Link-Adaptive Distributed Coding for Multi-Source Cooperation*

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***Abstract*— Combining multi-source cooperation and link-adaptive regenerative techniques, we develop a novel protocol capable of achieving diversity up to the number of cooperating users and larger coding gains without incurring the overhead of cyclic redundancy check (CRC) codes. The resulting protocol can be further optimized to take advantage of the information provided by CRC when available. Simulations confirm our theoretical assessments.**

I. INTRODUCTION

In distributed virtual antenna arrays (VAA) created by user cooperation, there is a distinction depending on how users relay information packets within the VAA. In a broad sense we can classify most relaying techniques as either analog forwarding (AF) or selective forwarding (SF) [1], [2]. In SF, prospective cooperators decode each source packet and, if correctly decoded, they cooperate by relaying a possibly re-encoded signal. In AF, cooperating terminals amplify the analog waveform they receive (transmitted signal plus noise). Both strategies achieve full diversity equal to the number of users forming the VAA, and in some sense their advantages and drawbacks are complementary. One of the major limitations of AF is that it requires storage of the analog-amplitude received waveform, implying that AF strains resources at relaying terminals, whereas SF implementation is definitely simpler. However, relaying decisions in SF are necessarily done on a per packet basis possibly leading to the dismissal of the entire packet because of a small number of erroneously decoded symbols. This drawback is sometimes obscured in analyses because it does not affect the diversity gain of the VAA. It does affect the coding gain, though, and in many situations SF does not improve performance of non-cooperative transmissions because the diversity advantage requires too high signal-to-noise ratios (SNR) to "kick-in" in practice.

Simple implementation with high diversity and coding gains are possible with link-adaptive regenerative (LAR) cooperation, whereby cooperators repeat packets based on the instantaneous SNR of the received signal [3]. In the LAR protocol of [3] relays re-transmit estimates of received symbols with power proportional to the instantaneous SNR in the source-to-relay link – available through, e.g., training – but never exceeding a given function of the average SNR in the relay-to-destination link – available through, e.g. low-rate feedback. With LAR-based cooperation, it suffices to perform maximum-ratio-combining (MRC) at the destination to achieve full diversity equal to the number of cooperators.

In the present paper we considerably broaden the scope of LAR cooperation to general distributed coding strategies, including as particular cases (LAR based) repetition coding, distributed complex field coding (DCFC) and distributed error control coding (DECC). For that matter, we consider a multi-source cooperation (MSC) setup relying on joint coding of multiple sources [4]. We show that assuming slow block fading Rayleigh channels with binary transmission, the maximum achievable diversity order d effected in LAR-MSC networks with N users is: i) $d = 2$ for repetition coding; ii) $d = N$ for DCFC and iii) $d = d_C$, where d_C is the diversity achieved by the ECC over a N point-to-point block-faded channels. As explained before, SF-MSC techniques require all cooperating terminals to correctly decode the transmitted information signal. Thus, while the diversity order of SF-MSC for different coding functions coincides with the ones summarized in i)-iii) for LAR-MSC, [5], the latter has a larger coding gain and can spare the overhead of CRC bits altogether.

In Section II we introduce the LAR-MSC protocol with a general coding function. We then move on to Section III where we present the main result of the paper characterizing the diversity gain in terms of the Hamming-distance properties of the distributed coding function. This general result is later specialized to repetition coding, DECC, and DCFC in Section IV. Section V presents corroborating simulations.

Notation: Upper (lower) bold face letters will be used for matrices (column vectors); $[\cdot]_k$ the kth entry of a vector; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{1}_N$ is the $N \times 1$ all-one vector; $\otimes$ denotes Kroneker product; $\|\cdot\|$ is the Frobenius norm; $\mathcal{R} \bigcup \mathcal{S}$ denotes the union of sets $\mathcal{R}$ and $\mathcal{S}$; $\mathcal{R} \bigcap \mathcal{S}$ denotes the intersection of sets $\mathcal{R}$ and $\mathcal{S}$; $|\mathcal{S}|$ is the cardinality of a set $\mathcal{S}$; $\emptyset$ is the empty set; and $\mathcal{CN}(\mu, \sigma^2)$ denotes the complex Gaussian distribution with mean μ and variance σ^2.

II. DISTRIBUTED COHERENT MODULATIONS

Consider a set of sources $\{S_n\}_{n=1}^N$ willing to communicate with a common access point or destination (D). Information bits of each source are modulated and carried over constellation symbols. Let $\mathbf{x}_n$ denote the $K \times 1$ block of symbols drawn from a constellation $\mathcal{A}_s$ to be transmitted by source S_n, $n \in [1, N]$. The protocol entails two phases. In Phase-1, $\{S_n\}_{n=1}^N$ transmit their symbols to D in separate time slots to avoid interference.

