Robust Stochastic Routing and Scheduling for Wireless Ad-Hoc Networks

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Abstract—**We discuss the design of robust protocols that despite poor knowledge about network connectivity achieve consistent performance. Optimal routes and schedules are obtained to (i) maximize a social network utility subject to a variance constraint; and (ii) minimize a variance cost subject to a minimum yield. Corresponding optimization problems are formulated and shown to be convex under mild conditions usually satisfied in practice. Protocols are obtained relying on dual decomposition algorithms that compute the solution of these optimization problems in a distributed manner. The resulting protocols yield utilities that come close to the prescribed requirement even when channel estimates are rough.**

I. Introduction

In a wireless ad-hoc network the determination of routes and schedules is complicated by the difficulty of acquiring knowledge about network connectivity. Because of the rapidly changing topology, it is likely that user terminals have access only to information regarding their immediate neighborhood. Even information about neighbors, e.g., achievable rates in a one-hop radius, is difficult to acquire. In practice, decisions about packet's transmission have to be made based on rough estimates of network connectivity. In this context, the design of robust protocols that despite poor knowledge about network connectivity achieve consistent performance becomes of interest.

In a wireless ad-hoc network, packet scheduling and routing are fundamental problems. The scheduling problem answers the question of how should terminals divide their transmission time among the different information flows they are serving. The routing problem deals with finding a convenient next hop for the packets of the scheduled flow. The landmark work in [7], [8] offers a joint solution to these three problems through the "back-pressure" algorithm whereby routing-scheduling decisions are based on the difference between queue lengths of adjacent terminals. The back-pressure algorithm, however, requires perfect knowledge of the achievable rates between any pair of terminals.

In [5] and [6] we have advocated stochastic routing and scheduling protocols. The idea is to forward packets at random according to probabilities that are then chosen to optimize pertinent criteria. The context in these works is to cope with channel reliability issues in line with the contributions of e.g., [3] and [4]. Nonetheless, it is apparent that random packet forwarding can be leveraged to deal with uncertain knowledge of network connectivity. By dividing traffic between different routes, stochastic routing-scheduling protocols reduce the effect of channel estimation errors. While complete cancellation of these is unlikely, it is certainly possible to make end-to-end communication rates less sensitive to channel estimation errors.

The goal of this paper is to design robust routing-scheduling protocols. In particular, we will define two different kinds of optimal routes and schedules:

Maximum utility. A social rate utility is maximized subject to a constraint in the maximum allowable variance. These protocols are rate optimal while guaranteeing a bounded departure from the optimal yield.

Minimum variance. A variance cost is minimized subject to a minimum required rate utility. These protocols aim at providing a minimum quality of service while minimizing the uncertainty brought about by channel estimation errors.

Both of these problems will be formulated (Section II) and their tractability discussed (Section III). We will show that under mild constraints in the utility functions the resulting optimization problems are convex (Section III-A). Even though convexity ensures problem tractability, the communication cost associated with collecting reliability estimates at a central location followed by percolation of the optimal routing matrix may be prohibitive. This motivates the introduction of routing protocols based on local communications only that as time progresses converge to the optimal routing matrix (Section IV). We finally present corroborating simulations (Section V) and conclude the paper (Section VI).

II. Problem formulation

Consider a wireless ad-hoc network with J terminals $\{U_i\}_{i=1}^J$ collaborating to support a set of ongoing communications. Without loss of generality, suppose that the first K terminals $\{U_k\}_{k=1}^K$ are destinations of packets randomly generated at other terminals, with ρ_{ki} denoting the rate at which U_i generates packets whose intended destination is U_k. To exemplify notation consider a network with $J/2$ bidirectional communications between pairs of nodes $U_i, U_{k(i)}$. In this case we would have that: i) every node is a destination, i.e., $K = J$; ii) the arrival rate is null except for communicating pairs, i.e., $\rho_{ki} = 0$ when $k \neq k(i)$; and iii) arrival rates $\rho_{ki} \neq 0$ if and only if $\rho_{ik} \neq 0$. In general, some nodes may not be receiving packets in which case $K < J$; some U_k node may receive packets from more than one source implying that $\rho_{ki} \neq 0$ for more than one i; and some U_i node may not be sending packets resulting in $\rho_{ki} = 0$, $\forall\ k$. We assume that the random processes generating packets are stationary and define the vectors $\boldsymbol{\rho}_k := [\rho_{k1}, \ldots, \rho_{kJ}]^T$ of rates with destination U_k. We further convene $\rho_{kk} = 0$.

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Network connectivity is described by the pairwise rates R_{ij} that are defined as the rate at which U_i can transmit packets to U_j. We arrange these probabilities in the reliability matrix $\mathbf{R}$ with (i,j)-th entry R_{ij}. The matrix $\mathbf{R}$ is a function of the transmitted power and other parameters pertaining to the physical and medium access layers but is assumed given for our purposes. With a certain margin of error, elements R_{ij} of $\mathbf{R}$ can be measured by channel probing as will be discussed in Section II-B.

A. Stochastic routing-scheduling and queue stability

At each time slot, a terminal U_i that decides to transmit a packet is faced with a scheduling and a routing decision. Of the intended destinations $\{U_k\}_{k=1,k\neq i}^{K}$, node U_i has to decide which one it is going to serve, i.e., schedule, in the current slot. Given that it chooses to send a packet whose final destination is U_k, node U_i chooses a convenient next hop $\{U_j\}_{j=1,j\neq i}^{J}$, i.e, U_i routes the packet through U_j.

