NON-PARAMETRIC DISTRIBUTED QUANTIZATION-ESTIMATION USING WIRELESS SENSOR NETWORKS*

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ABSTRACT

Wireless sensor networks deployed to perform surveillance and monitoring tasks have to operate under stringent energy and bandwidth limitations. These motivate well distributed estimation scenarios where sensors quantize and transmit only one, or a few bits per observation, for use in forming parameter estimators of interest. In a companion paper, we developed algorithms and studied interesting tradeoffs that emerge even in the simplest distributed setup of estimating a scalar location parameter in the presence of zero-mean additive white Gaussian noise of known variance. Herein, we derive distributed estimators based on binary observations along with their error-variance performance for unknown noise pdfs.

1. INTRODUCTION

Wireless sensor networks (WSNs) consist of low-cost energy-limited transceiver nodes spatially deployed in large numbers to accomplish monitoring, surveillance and control tasks through cooperative actions [4]. The potential of WSNs for surveillance has by now been well appreciated especially in the context of data fusion and distributed detection; e.g., [11, 12]. However, except for recent works where spatial correlation is exploited to reduce the amount of information exchanged [1, 2, 7, 8], use of WSNs for the equally important problem of distributed parameter estimation remains largely uncharted. When sensors have to quantize measurements in order to save energy and bandwidth, estimators based on quantized samples and pertinent tradeoffs have been studied [5, 6]. It is worth stressing that in these contributions as well as in the present work that deals with WSN-based distributed parameter acquisition under bandwidth constraints, the notions of quantization and estimation are intertwined. In fact, quantization becomes an integral part of estimation as it creates a set of *binary* observations based on which the estimator must be formed – a problem distinct from parameter estimation based on the unquantized observations.

In a companion paper we studied estimation of a scalar mean-location parameter in the presence of zero-mean additive white Gaussian noise [10]. We proved that when the dynamic range of the unknown parameter is comparable to the noise standard deviation, estimation based on sign quantization of the original observations exhibits variance almost equal to the variance of the (clairvoyant) estimator based on unquantized observations. We further established that under signal-to-noise ratio (SNR) conditions encountered with WSNs, even a single bit per sensor can have a variance close to the clairvoyant estimator. In this paper, we derive distributed parameter estimators based on binary observations along with their error-variance performance when the noise pdf is unknown. Interestingly, for this problem it is still true that transmitting a few bits (or even a single bit) per sensor can approach under realistic conditions the performance of the estimator based on unquantized data.

For comparison purposes we first analyze mean-location parameter estimation in the presence of known univariate but generally non-Gaussian noise pdfs (Section 3); and subsequently address mean-location parameter estimators based on binary observations when the noise pdf is unknown (Section 4). Simulations corroborate our theoretical findings in Section 5, and we conclude the paper in Section 6.

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2. PROBLEM STATEMENT

Consider a WSN consisting of N sensors deployed to estimate a deterministic parameter θ. The n^{th} sensor observation is

$$x(n) = \theta + w(n), \qquad n = 0, 1, \ldots, N-1 , \tag{1}$$

where $w(n)$ denotes zero-mean noise with pdf $p_w(w)$. We further assume that $w(n_1)$ is independent of $w(n_2)$ for $n_1 \neq n_2$; i.e., noise variables are independent across sensors.

Due to bandwidth limitations, the observations $x(n)$ have to be quantized and estimation of θ can only be based on these quantized values. We will henceforth think of quantization as the construction of a set of indicator variables

$$b_k(n) = \mathbf{1}\{x(n) \in B_k(n)\}, \qquad k = 1, \ldots, K , \tag{2}$$

taking the value 1 when $x(n)$ belongs to the region $B_k(n) = (\tau_k, \infty)$, and 0 otherwise. Estimation of θ will rely on this set of *binary* variables $\{b_k(n), k = 1, \ldots, K\}_{n=0}^{N-1}$. The latter are Bernoulli distributed with parameters $q_k(n)$ satisfying

$$q_k(n) := \Pr\{b_k(n) = 1\} = \Pr\{x(n) > \tau_k\}. \tag{3}$$

The problem addressed in this paper is the construction of estimators $\hat{\theta}$ based on the binary observations $\{b_k(n), k = 1, \ldots, K\}_{n=0}^{N-1}$, when the noise pdf, $p_w(w)$ is unknown. We will also study their variances and prove that for such pdf-unaware estimators it can come close to the clairvoyant Cramer-Rao Lower Bound (CRLB) based on $\{x(n)\}_{n=0}^{N-1}$ and complete knowledge of $p_w(w)$ in certain applications of practical interest.