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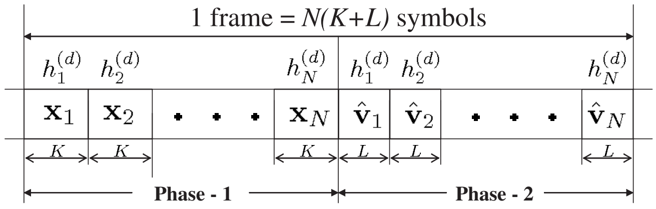

Fig. 1. Time-division multiplexing of N sources during Phase-1 and Phase-2

Thanks to the broadcast nature of wireless transmissions, symbols transmitted by S_n are received by the $N-1$ remaining sources $\{S_i\}_{i=1,i\neq n}^N$. After Phase-1, all sources have an estimate of other sources' blocks. Using these estimates, sources will *jointly* re-encode the received symbols and transmit again to the destination part of the re-encoded block in Phase-2.

We let $\mathbf{y}_n^{(d,1)}$ denote the $K \times 1$ block received at D when S_n transmits $\mathbf{x}_n$ in Phase-1. Likewise, we let $\mathbf{y}_{n,m}^{(s)}$ represent the $K \times 1$ block received at any S_m, $m \neq n$. Blocks $\mathbf{y}_n^{(d,1)}$ and $\mathbf{y}_{n,m}^{(s)}$ are given by

$$\mathbf{y}_n^{(d,1)} = h_n^{(d)}\mathbf{x}_n + \mathbf{w}_n^{(d,1)}, \quad n \in [1,N], \tag{1}$$

$$\mathbf{y}_{n,m}^{(s)} = h_{n,m}^{(s)}\mathbf{x}_n + \mathbf{w}_{n,m}^{(s)}, \quad n,m \in [1,N], n \neq m \tag{2}$$

where $h_n^{(d)} \sim \mathcal{CN}(0,(\sigma_n^{(d)})^2\bar{\gamma})$ and $h_{n,m}^{(s)} \sim \mathcal{CN}(0,(\sigma_{n,m}^{(s)})^2\bar{\gamma})$ are the fading coefficients corresponding to the $S_n - D$ and $S_n - S_m$ links; and $\mathbf{w}_n^{(d,1)}$, $\mathbf{w}_{n,m}^{(s)}$ are the noise terms, normalized to be $\mathcal{CN}(0,\mathbf{I}_K)$. For convenience, we define here the instantaneous output SNRs of each link $\gamma_n^{(d)} := |h_n^{(d)}|^2$ and $\gamma_{n,m}^{(s)} := |h_{n,m}^{(s)}|^2$; with expected values $\bar{\gamma}_n^{(d)} = (\sigma_n^{(d)})^2\bar{\gamma}$ and $\bar{\gamma}_{n,m}^{(s)} = (\sigma_{n,m}^{(s)})^2\bar{\gamma}$, respectively.

Let $\hat{\mathbf{x}}_{i,n}$ denote the estimate of the source block i formed at source n. Due to *unavoidable* communication errors, $\hat{\mathbf{x}}_{i,n}$ may differ from $\mathbf{x}_n$ or $\hat{\mathbf{x}}_{i,m}$ (estimated block at S_m), with $i \neq m$; also notice that $\hat{\mathbf{x}}_{n,n} = \mathbf{x}_n$. Source S_n, $n \in [1,N]$ collects all estimates in the super vector $\hat{\mathbf{x}}_n := [\hat{\mathbf{x}}_{1,n}^T, \ldots, \hat{\mathbf{x}}_{N,n}^T]^T$ of size $NK \times 1$ and constructs a new vector $\hat{\mathbf{v}}_n$ of size $L \times 1$, to be sent in Phase-2 based on the mapping

$$\hat{\mathbf{v}}_n = \psi_n(\hat{\mathbf{x}}_n). \tag{3}$$

Because no new information is conveyed during Phase-2, the bandwidth efficiency is $K/(K+L)$. Different from the MSC strategies in [6] and [4], we are encoding error-corrupted blocks $\hat{\mathbf{x}}_n$. The main challenge in this work is to show that through a suitable transmission protocol diversity is still enabled as in error-free MSC retransmissions. Toward this objective, we will resort to the LAR techniques in [3] to adjust the transmitted power of block $\hat{\mathbf{v}}_n$ at S_n according to the available channel knowledge. Specifically, if we define α_n to be the transmit power-weighting coefficient at S_n, we select

$$\alpha_n := \frac{\min\{\min_{i\neq n}\{\gamma_{i,n}^{(s)}\}, \bar{\gamma}_n^{(d)}\}}{\bar{\gamma}_n^{(d)}}. \tag{4}$$

Source S_n computes α_n using the instantaneous SNR of the links through which blocks $\mathbf{y}_{1,n}^{(d,1)}, \ldots, \mathbf{y}_{N,n}^{(d,1)}$ arrived (available e.g., by appending a training sequence) and the average SNR of its link to the destination, which is assumed to slowly fade at long scale, and thus is feasible to be fed back. These same conventions have also been adopted in the context of DF protocols in [3] and [7]. In [7], the average channel is assumed to be known for decoding at the destination, whereas in [3] average knowledge is assumed to be known at the relays; the latter has proved to be full-diversity achieving.