These are modelled jointly through T_{kij} that denotes the probability of U_i scheduling U_k, and routing the packet through U_j. Consequently, at any slot, say the n-th, U_i selects a final destination U_k *and* a next hop U_j with the pair (U_k, U_j) chosen with probability T_{kij}. Packets are then moved from U_i to U_j at a rate R_{ij}. Since we are assuming that in any slot U_i cannot serve more than one node we have that

$$\sum_{k=1}^{K}\sum_{i=1}^{J} T_{kij} \leq 1, \quad \forall\ j \neq k. \tag{1}$$

Note that the sum of probabilities is allowed to be less than one so that some probability can be assigned for not transmission at all. We also have $T_{kkj} = 0$, $\forall\ j$ and $T_{kii} = 0$, $\forall\ i$, respectively meaning that a destination U_k does not forward its own packets and that U_i does not route packets through itself. For future reference define the matrix $\mathbf{T}_k \in \mathbb{R}^{J\times J}$, with (i,j)-th element T_{kij}.

To characterize the evolution of packets through the network define a third matrix $\mathbf{K}_k$ with elements K_{kij} denoting the rate at which packets with final destination U_k move from U_i's to U_j's queue. This rate is the product of the rate T_{kij} at which the pair of destination and next hop (U_k, U_j) is chosen by U_i and the rate R_{ij} at which packets are communicated in such case. Therefore, for $i \neq j$ it holds

$$K_{kij} = T_{kij}R_{ij},\ i \neq j. \tag{2}$$

Given a set of rates $\{\boldsymbol{\rho}_k\}_{k=1}^{K}$ and a set of scheduling-routing matrices $\{\mathbf{T}_k\}_{k=1}^{K}$ the question arises of whether all queues in the network are stable. To ensure such stability it is sufficient to require that for all queues the aggregate arrival rates are smaller than the aggregate departure rate. Consider the queue at U_i for packets with final destination U_k. The aggregate departure rate of the queue is the sum of the rates at which packets are forwarded *to* other terminals, i.e., $\sum_{j=1,j\neq i}^{J} K_{kij}$. The aggregate arrival rate is the sum of the rates at which packets are forwarded *from* other terminals, i.e., $\sum_{j=1,j\neq i}^{J} K_{kji}$, plus the rate ρ_{ki} at which packets are generated at U_i. To ensure stable ques it thus must hold

$$\rho_{ki} + \sum_{j=1,j\neq i}^{J} K_{kji} = \sum_{j=1,j\neq i}^{J} K_{kij} \tag{3}$$

For a set of routing-scheduling matrices $\{\mathbf{T}_k\}_{k=1}^{K}$, the constraints in (1), (2) and (3) define a set of rates $\{\boldsymbol{\rho}_k\}_{k=1}^{K}$ that ensure stability of all queues. The $\{\boldsymbol{\rho}_k\}_{k=1}^{K}$ set is such that if any rate ρ_{kj} is increased, at least one queue in the network becomes unstable.

From (3) an expression for the rates $\{\boldsymbol{\rho}_k\}_{k=1}^{K}$ can be obtained. Ideally, this could be used as the basis to find routing matrices $\{\mathbf{T}_k\}_{k=1}^{K}$ satisfying some optimality criteria. Unfortunately, we do not have access to the rates R_{ij} but to estimates of it. This motivates the robust routing-scheduling problems that we formulate next.

B. Robust routes and schedules

To model the fact that the transmission rates R_{ij} are estimated, we consider that they are random with known mean and variance:

$$\begin{aligned} \hat{R}_{ij} &:= \mathrm{E}(R_{ij}) \\ \Sigma_{ij} &:= \mathrm{E}\left[\left(R_{ij} - \hat{R}_{ij}\right)^2\right] > 0 \end{aligned} \tag{4}$$

implying that the rate in (3) is also random. We further assume that reliability estimates are never perfect, i.e., $\Sigma_{ij} > 0$ whenever $\hat{R}_{ij} \neq 0$. Rate means are grouped in the matrix $\hat{\mathbf{R}}$ with elements $\hat{R}_{ij}$ and rate variances in the matrix $\boldsymbol{\Sigma}$ with elements Σ_{ij}.

The goal here is to design robust routing algorithms that are defined as follows:

(P1) Maximize a social utility function of the rates' expected value $\mathrm{E}(\rho_{ki})$ subject to a constraint in the maximum tolerable variance $\mathrm{var}(\rho_{ki})$

$$\begin{aligned} \{\mathbf{T}_k^*\}_{k=1}^{K} =& \arg\max f_0\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right] \\ & \text{s.t.}\ \ g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right] \leq g_{0m}\ \ m \in [1, M] \end{aligned} \tag{5}$$

where $f_0\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ denotes the mean social utility and $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ for $m \in [1, M]$ describe M prescribed tolerances on variance utilities. The constraints (1)-(3) as well as other ones describing mean and variances to be derived later on are implicit to the problem (6).

(P2) Minimize a social cost function of the variances $\mathrm{var}(\rho_{ki})$ subject to a minimum requirement on a function of the expected rate $\mathrm{E}(\rho_{ki})$:

$$\begin{aligned} \{\mathbf{T}_k^*\}_{k=1}^{K} =& \arg\min g_0\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right] \\ & \text{s.t.}\ \ f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right] \geq f_{0m}\ \ m \in [1, M] \end{aligned} \tag{6}$$

where $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ for $m \in [1, M]$ describe M pre-specified mean rate utility requirements and $g_0\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ the social variance cost.

The goal of this paper is to: (i) compute the means $\mathrm{E}(\rho_{ki})$ and variances $\mathrm{var}[\rho_{ki}]$ as functions of $\hat{\mathbf{R}}$, $\{\mathbf{T}_k\}_{k=1}^{K}$ and $\boldsymbol{\Sigma}$; (ii) establish cases in which (5) and (6) are convex optimization problems; and (iii) introduce a distributed implementation of (5)-(6).