3. KNOWN NOISE PDF

When the noise pdf is known, we will rely on a single region $B_1(n)$ in (2) to generate a single bit $b_1(n)$ per sensor, using a threshold τ_c

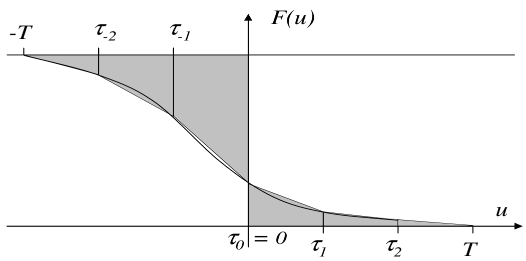

Fig. 1. When the noise pdf is unknown numerically integrating the CCDF using the trapezoidal rule yields an approximation of the mean.

common to all N sensors: $B_1(n) := B_c = (\tau_c, \infty), \forall n$. Based on these binary observations, $b_1(n) := \mathbf{1}\{\mathbf{x}(n) \in (\tau_c, \infty)\}$ received from all N sensors, the fusion center seeks estimates of θ.

Let $F_w(u) := \int_u^\infty p_w(w)\, dw$ denote the Complementary Cumulative Distribution Function (CCDF) of the noise. Using (3), we can express the Bernoulli parameter as, $q_1 = \int_{\tau_c-\theta}^\infty p_w(w)dw = F_w(\tau_c - \theta)$; and its Maximum Likelihood Estimator (MLE) as $\hat{q}_1 = N^{-1}\sum_{n=0}^{N-1} b_1(n)$. Invoking now the invariance property of MLE it follows readily that the MLE of θ is given by [10][1]:

$$\hat{\theta} = \tau_c - F_w^{-1}\left(\frac{1}{N}\sum_{n=0}^{N-1} b_1(n)\right). \tag{4}$$

Furthermore, it can be shown that the CRLB, that bounds the variance of any unbiased estimator $\hat{\theta}$ based on $\{b_1(n)\}_{n=0}^{N-1}$ is [10]

$$\mathrm{var}(\hat{\theta}) \quad \geq \quad \frac{1}{N}\frac{F_w(\tau_c-\theta)[1-F_w(\tau_c-\theta)]}{p_w^2(\tau_c-\theta)} \;:=\; B(\theta)\,. \tag{5}$$

If the noise is Gaussian, and we define the *σ-distance* between the threshold τ_c and the (unknown) parameter θ as $\Delta_c := (\tau_c - \theta)/\sigma$, then (5) reduces to

$$B(\theta) \;=\; \frac{\sigma^2}{N}\frac{2\pi Q(\Delta_c)[1-Q(\Delta_c]}{e^{-\Delta_c}} \;:=\; \frac{\sigma^2}{N}D(\Delta_c), \tag{6}$$

with $Q(u) := (1/\sqrt{2\pi})\int_u^\infty e^{-w^2/2}\, dw$.

The bound $B(\theta)$ is the variance of $\bar{x}$, scaled by the factor $D(\Delta_c)$; recall that $\mathrm{var}(\bar{x}) = \sigma^2/N$ [3, p.31]. Optimizing $B(\theta)$ with respect to Δ_c, yields the optimum at $\Delta_c = 0$ and the minimum value

$$B_{\min} = \frac{\pi}{2}\frac{\sigma^2}{N}. \tag{7}$$

Eq. (7) reveals something unexpected: relying on a single bit per $x(n)$, the estimator in (4) incurs a minimal (just a $\pi/2$ factor) increase in its variance relative to the clairvoyant $\bar{x}$ which relies on the unquantized data $x(n)$. But this minimal loss in performance corresponds to the ideal choice $\Delta_c = 0$, which implies $\tau_c = \theta$ and requires perfect knowledge of the unknown θ for selecting the quantization threshold τ_c.

