \ldots, R_N\}$; and for the sake of uniformity, the source will be named R_0, and the destination R_{N+1}. At node R_i the received signal will be named y_{i-1} and the transmitted signal x_i. Note that x_0 is the signal transmitted by the source and y_N is the signal received at destination. The fading coefficient between R_i and R_{i+1} will be denoted as h_i, the amplification factor at node R_i as A_i, and the AWGN as n_i. Given this nomenclature, the system equations are

$$\begin{aligned} y_i &= h_{i-1}x_{i-1} + n_i, \\ x_i &= A_i y_i. \end{aligned} \tag{22}$$

A detailed treatment of this model for A_i defined as in (3) seems infeasible. But as we emphasized in Section 2.1, the results for large average SNR are indistinguishable from the model using the simpler definition $A_i = 1/h_{i-1}$, with which we obtain the following input-output relationship:

$$y_N = h_N x_0 + h_N \sum_{i=0}^{n-1} A_i n_i + n_N, \tag{23}$$

Based on (23), we can show that the instantaneous SNR γ of the received variable y_N is,

$$\gamma = \frac{\gamma_0\gamma_1 \cdots \gamma_N}{\sum_{i=0}^{N} \gamma_0\gamma_1 \cdots \gamma_{i-1}\gamma_{i+1} \cdots \gamma_N}, \tag{24}$$

where $\{\gamma_i\}_{i=0}^{N}$ are the per-hop SNRs, defined by $\gamma_i := P_x/N_0|h_i|^2$. To study this SNR we require the following Proposition (proved in [5]).

Proposition 3 *Consider $N+1$ non-negative independent random variables $X_0, X_1, \ldots, X_N$, with unknown $pdfs$ $p_0(x_0), p_1(x_1), \ldots, p_N(x_N)$ except for their values at zero that are assumed strictly positive and known. If we define the random variable*

$$Z = g(\vec{X}) = \frac{X_0X_1 \ldots X_N}{\sum_{i=0}^{N} X_0X_1 \ldots X_{i-1}X_{i+1} \ldots X_N}, \tag{25}$$

then the pdf of Z at the origin, satisfies

$$p_Z(0) = \sum_{i=0}^{N} p_{X_i}(0). \tag{26}$$

Using (26), and (6) we obtain the following expression for the asymptotic average SEP of a multi-hop system:

$$\bar{P}_e \approx \frac{1}{2k} \sum_{k=0}^{N} p_{\gamma_i}(0). \tag{27}$$

For the case of Rayleigh fading, the latter reduces to

$$\bar{P}_e \approx \frac{1}{2k} \sum_{k=0}^{N} \frac{1}{\bar{\gamma}_i}. \tag{28}$$

From (28) we see that the multi-hop system is a diversity one system, and each new hop adds a new term to the probability of error. The advantage of multi-hop transmissions comes from the path loss gains associated with it. In a practical system, dividing the transmission path will result in a group of average SNRs whose sum of inverses is smaller than the inverse of the original path SNR. In fact, from (28) we can obtain a condition under which a multi-hop system offers advantages over a single-hop system. If $\bar{\Gamma}$ denotes the average SNR of the single hop system, the multi-hop system will be preferred if

$$\sum_{k=0}^{N} \frac{1}{\bar{\gamma}_i} < \frac{1}{\bar{\bar{\Gamma}}}. \tag{29}$$

Note that the sum in the left hand side of (29) is $1/N$ times the harmonic mean of the individual hop SNRs. Thus, we have established that for Rayleigh fading channels *multi-hop should be preferred over single hop if the harmonic mean of the average multi-hop SNRs is larger than the single-hop SNR divided by the number of hops* .

Worth mentioning, is that from (28) we can easily generalize the optimal relay placement design of Section 2.3. Regardless of the underlying path loss model, the result is that equispaced relays along the line that connects source with destination are SEP-optimal at sufficiently high SNR. This optimal design enjoys the same properties as that of Section 2.3 and points to the importance of CD networks having per-hop fading coefficients with balanced average power.

3.3. Multi-branch, Multi-hop CD

Relying on the results of Sections 3.1 and 3.2, we are ready to obtain an expression for the average SEP of multi-branch, multi-hop transmissions. The result of Section 3.1 applies to a sum of random variables regardless of their specific $pdfs$, provided that their values at the origin are nonzero. In particular, Proposition 2 applies when the $pdfs$ correspond to a multi-hop transmission as that of Section 3.2 for which the asymptotic value of the pdf at zero is given by (26).

Based on these two observations, let us consider a cooperative system with $M+1$ diversity branches $\{B_0, B_1, \ldots, B_M\}$, where by convention the diversity branch B_0 corresponds to the direct path. Each of the remaining M branches $\{B_1, \ldots, B_M\}$ is composed of N_i relays $\{R_1, \ldots, R_{N_i}\}$. The channel coefficients between the relays R_{ij} and $R_{i,j+1}$ of branch B_i are denoted by h_{ij}, with h_{i0} being the coefficient between the source and the first relay, and h_{iN_i} being that between the last relay and the destination.