III. Robust routing-scheduling optimization problems

To compute the mean and variance of $\boldsymbol{\rho}$ in terms of $\hat{\mathbf{R}}$ and $\boldsymbol{\Sigma}$, start by substituting (2) in (3) and reorder terms in the latter to obtain

$$\rho_{ki} = \sum_{j=1,j\neq i}^{J} R_{ij}T_{kij} - \sum_{j=1,j\neq i}^{J} R_{ji}T_{kji} \tag{7}$$

which shows that $\boldsymbol{\rho}$ is a linear function of the reliability matrix $\mathbf{R}$.

Since $\mathbf{R}$ is usually sparse many terms in the sum in (7) are null. To make this explicit, we define the set $c(i) := \{j : R_{ij} > 0;\ j \neq i,\ j \in [1, J]\}$, representing the indices of terminals $\{U_j\}_{j=1}^J$ that are able to receive packets from U_i. Likewise, define $r(i) := \{j : R_{ji} > 0;\ j \neq i,\ j \in [1, J]\}$ as the set of indices corresponding to terminals $\{U_j\}_{j=1}^J$ whose transmission can be received by U_i. We can thus rewrite (7) as

$$\rho_{ki} = \sum_{j \in c(i)} R_{ij} T_{kij} - \sum_{j \in r(i)} R_{ji} T_{kji} \tag{8}$$

To further simplify notation define $\boldsymbol{r}_i := \boldsymbol{R}_{ic(i)}$ and $\boldsymbol{s}_i := \boldsymbol{R}_{r(i)i}$ containing the non-zero elements of the i-th row and column of $\boldsymbol{R}$, respectively. In the same way define $\boldsymbol{t}_{ki} := \boldsymbol{T}_{k,i,c(i)}$ and $\mathbf{t}'_i := \boldsymbol{T}_{k,r(i),i}$ to write

$$\rho_{ki} = \mathbf{r}_i^T \mathbf{t}_{ki} - \mathbf{s}_i^T \mathbf{t}'_{ki}. \tag{9}$$

From (9) we can readily express $\mathrm{E}(\rho_{ki})$ in terms of the mean $\hat{\mathbf{R}} := \mathrm{E}(\mathbf{R})$ in (4). Noting that $\mathrm{E}(\mathbf{r}_i) = \mathrm{E}(\mathbf{R}_{i,c(i)}) = \hat{\mathbf{R}}_{i,c(i)}$ and $\mathrm{E}(\mathbf{s}_i) = \mathrm{E}(\mathbf{R}_{r(i),i}) = \hat{\mathbf{R}}_{r(i),i}$ we can take expected value in (9) to obtain

$$\mathrm{E}(\rho_{ki}) = \hat{\mathbf{R}}_{i,c(i)}^T \mathbf{t}_{ki} - \hat{\mathbf{R}}_{r(i),i}^T \mathbf{t}'_{ki} := \hat{\mathbf{r}}_i^T \mathbf{t}_{ki} - \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki} \tag{10}$$

where we defined $\hat{\mathbf{r}}_i := \hat{\mathbf{R}}_{i,c(i)}$ and $\hat{\mathbf{s}}_i := \hat{\mathbf{R}}_{r(i),i}$.

The rate variance $\mathrm{var}(\rho_{ki})$ can be analogously expressed in terms of the Σ_{ij} in (4). Indeed, using the variance definition $\mathrm{var}(\rho_{ki}) := \mathrm{E}\left[(\rho_{ki} - \mathrm{E}(\rho_{ki}))^2\right]$ and the fact that link rate estimates R_{ij} are assumed independent it follows readily that

$$\mathrm{var}[\rho_{ki}] = \sum_{j \in c(i)} T_{kij}^2 \Sigma_{ij} + \sum_{j \in r(j)} T_{kji}^2 \Sigma_{ji} \tag{11}$$

To simplify notation define the vectors $\mathbf{a}_i := \boldsymbol{\Sigma}_{i,c(i)}$ and $\mathbf{b}_i := \boldsymbol{\Sigma}_{r(i),i}$. Define then the diagonal matrices $\mathbf{A}_i := \mathrm{diag}(\mathbf{a}_i)$ and $\mathbf{B}_i := \mathrm{diag}(\mathbf{b}_i)$ so that (12) can be rewritten as

$$\mathrm{var}[\rho_{ki}] = \mathbf{t}_{ki}^T \mathbf{A}_i \mathbf{t}_{ki} + \mathbf{t'}_{ki}^T \mathbf{B}_i \mathbf{t}'_{ki} \tag{12}$$

A. Convexity of robust routing problems

Substituting (10) and (12) into (5) and (6), we obtain an optimization problem that can, in principle, be solved to obtain the optimal matrices $\{\mathbf{T}_k^*\}_{k=1}^K$. Solving these optimization problems might, or might not be tractable. Under proper conditions however, we can guarantee that (5) and (6) are convex, as we assert in the following proposition.

Proposition 1 *Consider the optimal robust routing problems in* (5) *and* (6) *and assume that (h1) the functions* $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ *are concave for* $m \in [0, M]$*; and (h2) the functions* $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ *are convex and nondecreasing in each argument for* $m \in [0, M]$*. Then, the optimization problems in* (5) *and* (6) *are convex.*

Proof: Since matrices $\mathbf{T}_k$ are constrained by a set of linear inequalities [cf. (1)], to prove that the problem in (5) is convex, it suffices to prove that: i) $g_0\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ is a convex function of the routing matrices $\{\mathbf{T}_k\}_{k=1}^K$; and ii) $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ for $m \in [1, M]$ is a concave function of $\{\mathbf{T}_k\}_{k=1}^K$. Correspondingly, (6) will be convex as long as: iii) $f_0\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ is a concave function of $\{\mathbf{T}_k\}_{k=1}^K$; and iv) $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ for $i \in [1, M]$ is a convex function of $\{\mathbf{T}_k\}_{k=1}^K$. Thus, the claim follows if (c1) $f_i\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ is a concave function of $\{\mathbf{T}_k\}_{k=1}^K$; and (c2) $g_i\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ is a convex function of $\{\mathbf{T}_k\}_{k=1}^K$ for $m \in [0, M]$.

