CROSS-LAYER OPTIMIZATION OF WIRELESS FADING AD-HOC NETWORKS*

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ABSTRACT

This paper develops near-optimal designs of wireless networks in the presence of fading. The novel approach optimizes jointly application level rates, routes, link capacities, power consumption and physical layer parameters. The physical layer is interference limited with terminals distributing their power budget among frequency tones, neighboring nodes and fading states. The present contribution builds on recent results establishing the optimality of layered architectures and develops physical layer resource allocation algorithms that are seamlessly integrated into layered architectures without loss of optimality.

***Index Terms*—** optimization methods, interference, ad-hoc networks, multi-hop, cross-layer designs

1. INTRODUCTION

It has long been recognized that proper design of wireless networks requires revamping the approach used in wireline networks [1]. At the core of this effort is a challenge to conventional layering that has led to the emergence of cross-layer wireless networking. One of the most promising approaches in this effort is optimal wireless network design. The best operating point is defined as the solution of an optimization problem and protocols follow from algorithms used for their solution; see e.g., [1], [2]. Interestingly, layers emerge naturally from the decomposition of the Lagrangian associated with optimal wireless networking formulations [3]. A recent development has shown that layered architectures not only emerge naturally but are in fact optimal [4]. The story has in some sense come full circle. The consequence of the optimality of layered architectures is that the difficulty in wireless networks is not in cross-layer design but in solving the resource allocation problem at the physical layer.

The challenge is thus to develop algorithms to find solutions to physical layer resource allocation problems. To this end, the present paper starts with a formulation to jointly optimize application level rates, routes, link capacities, power consumption and power allocation across frequency tones, neighboring terminals and fading states (Sec. 2). The optimality of layered architectures is then leveraged to introduce a subgradient descent algorithm that provides seamless and optimal integration of physical layer resource allocation with the remaining layers (Sec. 3). Focus then turns to the physical layer resource allocation problem. As this problem turns out to be intractable, approximate solutions are justified. The one developed here is inspired by the SCALE algorithm used for digital subscriber lines (DSL) [5]. Implementation of this algorithm is presented in Sec. 4 and corresponding numerical results in Sec. 5.

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2. PROBLEM STATEMENT

Consider a wireless ad-hoc network as in [2], [4], comprising a set of terminals (nodes) denoted by $\mathcal{V}$. Two nodes i, j form a link when they can communicate with each other; the communication between nodes is bidirectional. The set of nodes that a node $i \in \mathcal{V}$ can communicate with forms the neighborhood of i that is denoted by $\mathcal{N}(i)$.

Packets generated at each node correspond to possibly different applications and are destined for different nodes. Such packet streams from various applications are called commodities and are generically denoted by k. For simplicity and concreteness, consider the general scenario where: (i) each node is a destination of a single commodity (i.e., there are as many commodities as nodes); and (ii) each terminal accepts traffic (commodities) for all other terminals. The transport layer protocol of the network is responsible to maintain the end-to-end flow of packets from node i to the destination of every commodity k that enters the node. The average rate of such flow is denoted by a_i^k. Clearly, commodities that have as destination node i never actually enter node i. Hence, the set of commodities that can enter node i is $\mathcal{K}_i := \mathcal{V} \setminus \{i\}$.

Moreover, there are flows associated with the network layer. Specifically, each node i will transmit to its neighbors $j \in \mathcal{N}(i)$ packets of each commodity $k \in \mathcal{K}_i$ at an average rate denoted by r_{ij}^k, called network-layer rate or multicommodity flow. The r_{ij}^k are essentially routing variables, because they dictate how the packets from the various commodities are forwarded to the outgoing links of node i. As such, they pertain to the network layer. Hence, at node i there are exogenous packet arrivals with rate a_i^k (i.e., from the transport layer) and endogenous packet arrivals with rates r_{ji}^k from neighbors $j \in \mathcal{N}(i)$. Packets also leave node i with rates r_{ij}^k towards neighbors $j \in \mathcal{N}(i)$. Clearly, these rates satisfy a flow conservation constraint

$$a_i^k \leq \sum_{j \in \mathcal{N}(i)} r_{ij}^k - \sum_{j \in \mathcal{N}(i), j \neq k} r_{ji}^k \qquad \forall\, i,\ \forall\, k \in \mathcal{K}_i. \tag{1}$$