Considering (4), the received signal from S_n at D in Phase-2 is

$$\mathbf{y}_n^{(d,2)} = \sqrt{\alpha_n}h_n^{(d)}\hat{\mathbf{v}}_n + \mathbf{w}_n^{(d,2)}, \quad n \in [1,N], \tag{5}$$

where $\mathbf{y}_n^{(d,2)}$ has size $L \times 1$ and $h_n^{(d)}$ is the same fading coefficient as in the first phase, assumed to remain invariant for at least $N(K+L)$ symbols (coherence time). After Phases 1 and 2, the destination has $N(K+L)$ symbols, that we collect in two blocks $\mathbf{y}^{(d,1)} := [(\mathbf{y}_1^{(d,1)})^T, \ldots, (\mathbf{y}_N^{(d,1)})^T]^T$ and $\mathbf{y}^{(d,2)} := [(\mathbf{y}_1^{(d,2)})^T, \ldots, (\mathbf{y}_N^{(d,2)})^T]^T$, to form

$$\begin{bmatrix}\mathbf{y}^{(d,1)}\\ \mathbf{y}^{(d,2)}\end{bmatrix} = \begin{bmatrix}(\mathbf{D}_h^{(d)} \otimes \mathbf{I}_K) & \mathbf{0}_{NK\times NL}\\ \mathbf{0}_{NL\times NK} & ((\mathbf{D}_\alpha \mathbf{D}_h^{(d)}) \otimes \mathbf{I}_L)\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \hat{\mathbf{v}}\end{bmatrix} + \begin{bmatrix}\mathbf{w}_n^{(d,1)}\\ \mathbf{w}_n^{(d,2)}\end{bmatrix}, \tag{6}$$

where $\mathbf{x} := [\mathbf{x}_1^T, \ldots, \mathbf{x}_N^T]^T$, $\hat{\mathbf{v}} := [\hat{\mathbf{v}}_1^T, \ldots, \hat{\mathbf{v}}_N^T]^T$, have size $NK \times 1$ and $NL \times 1$, respectively; and the diagonal channel matrices $\mathbf{D}_h^{(d)} := \mathrm{diag}([h_1^{(d)}, \ldots, h_N^{(d)}])$ and $\mathbf{D}_\alpha := \mathrm{diag}([\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_N}])$ have sizes $NK \times NK$ and $NL \times NL$, respectively.

Assuming only knowledge of the $S_n - D$ links at D, we employ the following ML detection rule for decoding

$$\hat{\mathbf{x}}^{(d)} = \arg\min_{\mathbf{x}\in\mathcal{A}_s^{KL}} \left\{\|\mathbf{y}^{(d,1)} - \bar{\mathbf{D}}_h^{(d)}\mathbf{x}\|^2 + \|\mathbf{y}^{(d,2)} - \bar{\mathbf{D}}_\alpha\bar{\mathbf{D}}_h^{(d)}\mathbf{v}\|^2\right\}, \tag{7}$$

where $\bar{\mathbf{D}}_h^{(d)} := \mathbf{D}_h^{(d)} \otimes \mathbf{I}_K$ and $\bar{\mathbf{D}}_\alpha := \mathbf{D}_\alpha \otimes \mathbf{I}_K$. Upon recalling that $\mathbf{v}$ is constructed from $\mathbf{x}$ as $\mathbf{v} = [\mathbf{v}_1^T, \ldots, \mathbf{v}_N^T]^T = [(\psi_1(\mathbf{x}))^T, \ldots, (\psi_N(\mathbf{x}))^T]^T$, se deduce that the search in (7) is performed over the set of constellation codewords $\mathbf{x}$ of size $|\mathcal{A}_s|^{KN}$. We also notice that this is a general detector for performance-analysis purposes. Its complexity does not necessarily depend on the size of the codeword $\mathbf{x}$ and can be reduced depending on the specific coding function (3) that source S_n employs.

Looking closer into (6) and (7), we notice that this detector is actually looking for the Euclidean distance-minimizing vector $\mathbf{x}$ *assuming* the pair $\{\mathbf{x}, \mathbf{v}\}$ was sent. As we can infer from (6), errors at $\{S_n\}_{n=1}^N$ drive the pair $\{\mathbf{x}, \hat{\mathbf{v}}\}$ to be the *actual* block that was sent. Because $\hat{\mathbf{v}}$ is a concatenation of correct and erroneous estimates, it can happen that $\hat{\mathbf{v}}$ is confused to be a block built from a codeword different from $\mathbf{x}$, or it simply does not correspond to any codeword $\mathbf{x}$ in the constellation so that $\hat{\mathbf{v}} = [(\psi_1(\mathbf{x}))^T, \ldots, (\psi_N(\mathbf{x}))^T]^T$. Moreover, since D lacks knowledge of the probability of estimation error at $\{S_n\}_{n=1}^N$, the error performance of the detector (7) is undoubtedly degraded. The objective of the ensuing section is to show that the coefficients $\{\alpha_n\}_{n=1}^N$ chosen in (4) can improve this performance, and for a judicious re-encoding function ψ_n, it is surprisingly possible to collect diversity order up to the number of users N.