The latter follows from the composition rules of convex analysis [2, Sec.3.2.4]. Indeed, $\mathrm{E}[\rho_{ki}]$ is a linear function of $\mathbf{t}_{ki}$ and $\mathbf{t}'_{ki}$. Composition of the concave function $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ [cf. (h1)] with the linear functions $\mathrm{E}[\rho_{ki}]$ [cf. (10)], yields a concave function implying (c1). To prove (c2) recall that $\mathrm{var}(\rho_{ki})$ is a positive definite quadratic form with variables $\mathbf{t}_{ki}$ and $\mathbf{t}'_{ki}$, and thus convex (indeed, strictly convex). The composition of the convex and nondecreasing in each argument function $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$ [cf. (h2)] with the convex function $\mathrm{var}(\rho_{ki})$ [cf. (12)] is convex establishing (c2). ■

Under the (mild) restrictions (h1) and (h2) on the utility functions $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ and cost functions $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$, Proposition (1) ensures tractability of (5) and (6). Consequently, interior point methods can be used to solve these problems with affordable complexity in the order $O(J^{3.5})$.

The conditions (h1) and (h2) are satisfied in many practical cases. Some examples are given next.

Maximum rate utility with bounded variance. A typical example of a problem of the form in (P1) is to consider the maximization of a weighted sum of rates $\sum_{k,i} w_{ki}\mathrm{E}(\rho_{ki})$. The variance of the individual rates is further upper bounded by a certain tolerance v_{0j} yielding the problem

$$\begin{aligned} \max \quad & \sum_{k,i} w_{ki}\left(\hat{\mathbf{r}}_i^T \mathbf{t}_{ki} - \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}\right) \\ \text{s.t.} \quad & \mathrm{var}(\rho_{ki}) \leq v_{0ki}, \quad \mathbf{t}_{ki} \geq 0,\ \sum_k \mathbf{t}_{ki}^T \mathbf{1} \leq 1 \end{aligned} \tag{13}$$

The functions $f_0\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right] := \sum_{k,i} w_{ki}\mathrm{E}(\rho_{ki})$ and $g_{ki}\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right] = \mathrm{var}(\rho_{ki})$ satisfy the hypotheses (h1) and (h2) of Proposition 1 proving that the problem in (13) is convex. This can be verified by noting that the argument to be optimized is a linear function of the $\mathbf{t}_{ki}$ and $\mathbf{t}'_{ki}$, and that the constraint $\mathrm{var}(\rho_{ki}) \leq v_{0ki}$ is a positive definite quadratic form on $\mathbf{t}_{ki}$ and $\mathbf{t}'_{ki}$.

Different rate utilities can be used in the argument of (13). E.g., the minimum rate utility $\min_{k,i}[\mathrm{E}(\rho_{ki}] = \min_{k,i}\left(\hat{\mathbf{r}}_i^T \mathbf{t}_{ki} - \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}\right)$ is considered a fairer alternative since it maximizes the rate of the least favored terminal. The sum of logarithms utility $\sum_{k,i} \log[\mathrm{E}(\rho_{ki})] = \sum_{k,i} \log\left(\hat{\mathbf{r}}_i^T \mathbf{t}_{ki} - \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}\right)$, is regarded as an intermediate point between weighted sum and minimum rate.

Minimum variance with rate guarantees. Alternatively, we may aim to comply with a minimum rate requirement ρ_{0ki} for each source-destination pair U_i, U_k, while minimizing, e.g., the sum of variances. The problem in this case is

$$\begin{aligned} \min \quad & \sum_{i,k} \mathbf{t}_{ki}^T \mathbf{A}_i \mathbf{t}_{ki} + \mathbf{t'}_{ki}^T \mathbf{B}_i \mathbf{t}'_{ki} \\ \text{s.t.} \quad & \hat{\mathbf{r}}_i^T \mathbf{t}_{ki} - \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki} \geq \rho_{0ki}, \quad \mathbf{t}_{ki} \geq 0,\ \sum_k \mathbf{t}_{ki}^T \mathbf{1} \leq 1 \end{aligned} \tag{14}$$

It is easy to verify that the functions in (14) satisfy (h1) and (h2) the convexity of the problem then follows from Proposition 1. If the $\hat{\mathbf{R}}$ estimate were perfect, (14) would guarantee rates ρ_{0ki}. In the presence of estimation uncertainty, (14) attempts the same while, in some sense, maximizing the likelihood of this actually happening.

We have shown that finding the optimal solution to (5)-(6) incurs affordable computational complexity. However, it requires all reliability estimates $\hat{\mathbf{R}}$ and variances $\mathbf{\Sigma}$, to be available at a central location, so that the optimization problem can be solved and the optimal routing matrices $\{\mathbf{T}_k^*\}_{k=1}^K$ distributed to the individual nodes. The drawbacks of this centralized approach are: i) a large communication cost to collect $\hat{\mathbf{R}}$ and $\mathbf{\Sigma}$ and to distribute $\{\mathbf{T}_k^*\}_{k=1}^K$; ii) considerable delay to compute $\{\mathbf{T}_k^*\}_{k=1}^K$; and iii) mismatch with the lack of infrastructure typical of ad-hoc networks. These motivates distributed algorithms that we pursue next.