Link (i, j) carries total average rate $\sum_{k \in \mathcal{K}_i} r_{ij}^k$ that cannot exceed the capacity of the link, denoted by c_{ij}. This gives rise to the link capacity constraint

$$\sum_{k \in \mathcal{K}_i} r_{ij}^k \leq c_{ij} \qquad \forall\, i,\ \forall\, j \in \mathcal{N}(i). \tag{2}$$

Link capacities c_{ij} in wireless networks are not fixed, but depend on the specific physical and medium access control (MAC) layers, as well as on the allocation of network resources. In the considered network model, terminals have a set of tones $\mathcal{F} := \{1, \ldots, F\}$ available for transmission. It is further assumed that link (i, j) over tone f has power gain coefficient h_{ij}^f which captures fading effects. The focus in the present work is on non-orthogonal medium access, where different terminals are allowed to use the same frequency to transmit, and treat other terminals' transmissions as noise. Network resources here are power allocations $p_{ij}^f(\mathbf{h})$ for link (i, j) over tone f as a function of the power gains h_{ij}^f collected in the vector $\mathbf{h}$.

The key quantity dictating the link capacity is the signal-to-interference-plus-noise ratio (SINR). The (instantaneous) SINR of link (i,j) over tone f is given by

$$\gamma_{ij}^f := h_{ij}^f p_{ij}^f(\mathbf{h}) \Big/ \Big[\sigma_j^f + \sum\nolimits_{(k,l)\in\mathcal{I}_{ij}} h_{kj}^f p_{kl}^f(\mathbf{h})\Big] \tag{3}$$

where σ_j^f is the noise variance at node j over tone f, and $\mathcal{I}_{ij}$ is a set of links causing interference to (i,j). This set consists of the links carrying: (i) other incoming transmissions for j over f; (ii) outgoing transmissions from j; and (iii) transmissions originating from $\mathcal{N}(j)$. Note that the SINR depends on power allocations $p_{ij}^f(\mathbf{h})$ as well as on $\mathbf{h}$; this dependence is not shown for brevity.

Supposing capacity-achieving codebooks, and letting $\mathbb{E}_{\mathbf{h}}$ denote expectation over $\mathbf{h}$, the ergodic capacity c_{ij} of link (i,j) is

$$c_{ij} = \mathbb{E}_{\mathbf{h}}\left[\sum_{f\in\mathcal{F}} \ln\big(1+\gamma_{ij}^f\big)\right]. \tag{4}$$

Two kinds of power constraints will be considered. The first will be spectral mask or instantaneous power constraints, expressed as $0 \le p_{ij}^f(\mathbf{h}) \le p_{ij\,\max}^f$. The second refers to the average power consumed by a node in the network. Specifically, the average power is defined as $p_i := \mathbb{E}_{\mathbf{h}}\big[\sum_{j\in\mathcal{N}(i)}\sum_{f\in\mathcal{F}} p_{ij}^f(\mathbf{h})\big]$, and is assumed constrained by a power budget $p_{i\,\max}$, i.e., $0 \le p_i \le p_{i\,\max}$.

Intended arrival rates a_i^k in practical networks lie within application-specific bounds expressed as $a_{i\,\min}^k \le a_i^k \le a_{i\,\max}^k$. Furthermore, the network designer may wish to impose upper bounds on link capacities c_{ij} and multicommodity flows r_{ij}^k, that is $0 \le c_{ij} \le c_{ij\,\max}$ and $0 \le r_{ij}^k \le r_{ij\,\max}$. The notation $\mathbf{X}$ will be used for all average variables, i.e., $a_i^k, r_{ij}^k, c_{ij}, p_i$ for all $i \in \mathcal{V}$, $j \in \mathcal{N}(i)$, $k \in \mathcal{K}_i$. Then, all previously mentioned 'box' constraints can be conveniently written using the set notation

$$\begin{aligned}\mathcal{B} := \{&\mathbf{X}, \mathbf{p}(\mathbf{h}) : 0 \le p_{ij}^f(\mathbf{h}) \le p_{ij\,\max}^f, 0 \le p_i \le p_{i\,\max},\\ &a_{i\,\min}^k \le a_i^k \le a_{i\,\max}^k, 0 \le c_{ij} \le c_{ij\,\max}, 0 \le r_{ij}^k \le r_{ij\,\max}\}.\end{aligned} \tag{5}$$