III. Pairwise error probability analysis

The conditional pairwise error probability (PEP) between any two distinct transmitted blocks $\mathbf{x}$, $\tilde{\mathbf{x}}$ is denoted as

$$\Pr(\mathbf{x} \to \tilde{\mathbf{x}} | \mathcal{H}^{(s)}, \mathcal{H}^{(d)}), \tag{8}$$

where $\mathcal{H}^{(s)} := \{h_{i,n}^{(s)}\}_{i,n=1,i\neq n}^N$ and $\mathcal{H}^{(d)} := \{h_n^{(d)}\}_{n=1}^N$ are the sets of all fading coefficients in our set-up.

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Let $\hat{\mathbf{x}} := [\hat{\mathbf{x}}_1^T, \ldots, \hat{\mathbf{x}}_n^T]^T$ denote the $N^2K \times 1$ vector that concatenates the $NK \times 1$ vectors containing all estimated blocks by all sources. We emphasize that if no errors occur at the relays, then $\hat{\mathbf{x}} = \mathbf{1}_N \otimes \mathbf{x}$. Further, let $\Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)})$ represent the conditional probability for decoding $\hat{\mathbf{x}}$ at $\{S_n\}_{n=1}^N$ and $\Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}})$ the conditional probability of decoding $\tilde{\mathbf{x}} \neq \mathbf{x}$ at the destination given that $\hat{\mathbf{v}} := [\hat{\mathbf{v}}_1^T, \ldots, \hat{\mathbf{v}}_n^T]^T$ was sent. The conditional PEP in (8) can be expanded as

$$\Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}) = \sum_{\forall\, \hat{\mathbf{x}}} \Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)}) \Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}}). \quad (9)$$

For single-input single-output (SISO) block-fading channels, the diversity order depends on the Hamming distance between constellation codewords [8]. To make use of this fact, we will cast our VAA set-up in a SISO form. Given $\mathbf{x}$ and $\tilde{\mathbf{x}}$ in (8) and the corresponding coded blocks $\mathbf{v} = [(\psi_1(\mathbf{x}))^T, \ldots, (\psi_N(\mathbf{x}))^T]^T$ and $\tilde{\mathbf{v}} = [(\psi_1(\tilde{\mathbf{x}}))^T, \ldots, (\psi_N(\tilde{\mathbf{x}}))^T]^T$, we define

$$\begin{aligned} \mathcal{X} &:= \{n|\mathbf{x}_n \neq \tilde{\mathbf{x}}_n\}, \\ \mathcal{V} &:= \{n|\mathbf{v}_n \neq \tilde{\mathbf{v}}_n\} \end{aligned} \quad (10)$$

as the sets of distinct pairwise sub-block elements between $\mathbf{x}$ and $\tilde{\mathbf{x}}$, $\mathbf{v}$ and $\tilde{\mathbf{v}}$, respectively. With reference to Fig. 1, if we assume $\hat{\mathbf{v}}_n = \mathbf{v}_n$, the equivalent system can be seen as a SISO block-faded transmission and the achievable diversity order is the minimum Hamming distance $|\mathcal{X} \bigcup \mathcal{V}|$ over all possible pairs. We are now challenged to establish similar diversity claims when $\hat{\mathbf{v}}_n \neq \mathbf{v}_n$. The pertinent result is summarized in the following theorem.

Theorem 1 *Consider two distinct blocks $\mathbf{x}$, $\tilde{\mathbf{x}}$ sent in Phase-1 along with the corresponding coded blocks $\mathbf{v}$, $\tilde{\mathbf{v}}$ sent in Phase-2 and the power-weighting coefficients $\{\alpha_n\}_{n=1}^N$ defined in* (4) *employed for transmissions in Phase-2. Given the sets $\mathcal{X}$ and $\mathcal{V}$ defined in* (10)*, the average PEP in* (8) *behaves as*

$$-\lim_{\bar{\gamma} \to \infty} \frac{\log \Pr(\mathbf{x} \to \tilde{\mathbf{x}})}{\log \bar{\gamma}} = |\mathcal{X} \bigcup \mathcal{V}|, \quad (11)$$

meaning that diversity order $d := \min_{\forall \tilde{\mathbf{x}} \neq \mathbf{x}} \{|\mathcal{X} \bigcup \mathcal{V}|\}$ is achieved.