IV. Robust routing protocols

To develop a robust routing protocol, we introduce iterative algorithms to solve (P1) and (P2) in a distributed fashion. In such distributed algorithm U_i is interested in computing a sequence of iterates $\mathbf{t}_{ki}(n)$ such that as $n \to \infty$ we have $\lim_{n\to\infty} \mathbf{t}_{ki}(n) = \mathbf{t}_{ki}^*$, with $\{\mathbf{t}_{ki}^*\}_{k,i}$ denoting the solution of a problem of the form (P1) or (P2) for given functions $f_m\left[\left\{\mathrm{E}(\rho_{ki})\right\}_{k,i}\right]$ and $g_m\left[\left\{\mathrm{var}(\rho_{ki})\right\}_{k,i}\right]$.

Throughout this section we assume that

[A1] Terminal U_i can communicate with U_j if and only if U_j can communicate with U_i, i.e., $R_{ij} \neq 0$ if and only if $R_{ji} \neq 0$.

A distributed algorithm can be developed using dual decomposition techniques. For simplicity of exposition we concentrate on the problem in (14). A distributed optimization algorithm separates the original problem in J sub-problems involving variables $\mathbf{t}_j$ only. Even though the objective of the optimization problem in (14) is a summation of quadratic functions that depend on the local variables $\mathbf{t}_{ki}$ and the constraints $\mathbf{t}_{ki} \geq 0$ and $\sum_k \mathbf{t}_{ki}^T\mathbf{1} \leq 1$ depend on $\mathbf{t}_{ki}$ only, such separation cannot be achieved for the problem in (14). Indeed, the constraints $\hat{\mathbf{r}}_i^T\mathbf{t}_{ki} - \hat{\mathbf{s}}_i^t\mathbf{t}'_{ki} \geq \rho_{0ki}$ involve the local variables $\mathbf{t}_{ki}$, and the variables $\mathbf{t}'_{ki}$ that contains transmission probabilities corresponding to U_i's neighboring terminals.

This hurdle can be overcome by resorting to the dual problem. Define vectors of Lagrange multipliers $\boldsymbol{\lambda}_k := [\lambda_{k1}, \ldots, \lambda_{kJ}]^T$ with the multiplier λ_{ki} associated with the constraint $\hat{\mathbf{r}}_i^T\mathbf{t}_{ki} - \hat{\mathbf{s}}_i^t\mathbf{t}'_{ki} \geq \rho_{0ki}$ and write the Lagrangian:

$$\mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right) = \sum_{i,k} \mathbf{t}_{ki}^T\mathbf{A}_i\mathbf{t}_{ki} + {\mathbf{t}'}_{ki}^T\mathbf{B}_i\mathbf{t}'_{ki} + \lambda_{ki}\left(\rho_{0ki} - \hat{\mathbf{r}}_i^T\mathbf{t}_{ki} + \hat{\mathbf{s}}_i^T\mathbf{t}'_{ki}\right) \tag{15}$$

that is defined over the set $\{\mathbf{t}_{ki} : \mathbf{t}_{ki} \geq 0,\ \sum_k \mathbf{t}_{ki}^T\mathbf{1} \leq 1\}$.

The dual function is then defined as

$$q\left(\{\boldsymbol{\lambda}_k\}_k\right) = \min_{\left\{\mathbf{t}_{ki}:\mathbf{t}_{ki}\geq 0,\ \sum_k \mathbf{t}_{ki}^T\mathbf{1}\leq 1\right\}} \mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right). \tag{16}$$

Since the problem in (14) is convex strong duality holds and the optimal value of (14) can be found as $\max_{\{\boldsymbol{\lambda}_k \geq \mathbf{0}\}} q\left(\{\boldsymbol{\lambda}_k\}_k\right)$.

An interesting observation is that the Lagrangian can be separated in J "local" Lagrangian containing $\mathbf{t}_{ki}$ variables only. This property also allows a separable computation of the gradient of the dual function. These two properties are introduced in the following proposition.

Proposition 2 *Define the vectors $\boldsymbol{\lambda}'_{ki} := \boldsymbol{\lambda}_{k,c(i)}$ containing the multipliers of U_i's neighbors associated with final destination U_k and the diagonal matrix $\boldsymbol{\Lambda}_{ki} := \lambda_{ki}\mathbf{I} - \mathrm{diag}(\boldsymbol{\lambda}'_{ki})$. Define then the local Lagrangian:*

$$\mathcal{L}_i\left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right) = \sum_k 2\mathbf{t}_{ki}^T\mathbf{A}_i\mathbf{t}_{ki} - \hat{\mathbf{r}}_j^T\boldsymbol{\Lambda}_{ki}\mathbf{t}_{ki} + \lambda_{ki}\rho_{0ki}. \tag{17}$$

We then have that

(i) The (global) Lagrangian $\mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right)$ in (16) is the sum of the local Lagrangians $\mathcal{L}_i\left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right)$ in (17), i.e.,

$$\mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right) = \sum_i \mathcal{L}_i\left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right). \tag{18}$$

(ii) Let

$$\mathbf{t}_{ki}(\{\boldsymbol{\lambda}_k\}_k) := \arg \min_{\left\{\mathbf{t}_{ki}\geq 0,\ \sum_k \mathbf{t}_{ki}^T\mathbf{1}\leq 1\right\}_{k,i}} \mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right) \tag{19}$$

be the arguments minimizing the Lagrangian in (16) for given multipliers $\{\boldsymbol{\lambda}_k\}_k$. Then $\mathbf{t}_{ki}(\{\boldsymbol{\lambda}_k\}_k)$ is the argument minimizing the local Lagrangian, i.e.,

$$\mathbf{t}_{ki}\left(\{\boldsymbol{\lambda}_k\}_k\right) = \arg \min_{\left\{\mathbf{t}_{ki}\geq 0,\ \sum_k \mathbf{t}_{ki}^T\mathbf{1}\leq 1\right\}_k} \mathcal{L}_i\left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right). \tag{20}$$

(iii) The derivative of the dual function with respect to λ_{ki} is given by

$$\frac{\partial q\left(\{\boldsymbol{\lambda}_k\}_k\right)}{\partial \lambda_{ki}} = \rho_{0ki} - \hat{\mathbf{r}}_i^t\mathbf{t}_{ki}(\{\boldsymbol{\lambda}_k\}_k) + \hat{\mathbf{s}}_i^T\mathbf{t}'_{ki}(\{\boldsymbol{\lambda}_k\}_k) \tag{21}$$

with $\mathbf{t}_{ki}(\{\boldsymbol{\lambda}_k\}_k)$ and $\mathbf{t}'_{ki}(\{\boldsymbol{\lambda}_k\}_k)$ the solutions of (20).