Higher exogenous arrival rates a_i^k are in general more desirable; this is captured by utility functions $U_i^k(a_i^k)$ that are selected strictly increasing and concave. Moreover, average power consumption may be penalized by cost functions $V_i(p_i)$, selected strictly increasing and convex. Optimal wireless networking refers to selecting (average) variables a_i^k, r_{ij}^k, c_{ij}, p_i and instantaneous power allocations $p_{ij}^f(\mathbf{h})$, such that constraints (1), (2), (4), (5) are respected, while the total network utility is maximized and the total cost minimized:

$$\max_{\{\mathbf{X},\mathbf{p}(\mathbf{h})\}\in\mathcal{B}} \sum_{i,k\in\mathcal{K}_i} U_i^k(a_i^k) - \sum_i V_i(p_i) \tag{6a}$$

$$a_i^k \le \sum_{j\in\mathcal{N}(i)} r_{ij}^k - \sum_{j\in\mathcal{N}(i), j\neq k} r_{ji}^k \quad \forall k \in \mathcal{K}_i,\ \forall i \tag{6b}$$

$$\sum_{k\in\mathcal{K}_i} r_{ij}^k \le c_{ij} \qquad \forall j \in \mathcal{N}(i),\ \forall i \tag{6c}$$

$$c_{ij} \le \mathbb{E}_{\mathbf{h}}\left[\sum_f \ln\big(1+\gamma_{ij}^f\big)\right] \quad \forall j \in \mathcal{N}(i),\ \forall i \tag{6d}$$

$$\mathbb{E}_{\mathbf{h}}\left[\sum_{j\in\mathcal{N}(i)}\sum_{f\in\mathcal{F}} p_{ij}^f(\mathbf{h})\right] \le p_i \quad \forall i. \tag{6e}$$

Let ν_i^k, ξ_{ij}, λ_{ij}, μ_i be Lagrange multipliers for constraints (6b), (6c), (6d) and (6e), respectively. The Lagrangian of (6) reduces after straightforward rearrangements to ($\mathbf{\Lambda}$ collectively denotes all multipliers)

$$\begin{aligned}L(\mathbf{\Lambda}, \mathbf{X}, \mathbf{p}(\mathbf{h})) &= \sum_{i,k\in\mathcal{K}_i}\Big(U_i^k(a_i^k) - \nu_i^k a_i^k\Big) + \sum_i (\mu_i p_i - V_i(p_i))\\ &+ \sum_{i,j\in\mathcal{N}(i)} (\xi_{ij} - \lambda_{ij})\, c_{ij} + \sum_{\substack{i,j\in\mathcal{N}(i)\\ f\in\mathcal{F}}} \mathbb{E}_{\mathbf{h}}\big[\lambda_{ij}\ln\big(1+\gamma_{ij}^f\big) - \mu_i p_{ij}^f(\mathbf{h})\big]\\ &+ \sum_{\substack{i,j\in\mathcal{N}(i)\\ k\in\mathcal{K}_i, j\neq k}} (\nu_i^k - \nu_j^k - \xi_{ij}) r_{ij}^k + \sum_{\substack{i,j\in\mathcal{N}(i)\\ j\in\mathcal{K}_i}} (\nu_i^j - \xi_{ij}) r_{ij}^j.\end{aligned} \tag{7}$$