Proof. Let

$$\mathcal{E} := \{n|\mathbf{x} \neq \hat{\mathbf{x}}_n\} \quad (12)$$

be the set of sources that failed to correctly estimate $\mathbf{x}$. Notice that because $\hat{\mathbf{v}}_n = \psi_n(\hat{\mathbf{x}}_n)$, the set $\mathcal{E}$ can be equivalently defined as $\mathcal{E} := \{n|\mathbf{v}_n \neq \hat{\mathbf{v}}_n\}$. Obviously, the complementary set $\bar{\mathcal{E}}$ contains all sources with correct estimates. Let also

$$\mathcal{C} := \left(\mathcal{X} \bigcup \mathcal{V}\right) \bigcap \bar{\mathcal{E}}, \quad (13)$$

be the set of distinct pairwise sub-blocks with correct estimates, and cardinality $|\mathcal{C}| \geq |\mathcal{X} \bigcup \mathcal{V}| - |\mathcal{E}|$. Notice that $\mathcal{C} \bigcap \mathcal{E} = \emptyset$. Based on these facts, the following lemma[1]constructs suitable bounds for the products on the right-hand-side of (9).

Lemma 1 *Let $\mathcal{E}$ and $\mathcal{C}$ respectively denote the sets of terminals that failed to correctly decode the Phase-1 packets and the set of distinct pairwise sub-blocks restricted to correct estimations in Phase 1, as defined in* (12) *and* (13)*, respectively. Let $\gamma_{i,n}^{(s)} = |h_{i,n}^{(s)}|^2$ and $\gamma_n^{(s)} = |h_n^{(d)}|^2$ be the instantaneous SNRs in the $S_i - S_n$ and $S_n - D$ links, respectively, and denote by α_n the power scaling constant in* (4)*. We then have that:*

[1a] *the probability of incorrect detection,* $\Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)})$ *in* (9) *can be bounded as*

$$\Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)}) \leq \kappa_1 \exp\left(-\kappa_2 \sum_{n \in \mathcal{E}} \min_{i \neq n}\{\gamma_{i,n}^{(s)}\}\right), \quad (14)$$

for some finite constants κ_1, κ_2; and

[1b] *the conditional error probability at the destination,* $\Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}})$ *in* (9) *can be bounded as*

$$\Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}}) \leq Q\left(\frac{\kappa_3 \sum_{n \in \mathcal{C}} \alpha_n \gamma_n^{(d)} - \kappa_4 \sum_{n \in \mathcal{E}} \alpha_n \gamma_n^{(d)}}{\sqrt{\kappa_3 \sum_{n \in \mathcal{C}} \alpha_n \gamma_n^{(d)} + \kappa_4 \sum_{n \in \mathcal{E}} \alpha_n \gamma_n^{(d)}}}\right), \quad (15)$$

for some finite constants κ_3, κ_4.

Before proceeding with the proof, we illustrate the roles of Lemmas 1a and 1b. If we bound $\Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}}) \leq 1$ in (9), Lemma 1a demonstrates that the average probability of $|\mathcal{E}|$ errors in (14) can be bounded by $k_1 \bar{\gamma}^{-|\mathcal{E}|}$, for some constant k_1. On the other hand, when $\mathcal{E} = \emptyset$, we can bound $\Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)}) \leq 1$, and Lemma 1b takes the form of the error-probability of a multi-antenna scenario and (ignoring for now α_n) it is not difficult to prove that in this case (15) is bounded by $k_2 \bar{\gamma}^{-|\mathcal{C}|} = k_2 \bar{\gamma}^{-|\mathcal{X} \bigcup \mathcal{V}|}$, for some constant k_2.

Unfortunately, the aforementioned bounds are only useful when either $\mathcal{E} = \emptyset$ or $|\mathcal{E}| \geq |\mathcal{X} \bigcup \mathcal{V}|$. When $0 < |\mathcal{E}| < |\mathcal{X} \bigcup \mathcal{V}|$, these bounds are too loose and we need to consider the joint expectation of

$$P_e(\underline{\gamma}) := \kappa_1 \exp\left(-\kappa_2 \sum_{n \in \mathcal{E}} \min_{i \neq n}\{\gamma_{i,n}^{(s)}\}\right) \times Q\left(\frac{\kappa_3 \sum_{n \in \mathcal{C}} \alpha_n \gamma_n^{(d)} - \kappa_4 \sum_{n \in \mathcal{E}} \alpha_n \gamma_n^{(d)}}{\sqrt{\kappa_3 \sum_{n \in \mathcal{C}} \alpha_n \gamma_n^{(d)} + \kappa_4 \sum_{n \in \mathcal{E}} \alpha_n \gamma_n^{(d)}}}\right) \quad (16)$$

with $P_e(\underline{\gamma}) \geq \Pr((\mathbf{1}_N \otimes \mathbf{x}) \to \hat{\mathbf{x}}|\mathcal{H}^{(s)}) \Pr(\mathbf{x} \to \tilde{\mathbf{x}}|\mathcal{H}^{(s)}, \mathcal{H}^{(d)}, \hat{\mathbf{v}})$ and $\underline{\gamma} := \{\{\gamma_{i,n}^{(s)}\}_{i,n=1, i \neq n}^N, \{\gamma_n^{(d)}\}_n^N\}$ denoting the set of all instantaneous SNRs.