Proof: Consider the sum of the local Lagrangians in (17) and use the definition of $\boldsymbol{\Lambda}_{ki} := \lambda_{ki}\mathbf{I} - \mathrm{diag}(\boldsymbol{\lambda}'_{ki})$ to write.

$$\sum_i \mathcal{L}_i\left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right) = \sum_{k,i} 2\mathbf{t}_{ki}^T\boldsymbol{A}_i\mathbf{t}_{ki} + \lambda_{ki}\rho_{0ki} - \lambda_{ki}\hat{\mathbf{r}}_i^T\mathbf{t}_{ki} + \hat{\mathbf{r}}_{ki}^T\mathrm{diag}(\boldsymbol{\lambda}'_{ki})\mathbf{t}_{ki} \tag{22}$$

The sum of the first terms in (22) is equivalent to

$$\begin{aligned}\sum_{k,i} 2\mathbf{t}_{ki}^T\boldsymbol{A}_i\mathbf{t}_{ki} &= \sum_{k,i}\sum_{j\in c(i)} 2\Sigma_{ij}T_{kij}^2 \\ &= \sum_{k,i}\sum_{j\in c(i)} \Sigma_{ij}T_{kij}^2 + \sum_{k,i}\sum_{j\in c(i)} \Sigma_{ji}T_{kji}^2 \\ &= \sum_{k,i} \mathbf{t}_{ki}^T\boldsymbol{A}_i\mathbf{t}_{ki} + {\mathbf{t}'}_{ki}^T\boldsymbol{B}_i\mathbf{t}'_{ki}\end{aligned} \tag{23}$$

where in the first equality we used the definition of $\mathbf{A}_i$; in the second equality we separated the sums and used the assumption $c(i) = r(i)$; and in the third equality we used the definitions of $\mathbf{A}_i$ and $\mathbf{B}_i$.

The sum of the last terms in (22) can be written as

$$\sum_{k,i} \hat{\mathbf{r}}_i^T \mathrm{diag}(\boldsymbol{\lambda}'_{ki})\mathbf{t}_{ki} = \sum_{k,i} \sum_{j\in c(i)} \lambda_{kj} \hat{R}_{ij} T_{kij} = \sum_{k,j} \lambda_{kj} \sum_{i\in c(j)} \hat{R}_{ij} T_{kij} = \sum_{k,j} \lambda_{kj} \hat{\mathbf{s}}_j^T \mathbf{t}'_{kj} \tag{24}$$

where in the first equality we use the definitions of $\hat{\mathbf{r}}_i^t$ and $\mathbf{t}_{ki}$; in the second equality we interchange the order of summations and use the assumption $c(i) = r(i)$; and in the last equality we use the definitions of $\hat{\mathbf{s}}_j^t$ and $\mathbf{t}'_j$.

Substituting (23) and (24) in (22) we obtain

$$\sum_i \mathcal{L}_i \left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right) = \sum_{k,i} \mathbf{t}_{ki}^T \boldsymbol{A}_i \mathbf{t}_{ki} + \mathbf{t'}_{ki}^T \boldsymbol{B}_i \mathbf{t}'_{ki} + \lambda_{ki}\rho_{0ki} - \lambda_{ki}\hat{\mathbf{r}}_i^T \mathbf{t}_{ki} + \lambda_{kI}\hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}. \tag{25}$$

Since the right hand sides of (25) and (16) coincide, (18) follows.

To prove (20) use (18) to write the optimal argument $\mathbf{t}_{ki}\left(\{\boldsymbol{\lambda}_k\}_k\right)$ as

$$\mathbf{t}_{ki}\left(\{\boldsymbol{\lambda}_k\}_k\right) = \arg \min_{\left\{\mathbf{t}_{ki}\geq 0,\ \sum_k \mathbf{t}_{ki}^T \mathbf{1}\leq 1\right\}_{k,i}} \sum_i \mathcal{L}_i \left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right). \tag{26}$$

The only term in the summation in (26) that contains $\mathbf{t}_{ki}$ is $\mathcal{L}_i \left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right)$ from where (20) follows.

To obtain the result in (21) note that since $\mathbf{A}_i$ is positive definite ($\mathbf{A}_i$ is a diagonal matrix with strictly positive elements) it is invertible. Thus, for any set $\{\boldsymbol{\lambda}_k\}_k$ there exists unique minimizers $\mathbf{t}_{ki}\left(\{\boldsymbol{\lambda}_k\}_k\right)$ of $\mathcal{L}\left(\{\mathbf{t}_{ki}\}_{k,i}, \{\boldsymbol{\lambda}_k\}_k\right)$. The result then follows from Danskin's Theorem, [1, pp. 737] stating that: i) if there is a unique Lagrangian minimizer – i.e., a unique set $\{\mathbf{t}_{ki}\left(\{\boldsymbol{\lambda}_k\}_k\right)\}_{k,i}$ solving (16) – the dual function is differentiable; and ii) the derivative of the dual function with respect to λ_{ki} is given by the constraint violations $\rho_{0ki} - \hat{\mathbf{r}}_i^t \mathbf{t}_{ki}(\{\boldsymbol{\lambda}_k\}_k) + \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}(\{\boldsymbol{\lambda}_k\}_k)$. ■