A closer look at $B(\theta)$ in (5) will confirm that the loss can be huge if $\tau_c - \theta \gg 0$. Indeed, as $\tau_c - \theta \to \infty$ the denominator in (5) goes to zero faster than its numerator, since F_w is the integral of the non-negative pdf p_w; and thus, $B(\theta) \to \infty$ as $\tau_c - \theta \to \infty$. The implication of the latter is twofold: i) since it shows up in the CRLB, the potentially high variance of estimators based on quantized observations is inherent to the possibly severe bandwidth limitations of the problem itself and is not unique to a particular estimator; ii) for any choice of τ_c, the fundamental performance limits in (5) are dictated by the end points $\tau_c - \Theta_1$ and $\tau_c - \Theta_2$ when θ is confined to the interval $[\Theta_1, \Theta_2]$. On the other hand, how successful the τ_c selection is depends on the dynamic range $|\Theta_1 - \Theta_2|$. Notice that in such joint quantization-estimation problems one faces two sources of error: quantization and noise. To account for both, the proper figure of merit for estimators based on binary observations is what we will term quantization signal-to-noise ratio (Q-SNR):

$$\gamma := \frac{|\Theta_1 - \Theta_2|^2}{\sigma^2}; \tag{8}$$

which for WSN is expected to be in the low to moderate range.

[1]Although related results are derived in [10, Prop.1] for Gaussian noise, it is straightforward to generalize the referred proof to cover also non-Gaussian noise pdfs.

4. UNKNOWN NOISE PDF

In certain applications it may not be reasonable to assume knowledge about the noise pdf $p_w(w)$. These cases require *non - parametric* approaches as the one pursued in this section.

We assume that $p_w(w)$ has zero mean so that θ in (1) is identifiable. Let $p_x(x)$ and $F_x(x)$ denote the pdf and CCDF of the observations $x(n)$. As θ is the mean of $x(n)$, we can write

$$\begin{aligned}\theta \;&:=\; \int_{-\infty}^{+\infty} x p_x(x)\, dx = -\int_{-\infty}^{+\infty} x\frac{\partial F_x(x)}{\partial x}\, dx \\ &=\; \int_0^1 F_x^{-1}(v)\, dv\,,\end{aligned} \tag{9}$$

where in establishing the second equality we used the fact that the pdf is the negative derivative of the CCDF, and in the last equality we introduced the change of variables $v = F_x(x)$. But note that the integral of the inverse CCDF can be written in terms of the integral of the CCDF as (see also Fig. 1)

$$\theta \;=\; -\int_{-\infty}^{0}[1-F_x(u)]\, du + \int_0^{+\infty} F_x(u)\, du, \tag{10}$$

allowing one to express the mean θ of $x(n)$ in terms of its CCDF. To avoid carrying out integrals with infinite range, let us assume that $x(n) \in (-T, T)$ which is always practically satisfied for T sufficiently large, so that we can rewrite (10) as

$$\theta = \int_{-T}^{T} F_x(u)\, du \;-\; T. \tag{11}$$

Numerical evaluation of the integral in (11) can be performed using a number of known techniques. Let us consider an ordered set of interior points $\{\tau_k\}_{k=1}^K$ along with end-points $\tau_0 = -T$ and $\tau_{K+1} = T$. Relying on the fact that $F_x(\tau_0) = F_x(-T) = 1$ and $F_x(\tau_{K+1}) = F_x(T) = 0$, application of the trapezoidal rule for numerical integration yields (see also Fig. 1),

$$\theta = \frac{1}{2}\sum_{k=1}^{K}(\tau_{k+1} - \tau_{k-1})F_x(\tau_k) \;-\; T + e_a, \tag{12}$$

with e_a denoting the approximation error. Certainly, other methods like Simpson's rule, or the broader class of Newton-Cotes formulas, can be used to further reduce e_a.