Notice that the difficulty in taking expectation over $\underline{\gamma}$ lies in that α_n in (4) depends on $\min_{i \neq n}\{\gamma_{i,n}^{(s)}\}$ and is present in both factors. The next lemma helps us to address this challenge.

Lemma 2 *For some error probability $P_e'(\gamma_c, \gamma_e, \eta_c, \eta_e)$ satisfying*

$$P_e'(\gamma_c, \gamma_e, \eta_c, \eta_e) \leq \exp(-\gamma_e) Q\left[\frac{\gamma_c \eta_c - \gamma_e \eta_e}{\sqrt{\gamma_c \eta_c + \gamma_e \eta_e}}\right], \quad (17)$$

where $\gamma_c \sim Gamma(|\mathcal{C}|, 1/\bar{\gamma})$, $\gamma_e \sim Gamma(|\mathcal{E}|, 1/\bar{\gamma})$; γ_c, η_c, γ_e, and η_e are nonnegative and independent of each other, if the probability density functions $p(\eta_c)$ and $p(\eta_e)$ do not depend on $\bar{\gamma}$, the expectation over γ_c, γ_e, η_c and η_e is bounded as

$$E[P_e'] \leq k \bar{\gamma}^{-|\mathcal{C}| - |\mathcal{E}|}, \quad (18)$$

with $k := E[k(\eta_c, \eta_e)]$ a constant not dependent on $\bar{\gamma}$.

[1]Omitted due to space limitations, proofs for all the lemmas in this paper can be found in [9].

It is possible to derive bounds for $P_e(\underline{\gamma})$ in (16) to match the requirements of (17) [9]. In this process, the definition of α_n in (4) plays an instrumental role. Finally, because $|\mathcal{C}| \geq |\mathcal{X} \bigcup \mathcal{V}| - |\mathcal{E}|$, we have

$$P_e \leq k\bar{\gamma}^{-|\mathcal{C}|-|\mathcal{E}|} \leq k\bar{\gamma}^{-|\mathcal{X} \cup \mathcal{V}|+|\mathcal{E}|-|\mathcal{E}|} = k\bar{\gamma}^{-|\mathcal{X} \cup \mathcal{V}|}. \quad (19)$$

Because (19) holds $\forall\ \hat{\mathbf{v}}$ in (9), the proof is complete. ■

IV. Distributed coding strategies

Theorem 1 not only quantifies error performance bounds for our system, but also provides insight on how to design diversity-enabling re-encoding functions $\psi_n(\cdot)$. To illustrate this fact, we present next three coding paradigms and outline their corresponding diversity performance in view of Theorem 1.

A. Repetition Coding

Assume $K = 1$ (one symbol is only transmitted from each source per transmission phase). The block of estimated symbols at S_n reduces to $\hat{\mathbf{x}}_n = [\hat{x}_{1,n}, \ldots, \hat{x}_{N,n}]^T$ and $\psi_n(\cdot)$ codes a scalar $\hat{v}_n$ ($L = 1$) as

$$\hat{v}_n = \psi_n(\hat{\mathbf{x}}_n) = [\hat{\mathbf{x}}_n]_{\tilde{n}}, \quad (20)$$

where $\tilde{n} := \mathrm{mod}\,[n, N] + 1$. We can re-write the optimum receiver in (6) as:

$$[\hat{\mathbf{x}}^{(d)}]_n = \arg\min_{x\in\mathcal{A}_s} \left\{ |[\mathbf{y}^{(d,1)}]_n - h_n^{(d)} x|^2 + |[\mathbf{y}^{(d,2)}]_{\tilde{n}} - \sqrt{\alpha_{\tilde{n}}} h_{\tilde{n}}^{(d)} x|^2 \right\}. \quad (21)$$

Assume the worst-case event in which, $\mathbf{x}$ differs from $\tilde{\mathbf{x}}$ in one unique position, say the nth. In this case, we clearly have $\mathcal{X} = \{n\}$. If we permute symbols in one position, $\mathcal{V} = \{\tilde{n}\}$ with $\tilde{n} \neq n$, then the union of $\mathcal{X}$ and $\mathcal{V}$ has at least two elements and the proposed system achieves diversity 2.

Because information is forwarded without modification, this scheme can be interpreted as a relaying scenario similar to the ones proposed in [1], [2], [3]; thus, Theorem 1 demonstrates diversity for *classical* relaying scenarios based on repetition coding. In [3], the diversity order of such a scheme has also been shown to be 2. The diversity order of these protocols can be improved without decreasing bandwidth efficiency. As an example, consider the next two distributed coding strategies.