Using the separability properties described by Proposition 2 we can propose a distributed routing protocol. Indeed, since a gradient of the dual function can be computed using local and neighboring iterates, we can define a routing protocol by using a gradient ascent algorithm on the dual function. The routing protocol and the corresponding optimality claim is presented in the next proposition

Proposition 3 *Consider a routing protocol in which terminal U_j updates local primal and dual iterates denoted by $\mathbf{t}_{ki}(n)$ and $\lambda_{ki}(n)$ respectively. The iterates are updated according to the following rules:*

[P1] *Receive dual iterates $\boldsymbol{\lambda}'_{ki}(n)$ from neighboring terminals $\{U_j\}_{j\in c(i)}$.*

[P2] *Update the local primal iterates $\mathbf{t}_{ki}(n)$ using*

$$\mathbf{t}_{ki}(n) = \left[\boldsymbol{A}_i^{-1}\left(\delta_i(n)\mathbf{1} + \boldsymbol{\Lambda}_{ki}\right)\right]^+. \tag{27}$$

where $[\cdot]^+$ denotes projection to the nonnegative orthant and $\delta_i(n) \geq 0$ is chosen so that $\sum_k \boldsymbol{t}_{ki}^T(n)\mathbf{1} = 1$. If for making $\sum_k \boldsymbol{t}_{ki}^T(n)\mathbf{1} = 1$ it is required that $\delta_i(n) < 0$ we set $\delta_i(n) = 0$.

[P3] *Transmit the primal iterates $\mathbf{t}_{ki}(n)$ to neighboring terminals $\{U_j\}_{j\in c(i)}$.*

[P4] *Receive primal iterates $\mathbf{t}'_i(n)$ from neighboring terminals $\{U_j\}_{j\in c(i)}$.*

[P5] *Update the local dual iterates using*

$$\lambda_{ki}(n{+}1) = \left[\lambda_{ki}(n) + c\left(\rho_{0ki} - \hat{\mathbf{r}}_i^t \mathbf{t}_{ki}(n) + \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}(n)\right)\right]^+. \tag{28}$$

with $c > 0$ a properly selected step size.

[P6] *Transmit the dual iterates $\lambda_{ki}(n+1)$ to neighboring terminals $\{U_j\}_{j\in c(i)}$.*

For sufficiently small step size c, as $n \to \infty$, the local iterates $\mathbf{t}_{ki}(n)$ converge to the optimal robust routes $\mathbf{t}_{ki}^*$ solving the optimization problem in (14)*, i.e.,*

$$\lim_{n\to\infty} \mathbf{t}_{ki}(n) = \mathbf{t}_{ki}^* \tag{29}$$

Proof: Start by noting that (27) is the solution of the local Lagrangian optimization in (17), i.e., $\mathbf{t}_{ki}(n)$ in (27) is such that

$$\mathbf{t}_{ki}(n) = \arg \min_{\left\{\mathbf{t}_{ki}\geq 0,\ \sum_k \mathbf{t}_{ki}^T \mathbf{1}\leq 1\right\}_k} \mathcal{L}_i \left(\{\mathbf{t}_{ki}, \lambda_{ki}, \boldsymbol{\lambda}'_{ki}\}_k\right). \tag{30}$$

Thus, according to (21) we have

$$\frac{\partial q\left(\{\boldsymbol{\lambda}_k\}_k\right)}{\partial \lambda_{ki}} = \rho_{0ki} - \hat{\mathbf{r}}_i^t \mathbf{t}_{ki}(n) + \hat{\mathbf{s}}_i^T \mathbf{t}'_{ki}(n). \tag{31}$$

Consequently, the iteration in (28) is tantamount to gradient ascent for optimizing $q\left(\{\boldsymbol{\lambda}_k\}_k\right)$, implying that for sufficiently small c

$$\lim_{n\to\infty} \lambda_{ki}(n) = \lambda_{ki}^* \tag{32}$$

with $\{\boldsymbol{\lambda}_k^*\}_k = \arg\max_{\{\boldsymbol{\lambda}_k \geq \mathbf{0}\}_k} q\left(\{\boldsymbol{\lambda}_k\}_k\right)$. But since the dual function is differentiable, convergence of $\lambda_{ki}(n)$ implies convergence of $\mathbf{t}_{ki}(n)$.

Steps [P1] and [P6] simply ensure that dual iterates are properly communicated and received. Steps [P3] and [P4] do the same for the primal iterates. ■

The distributed routing protocol [P1]-[P6] overcomes the limitations of a centralized implementation detailed at the end of Section III. Indeed, note that the proposed protocol i) requires communication with one-hop neighbors only, and ii) relies on knowledge of R_{ij} estimates and variances that either U_i or U_i's neighbors have available. Interestingly, there is no optimality penalty associated with this reduction in communication cost. The optimal routing matrix solving (14) and its corresponding optimal utility are eventually achieved by [P1]-[P6].