Whichever the choice, the key is that binary observations constructed from the region $B_k := (\tau_k, \infty)$ have Bernoulli parameters

$$q_k := \Pr\{x(n) > \tau_k\} = F_x(\tau_k). \tag{13}$$

Inserting the non-parametric estimators $\hat{F}_x(\tau_k) = \hat{q}_k$ in (12), our parameter estimator when the noise pdf is unknown takes the form:

$$\hat{\theta} = \frac{1}{2}\sum_{k=1}^{K} \hat{q}_k(\tau_{k+1} - \tau_{k-1}) \, - \, T. \tag{14}$$

Since $\hat{q}_k$'s are unbiased, (12) and (14) imply that $\mathrm{E}(\hat{\theta}) = \theta + e_a$. Being biased, the proper performance indicator for $\hat{\theta}$ in (14) is the Mean Squared Error (MSE), not the variance.

Maintaining the bandwidth constraint of 1 bit per sensor (i.e. $K = 1$), let us divide the N sensors in K subgroups containing N/K sensors each, and define the regions

$$B_1(n) := B_k = (\tau_k, \infty), \;\; n = (k-1)(N/K), \ldots, k(N/K)-1; \tag{15}$$

the region $B_1(n)$ will be used by sensor n to construct and transmit the binary observation $b_1(n)$. Herein, the unbiased estimators of the Bernoulli parameters q_k are

$$\hat{q}_k = \frac{1}{(N/K)} \sum_{n=(k-1)(N/K)}^{k(N/K)-1} b_1(n), \quad k = 1, \ldots, K, \tag{16}$$

and are used in (14) to estimate θ. It is easy to verify that $\mathrm{var}(\hat{q}_k) = q_k(1-q_k)/(N/K)$, and that $\hat{q}_{k_1}$ and $\hat{q}_{k_2}$ are independent for $k_1 \neq k_2$.

The resultant MSE, $\mathrm{E}[(\theta - \hat{\theta})^2]$, will be bounded as follows[2].

Proposition 1 *Consider the estimator $\hat{\theta}$ given in* (14)*, with $\hat{q}_k$ as in* (16)*. Assume that for T sufficiently large and known $p_x(x) = 0$, for $|x| \geq T$; the noise pdf has bounded derivative $\dot{p}_w(u) := \partial p_w(w)/\partial w$; and define $\tau_{\max} := \max_k\{\tau_{k+1} - \tau_k\}$ and $\dot{p}_{\max} := \max_{u \in (-T,T)}\{\dot{p}_w(u)\}$. The MSE is given by,*

$$\mathrm{E}[(\theta - \hat{\theta})^2] = |e_a|^2 + \mathrm{var}(\hat{\theta}), \tag{17}$$

with the approximation error e_a and $\mathrm{var}(\hat{\theta})$, satisfying

$$|e_a| \;\; \leq \;\; \frac{T\dot{p}_{\max}}{6}\tau_{\max}^2, \tag{18}$$

$$\mathrm{var}(\hat{\theta}) \;\; = \;\; \sum_{k=1}^{K} \frac{(\tau_{k+1} - \tau_{k-1})^2}{4} \frac{q_k(1-q_k)}{N/K}, \tag{19}$$

with $\{\tau_k\}_{k=1}^K$ a grid of thresholds in $(-T, T)$ and $\{q_k\}_{k=1}^K$ as in (13)*.*

Note from (19) that the larger contributions to $\mathrm{var}(\hat{\theta})$ occur when $q_k \approx 1/2$, since this value maximizes the coefficients $q_k(1 - q_k)$; equivalently, this happens when the thresholds satisfy $\tau_k \approx \theta$ [c.f. (13)]. Thus, as with the case where the noise pdf is known, when θ belongs to an a priori known interval $[\Theta_1, \Theta_2]$, this knowledge must be exploited in selecting thresholds around the likeliest values of θ.