B. Distributed Complex Field Coding

Assume again $K = 1$ and use a space-time CFC matrix $\mathbf{\Theta}$ as in [10]. This matrix is used as the mapping function for a scalar $\hat{v}_n$ ($L = 1$) as

$$\hat{v}_n = \psi(\hat{\mathbf{x}}_n) = \boldsymbol{\theta}_n^T \hat{\mathbf{x}}_n, \quad (22)$$

where $\boldsymbol{\theta}_n^T$ is the nth row of matrix $\mathbf{\Theta}$ and guarantees that $\boldsymbol{\theta}_n^T \mathbf{x} \neq \boldsymbol{\theta}_n^T \tilde{\mathbf{x}}$ for any $\mathbf{x} \neq \tilde{\mathbf{x}}$. The receiver now becomes

$$\hat{\mathbf{x}}^{(d)} = \arg\min_{\mathbf{x}\in\mathcal{A}_s^N} \left\{ \|\mathbf{y}^{(d,1)} - \mathbf{D}_h^{(d)}\mathbf{x}\|^2 + \|\mathbf{y}^{(d,2)} - \mathbf{D}_\alpha \mathbf{D}_h^{(d)} \mathbf{\Theta}\mathbf{x}\|^2 \right\}. \quad (23)$$

Using the maximum distance separability (MDS) property of CFC schemes, for any $\mathbf{x} \neq \tilde{\mathbf{x}}$, the difference vector $\mathbf{v} - \tilde{\mathbf{v}} = \mathbf{\Theta}(\mathbf{x} - \tilde{\mathbf{x}})$ has all entries different from zero [10], meaning that $\mathcal{V} = \{1, \ldots, N\}$. According to Theorem 1, the latter implies that the system achieves diversity order N.

Since $L = 1$, CFC maintains the same bandwidth efficiency as repetition coding while increasing diversity because each symbol $\hat{v}_n$ carries information from all N sources.

C. Distributed Error Control Coding

Consider the general transmission scheme in (3) and suppose that $\hat{\mathbf{v}}_n$ now carries parity check bits of the block $\hat{\mathbf{x}}_n$. In this case, we are basically implementing the distributed channel coding strategies in [4] and [6]. As depicted in Fig. 1, the complete sequence sent to the receiver is $[\mathbf{x}_1, \ldots, \mathbf{x}_N, \hat{\mathbf{v}}_1, \ldots, \hat{\mathbf{v}}_N]$ with size $N(K+L)$. The first NK symbols sent during Phase-1 then correspond to the systematic symbols and the NL symbols sent during Phase-2 comprise the parity-check portion of a generic ECC scheme.

Theorem 1 establishes the diversity order as a function of the Hamming distance between codewords $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_N^T]^T$ and $\mathbf{v} = [\mathbf{v}_1^T, \ldots, \mathbf{v}_N^T]^T$. To illustrate that this result is applicable over coded transmissions, assume $\mathbf{x}^T$ and $\mathbf{v}^T$ are, respectively, the $NK \log_2 |\mathcal{A}_s|$ systematic bits and $NL \log_2 |\mathcal{A}_s|$ parity bits of a coded system over a (SISO) block-faded channel as drawn in Fig. 1. Suppose this system is designed to enable diversity order d_C. Then, for any two blocks $\mathbf{x}$ and $\tilde{\mathbf{x}}$, this system obeys $d_C = |\mathcal{X} \bigcup \mathcal{V}|$, [8]. Using Theorem 1, we can thus conclude that the proposed coded transmission enables diversity d_C in the distributed set-up.

Notice that in order to achieve diversity d_C, one has to judiciously design interleavers provided that systematic and parity bits are sent as shown in Fig. 1. For this purpose, we resort to those designed in [6] for error-free MSC transmissions.

We further remark that the diversity order of coded systems over block-fading channels is limited by the Singleton bound [8] as

$$d_C \leq \min\left\{ d_{\min}, 1 + \left\lfloor N\left(1 - \frac{K}{(K+P)\log_2|\mathcal{A}_s|}\right)\right\rfloor \right\} \quad (24)$$

where $d_{\min}$ is the minimum Hamming (free) distance of the ECC. Eq. (24) implies that the code rate and the constellation employed affect the maximum achievable diversity order.

V. Simulations

In this section we present numerical simulations to test error performance of the novel protocols. We employ binary phase-shift-keying (BPSK) and suppose that all inter-source and source-destination links have the same average output SNR $\bar{\gamma}_{m,n}^{(s)} = \bar{\gamma}_n^{(d)} = \bar{\gamma}$. The CFC matrix in (22) is chosen from [10].

A. Distributed coding strategies

We will compare the diversity order achieved by the encoding schemes in subsections IV-A, IV-B and IV-C for different number of cooperating sources N. As already mentioned, repetition coding can be seen as the well-known DF-relaying. This motivates us to also include comparisons with the coherent piecewise-linear (PL) detector of [7], which assumes that the average inter-source SNR is known at D.

Fig. 2 shows bit-error-rate (BER) as a function of the average SNR $\bar{\gamma}$ when employing distributed CFC, repetition coding and the PL detector in [7] for $N = 1, 2, 3$ cooperating sources. For CFC and repetition-based transmissions, we employ the detectors in (23) and (21). For reference, we also depict the BER when sources are not cooperating. We can verify that, as established in Theorem 1, the slope of the BER varies with N when employing CFC and remains fixed at 2 when employing repetition coding (two curves coincide for $N = 2$).