V. Simulations

We consider a wireless ad-hoc network with $J = 100$ nodes randomly deployed in a rectangle of dimensions $5\text{Km.} \times 3\text{Km.}$. To determine the average rate at which terminals can communicate with each other we let terminals transmit at random with probability 0.2. In every slot, consider the indicator variable $e_i(n) = 1$ if U_i transmitted in the n-th slot and $e_i(n) = 0$ otherwise. Letting p_i denote the transmission power of U_i and $h_{ij}(n)$ the gain in the channel $U_i \to U_j$ at the n-th time slot. We have

$$\gamma_{ij}(n) = \frac{h_{ij}(n)p_i}{\sigma_j + (1/S)\sum_{l=1,l\neq i}^{J} e_l(n)h_{lj}(n)p_l} \tag{33}$$

where $\gamma_{ij}(n)$ denotes the instantaneous SINR in the $U_i \to U_j$ link for the slot n, $\sigma_j = -90$db the noise power at U_j, and $S = 32$ the spreading gain common to all nodes in the network. The channels $h_{ij}(n)$ are assumed Rayleigh distributed with mean $\bar{h}_{ij}$,

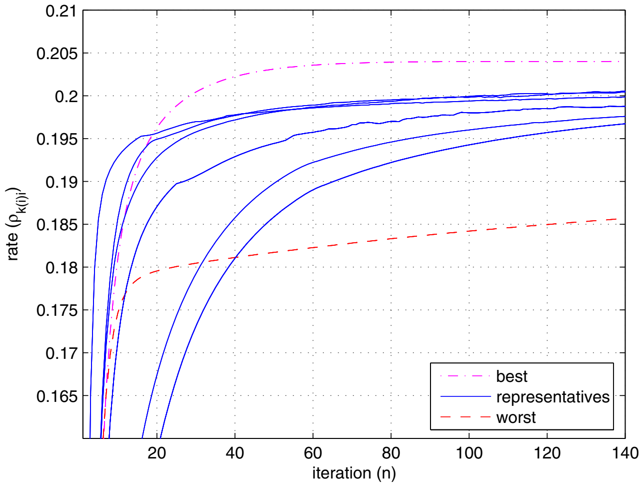

Fig. 1. Convergence of the algorithms in Proposition 3 to the required rates $\rho_{k(i),i} = 0.2$ for a set of 10 representative communicating pairs. Not all of the rates converge to $\rho_{k(i),i} = 0.2$ because of the mismatch between the actual $\mathbf{R}$ and the measured $\hat{\mathbf{R}}$. However, while the estimates have a 25% error, the achieved rates are within 5% of the required rate.

known at the receiver end, and independent across terminals and time. The mean channel power obeys an exponential pathloss law $\bar{h}_{ij} = \kappa d(U_i, U_j)^\alpha$ where $d(U_i, U_j)$ denotes the distance between U_i and U_j, and $\kappa = 1$ and $\alpha = 3.4$ are constants. By convention, $h_{ii}(n) = +\infty$ to ensure that U_i does not transmit and receive simultaneously.

With the SINR $\gamma_{ij}(n)$ in (33) a capacity achieving code achieves a rate $R_{ij}(n) = \log[1 + \gamma_{ij}(n)]$. Each R_{ij} element of the matrix $\mathbf{R}$ is the time average of $R_{ij}(n)$, i.e., $R_{ij} = \lim_{N\to\infty}(1/N)\sum_{n=1}^{N} R_{ij}(n)$. The rates are then normalized so that the maximum rate R_{ij} is equal to one. Every U_i node wants to deliver packets to a randomly chosen destination at a rate $\rho_{k(i),i} = 0.2$.

The $\hat{R}_{ij}$ estimates are randomly generated according to a uniform distribution in $[(1-0.25)R_{ij}, (1+0.25)R_{ij}]$. This entails a 25% uncertainty in reliability estimates. The variance of the estimates is $\Sigma_{ij} = (0.5R_{ij}^2)/12$.

The convergence of the algorithms in Proposition 3 to the required rates $\rho_{k(i),i} = 0.2$ is illustrated in Fig. 1 for a set of 10 representative communicating pairs. Interpreting convergence as the point at which the achieved rate is 90% of the required rate ($\rho_{k(i),i}(n) = 0.18$ in the example) the protocol converges in between 20 to 40 iterations.

Note that not all of the rates converge to $\rho_{k(i),i} = 0.2$. This is because of the mismatch between the actual $\mathbf{R}$ and the measured $\hat{\mathbf{R}}$. However, by minimizing the sum of the variances of $\rho_{k(i),i}$ the variation of the achieved rates is greatly reduced. While the estimates have a 25% error, the achieved rates are within 5% of the required rate.

The type of routes achieved by the protocol in Proposition 3 is illustrated in Fig. 2. Note how the packets are divided among a large number of neighbors.

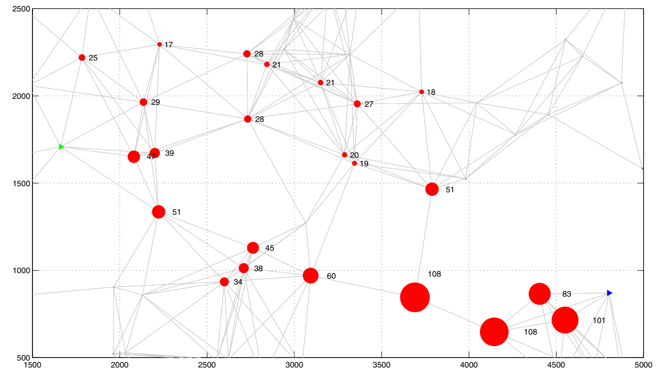

Fig. 2. Number of packets handled by each nodes in 500 time slots. The communication between the node marked with the blue (left) triangle to the node marked by the green (right) triangle is showcased.

VI. Conclusions

We introduced robust routing-scheduling protocols for wireless ad-hoc networks that reduce the uncertainty on network utility yield due to channel estimation errors. The protocols were defined as solutions of optimization problems that either maximize an average social network utility subject to a variance constraint; or, alternatively, minimize a variance cost subject to a minimum required rate yield. Conditions under which these problems are convex were found and shown to be not very restrictive. A distributed implementation based on dual decomposition techniques was then proposed. Although the communication cost to compute the optimal routes is thus significantly reduced, it was shown that there is no performance penalty with respect to optimal routes computed by a centralized algorithm.

Simulations corroborated that even with rough channel estimates – with up to 25% error – actual and prescribed utility yields turn out very close – within 5% of each other.

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