On the other hand, note that the $\mathrm{var}(\hat{\theta})$ term in (17) will dominate $|e_a|^2$, because $|e_a|^2 \;\propto\; \tau_{\max}^4$ as per (18). To clarify this point, consider an equispaced grid of thresholds with $\tau_{k+1} - \tau_k = \tau = \tau_{\max}, \forall k$, such that $\tau_{\max} = 2T/(K+1) < 2T/K$. Using the (loose) bound $q_k(1 - q_k) \leq 1/4$, the MSE is bounded by [c.f. (17) - (19)]

$$\mathrm{E}[(\theta - \hat{\theta})^2] < \frac{4T^6\dot{p}_{\max}^2}{9K^4} + \frac{T^2}{N}. \tag{20}$$

The bound in (20) is minimized by selecting $K = N$, which amounts to having *each sensor use a different region* to construct its binary observation. In this case, $|e_a|^2 \propto N^{-4}$ and its effect becomes practically negligible. Moreover, most pdfs have relatively small derivatives; e.g., for the Gaussian pdf we have $\dot{p}_{\max} = (2\pi e\sigma^4)^{-1/2}$. The integration error can be further reduced by resorting to a more powerful numerical integration method, although its difference with respect to the trapezoidal rule will not have any impact in practice.

Since $K = N$, the selection $\tau_{k+1} - \tau_k = \tau, \forall k$, yields

$$\hat{\theta} \; = \; \tau \sum_{n=0}^{N-1} b_1(n) - T \; = \; T\left[\frac{2}{N+1}\sum_{n=0}^{N-1} b_1(n) - 1\right], \tag{21}$$

that *does not require knowledge of the threshold* used to construct the binary observation at the fusion center of a WSN. This feature allows for each sensor to randomly select its threshold without using values pre-assigned by the fusion center; see also [5] for related random quantization algorithms.

Remark 1: While $e_a^2 \;\propto\; T^6$ seems to dominate $\mathrm{var}(\hat{\theta}) \;\propto\; T^2$ in (20), this is not true for the operational low-to-medium Q-SNR range for distributed estimators based on binary observations. This is because the support $2T$ over which $F_x(x)$ in (11) is non-zero depends on σ and the dynamic range $|\Theta_1 - \Theta_2|$ of the parameter θ. And as the Q-SNR decreases, $T \propto \sigma$. But since $\dot{p}_{\max} \propto \sigma^{-2}$, $e_a^2 \propto \sigma^2/N^4$ which is negligible when compared to the term $\mathrm{var}(\hat{\theta}) \propto \sigma^2/N$.

Remark 2: Pdf-unaware bandwidth-constrained distributed estimation was introduced in [5], where it was referred to as universal. At the (relatively minor) restriction of deterministically-assigned thresholds, the estimator in (21) achieves a four times smaller variance than the universal estimator in [5] which can afford randomly assigned thresholds – though it is true that $\hat{\theta}$ in (21) can also be implemented with randomly assigned thresholds, its MSE in (20) has been derived for deterministically assigned ones. The reason behind this noticeable performance improvement is that the approach here implicitly utilizes the data pdf (through the numerical approximation of the CCDF) in constructing the asymptotic MLE of (14). The only extra condition required over [5] is for the pdf to be differentiable, which is typically satisfied in practice. Also, the approach herein is readily generalizable to estimation of vector parameters – a practical scenario where universal estimators like those in [5] are yet to be found.

Apart from providing useful bounds on the finite-sample performance, eqs. (18), (19), and (20) establish asymptotic optimality of the $\hat{\theta}$ estimators in (14) and (21) as summarized in the following:

Corollary 1 *Under the assumptions of Propositions 1 and the conditions: i) $\tau_{\max} \propto K^{-1}$; and ii) $T^2/N, T^6/K^4 \to 0$ as $T, K, N \to \infty$, the estimators $\hat{\theta}$ in* (14) *and* (21) *are asymptotically (as $K, N \to \infty$) unbiased and consistent in the mean-square sense.*

The estimators in (14) and (21) are consistent even if the support of the data *pdf* is infinite, as long as we guarantee a proper rate of convergence relative to the number of sensors and thresholds..