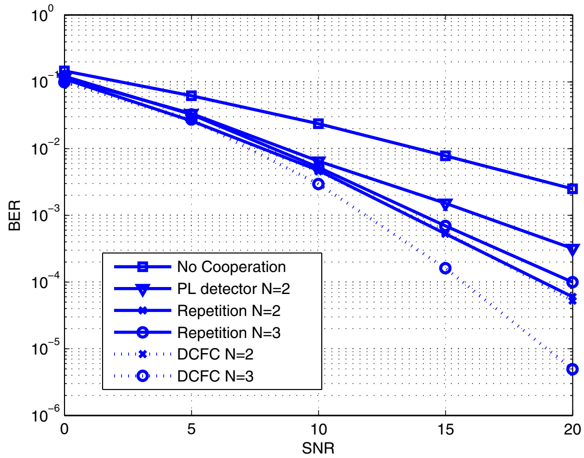

Fig. 2. BER of CFC vs. repetition and PL relaying strategies for $N = 2, 3$ sources.

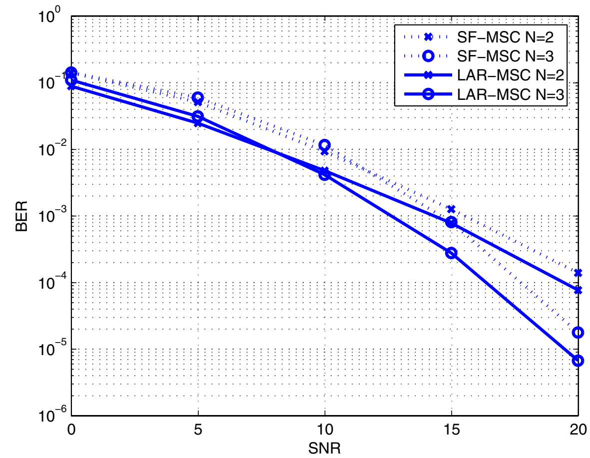

Fig. 4. CRC-aided retransmissions with packet length $K = 1024$ bits.

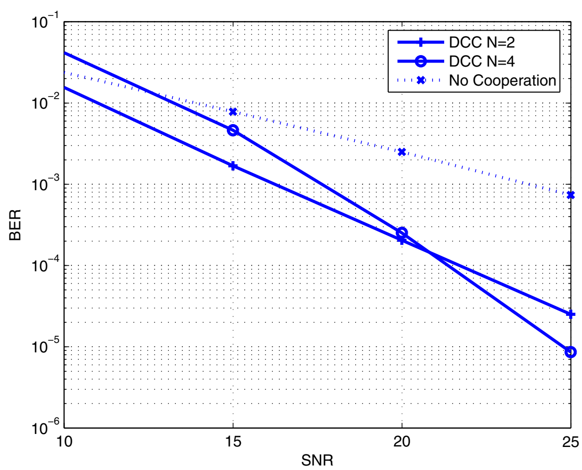

Fig. 3. BER of DCC for $N = 2, 4$ sources.

As a byproduct, we also outline the advantages of repetition-based link adaptation compared to [7]; whereas the former achieves diversity 2 in any scenario, the latter loses diversity when sources are sufficiently separated.

Fig. 3 illustrates BER performance when employing the distributed convolutional codes (DCC) of [6] for $N = 2$ and $N = 4$ users. We employ blocks of size $K = 52$ bits encoded through a rate 1/2 convolutional code ($K = L$) with generator in octal form with Hamming (free) distance $d_{\text{free}} = 5$ [6], [8]. According to (24), the achievable diversity orders are $d_C = 2$ for $N = 2$ and $d_C = 3$ for $N = 4$. Fig. 3 confirms that diversity orders are achieved as predicted by Theorem 1. From the same figure we can also observe that coding gain is reduced. This is due to the fact that highly-corrupted frames pass through the Viterbi decoder degrade its optimality.

B. *CRC-aided re-transmissions vs. adaptive techniques*

The advantages of MSC schemes hinge upon the assumption of either error-free links between sources or, as is the case in practice, on correct error-detection decoding per frame. In this practical case, frames with errors are discarded and no signal is re-transmitted. This strategy, however, can be inefficient at low SNR and/or when the CRC block size is large, because a single erroneous bit leads one to discard the entire block. To delineate this assessment, we set both strategies to use the same error-correction strategy. For the LAR-MSC, we set $\alpha_n = 1$ if no error is detected at user n; otherwise, the block is transmitted with α_n as in (4). This slight modification of our protocol, although not proven here, can be reasonably expected to achieve full diversity. On the other hand, and for the sake of a fair comparison, we increase the average power of SF to match that of adaptive transmissions.

Fig. 4 compares the BER of these strategies for a block size $K = 1024$ bits. As expected, both systems achieve full diversity. Moreover, link-adaptive transmissions exhibit larger coding gain, which corroborates the fact that discarding large packets renders SF strategies inefficient.

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