We define the average per-hop SNRs as usual $\gamma_{ij} := E|h_{ij}|^2 P_x/N_0$, and we also define $p_{ij}(\gamma_{ij})$ to be the pdf of γ_{ij}. With these definitions and combining the results of (20) and (26) we arrive at

$$\bar{P}_e \approx \frac{C(M)}{k^{M+1}}\, p_{00}(0) \prod_{i=1}^{M} \left(\sum_{j=0}^{N_i} p_{ij}(0) \right). \tag{30}$$

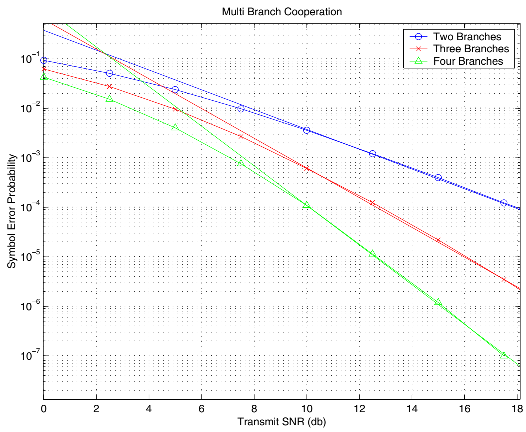

Figure 4. Multi-branch cooperation.

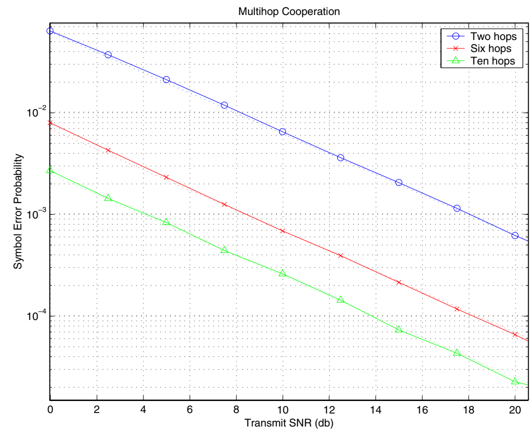

Figure 5. Multi-hop cooperation.

Restricting (30) to Rayleigh fading, we obtain,

$$\bar{P}_e \approx \frac{C(M)}{k^{M+1}} \frac{1}{\bar{\gamma}_{00}} \prod_{i=1}^{M} \left(\sum_{0}^{N_i} \frac{1}{\bar{\gamma}_{ij}} \right), \qquad (31)$$

which is neat in its simplicity given its applicability to quite general cooperative networks.

4. Simulations and numerical results

A first concern is how tight the asymptotic results are with respect to pragmatic SNR values. Several simulations ran with this goal confirmed that the asymptotic results provide a very good approximation not only for large but also for moderate SNR values. We tested BPSK modulation, Rayleigh fading and parameterized the results against the transmit SNR defined as $SNR := P_x/N_0$. Fig. 4 presents simulated values for various number of cooperating branches. In this case, we consider the channels g_i and h_i as having equal power and compare the simulation results with the analytical lines predicted by (21). For SNR values as low as 10dB, the difference between the observed SEP and the asymptotic SEP is less than 9%.

Similar tightness is observed in Fig. 5 for the multi-hop case. In this case the quality approximation remains good for even large number of cooperating hops. different from the case of Fig. 4 where the approximations are not good for more than four or five cooperating branches.

It is apparent from Figs. 4 and 5, and can be confirmed from (31), that in general *relay power is better used when it adds a cooperative branch than when it adds a hop in an existent branch* .

5. Concluding Remarks

We analyzed the average error probability performance for networks with cooperative terminals amplifying and forwarding their received signals from the source, when the average SNR is sufficiently high. Our performance analysis is applicable to cooperative links with any number of hops and branches; and remains valid for a large class of fading models, whose *pdf s* have nonzero values at the origin, including Rayleigh and Rician fading channels. While our error probability formulas were derived for high average SNR, our simulations testified that they match well the simulated error probability even at moderate SNR values.

Our error probability analysis revealed that the error gain of multi-hop systems stems from the reduced path loss, while that of multi-branch systems comes from both the reduced path loss and the diversity. Furthermore, the simplicity of our error probability expressions can be used to design cooperative relays optimally in the sense of minimizing error probability. This may have interesting applications to routing algorithms, relay placement, and power allocation among different terminals in wireless networks, directions we have marked in our future research agenda[1].

References

[1] P. Anghel, G. Leus, and M. Kaveh, “Multi-user space-time coding in cooperative networks,” in *Proc. of Intl. Conf. on Acoustics, Speech, and Signal Processing*, vol. 4, pp. 73–76, Hong Kong, April 2003.

[2] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, “A theoretical characterization of the multihop wireless communications channel with diversity,” in *Proc. of IEEE Global Telecommunications Conference*, pp. 841–845, San Antonio, Texas, November 2001.

[3] J. N. Laneman, “Cooperative diversity in wireless networks: algorithms and architectures,” *Ph.D. Thesis, Massachusets Institute of Technology*, September 2002.

[4] T. S. Rappaport, *Wireless Communications*. Prentice Hall, 1996.

[5] A. Ribeiro, X. Cai, and G. B. Giannakis, “Symbol error probabilities for general cooperative links,” *IEEE Transactions on Wireless Communications*, submitted 2003.

[6] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity - part I: system description,” *IEEE Transactions on Communications*, accepted 2002.

[7] M. K. Simon and M.-S. Alouini, *Digital Communications over Fading Channels*. Wiley-Interscience, 2000.

[8] Z. Wang and G. B. Giannakis, “A simple and general parameterization quantifying performance in fading channels,” *IEEE Transactions on Communications*, vol. 51, pp. 1389–1398, Aug. 2003.

1 The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U. S. Government.