[2]Omitted due to space considerations, proofs pertaining to claims in this work can be found in [9]

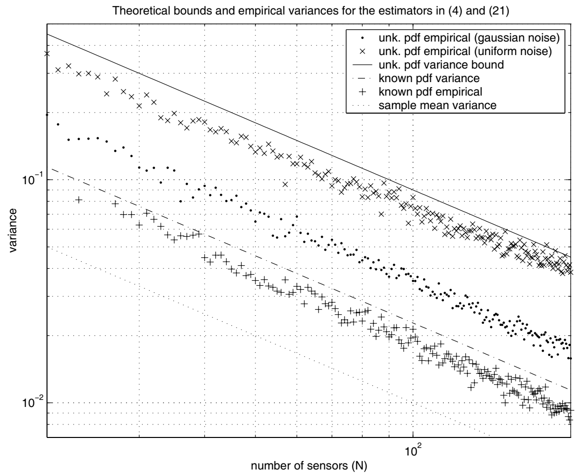

Fig. 2. The variance of the estimators in (4) and (21) are close to the sample mean estimator variance ($\sigma^2 := \mathrm{E}[w^2(n)] = 1, T = 3, \theta \in [-1, 1]$).

Remark 3: To compare the estimators in (4) and (21), consider that $\theta \in [\Theta_1, \Theta_2] = [-\sigma, \sigma]$, and that the noise is Gaussian with variance σ^2, yielding a Q-SNR $\gamma = 4$. No estimator can have variance smaller than $\mathrm{var}(\bar{x}) = \sigma^2/N$; however, for the (medium) $\gamma = 4$ Q-SNR value they can come close. For the known pdf estimator in (4), the variance is $\mathrm{var}(\hat{\theta}) \approx 2\sigma^2/N$. The unknown pdf estimator in (21) requires an assumption about the essentially non-zero support of the Gaussian pdf. If we suppose that the noise pdf is non-zero over $[-2\sigma, 2\sigma]$, the corresponding variance becomes $\mathrm{var}(\hat{\theta}) \approx 9\sigma^2/N$. The penalties due to the transmission of a single bit per sensor with respect to $\bar{x}$ are approximately 2 and 9. While the increasing penalty is expected as the uncertainty about the noise pdf increases, the relatively small loss is rather unexpected.

5. SIMULATIONS

Fig. 2 depicts theoretical bounds and simulated variances for the estimators (4) and (21) for an example Q-SNR $\gamma = 4$. The sample mean estimator variance, $\mathrm{var}(\bar{x}) = \sigma^2/N$, is also depicted for comparison purposes. The simulations corroborate the implications of Remark 3, reinforcing the idea that for low to medium Q-SNR problems quantization to a single bit per observation leads to minimal losses in variance performance. Note that for this particular example the unknown pdf variance bound, (20), overestimates the variance by a factor of roughly 1.2 for the uniform case and roughly 2.6 for the Gaussian case.

6. CONCLUSIONS

We were motivated by the need to effect energy savings in a wireless sensor network deployed to estimate parameters of interest in a decentralized fashion. To this end, we developed parameter estimators and derived their fundamental variance limits under bandwidth constraints. The latter were adhered to by quantizing each sensor's observation to one or a few bits. By jointly accounting for the unique quantization-estimation tradeoffs present, these bit(s) per sensor were first used to derive distributed maximum likelihood estimators (MLEs) for scalar mean-location parameters in the presence of generally non-Gaussian noise when the noise pdf is completely known. We latter derived estimators when the noise pdf is unknown through a non-parametric estimator of the unknown complementary cumulative distribution function based on quantized (binary) observations.

In both cases, the resulting estimators turned out to exhibit comparable variances that can come surprisingly close to the variance of the clairvoyant estimator which relies on unquantized observations. This happens when the SNR capturing both quantization and noise effects assumes low-to-moderate values[3].

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