The dual function and the dual problem are, respectively,

$$g(\mathbf{\Lambda}) := \max_{(\mathbf{X},\mathbf{p}(\mathbf{h}))\in\mathcal{B}} L(\mathbf{\Lambda}, \mathbf{X}, \mathbf{p}(\mathbf{h})) \tag{8}$$

$$\min_{\mathbf{\Lambda}\ge\mathbf{0}} g(\mathbf{\Lambda}). \tag{9}$$

Due to constraint (6d), problem (6) is non-convex. Remarkably though, problem (6) has zero duality gap, whenever the cumulative distribution function (cdf) of $\mathbf{h}$ is continuous, i.e., under all practical fading models [4]. (See references in [4] for related results in other contexts.) This result means that one may solve the dual problem (9) without loss of optimality. This is desirable, because the dual problem is a convex optimization problem [6, Sec. 5.1], and also its solution effects separation into conventional layers (see [4] and the discussion in the next section).

On the other hand, a task needed in the solution of the dual problem is maximization of the Lagrangian (cf. (8)). Such maximization is computationally intractable (in terms of seeking an exact maximizer), due to the term $\ln\big(1+\gamma_{ij}^f\big)$ in (7). The present paper's main contribution is to tackle the solution of (9) (and hence of (6)) for a network whose physical layer is characterized by link capacities as in (4), i.e., when terminals use the simple strategy of treating non-intended transmissions as noise.

This will be achieved by utilizing a subgradient descent algorithm for the solution of (9), described in Sec. 3. The subgradient iterations will be paired in Sec. 4 with a suitable approximation algorithm, aiming at evaluating (8).

3. SUBGRADIENT METHOD

Subgradient iterations will be used for the solution of (9), as the dual problem is typically non-differentiable [6, Sec. 6.3]. Specifically, rewrite all constraints in (6) with 0 on the right-hand side of the inequalities, and let all left-hand side functions be collectively denoted by $\mathbf{v}(\mathbf{X}, \mathbf{p}(\mathbf{h}))$; also let t be the iteration index. Then the iterates $\mathbf{\Lambda}(t)$ obtained via the subgradient method, with initial $\mathbf{\Lambda}(0) \ge \mathbf{0}$ are

$$(\mathbf{X}(t), \mathbf{p}(\mathbf{h};t)) \in \operatorname*{argmax}_{(\mathbf{X},\mathbf{p}(\mathbf{h}))\in\mathcal{B}} L(\mathbf{\Lambda}(t), \mathbf{X}, \mathbf{p}(\mathbf{h})) \tag{10a}$$

$$\mathbf{\Lambda}(t+1) = \big[\mathbf{\Lambda}(t) + \epsilon_t \mathbf{v}(\mathbf{X}(t), \mathbf{p}(\mathbf{h};t))\big]^+ \tag{10b}$$

where ϵ_t is the stepsize (depending on t in general), and $[.]^+$ denotes elementwise projection onto the nonnegative reals.

Due to the separable structure of the Lagrangian, (10a) becomes

$$a_i^k(t) \in \operatorname*{argmax}_{a_{i\,\min}^k \le a_i^k \le a_{i\,\max}^k} \big[U_i^k(a_i^k) - \nu_i^k(t) a_i^k\big] \tag{11a}$$

$$r_{ij}^k(t) \in \operatorname*{argmax}_{0\le r_{ij}^k \le r_{ij\,\max}} \begin{cases} \big(\nu_i^k(t) - \nu_j^k(t) - \xi_{ij}(t)\big) r_{ij}^k & \text{if } j \neq k\\ \big(\nu_i^j(t) - \xi_{ij}(t)\big) r_{ij}^j & \text{o/w}\end{cases} \tag{11b}$$

$$c_{ij}(t) \in \operatorname*{argmax}_{0\le c_{ij}\le c_{ij\,\max}} \big[(\xi_{ij}(t) - \lambda_{ij}(t)) c_{ij}\big] \tag{11c}$$

and

$$p_i(t) \in \underset{0\le p_i \le p_{i\,\max}}{\operatorname{argmax}} \left[\mu_i(t)p_i - V_i(p_i)\right] \tag{11d}$$

$$\mathbf{p}(\mathbf{h};t) \in \underset{0\le p_{ij}^f(\mathbf{h})\le p_{ij\,\max}^f}{\operatorname{argmax}} \sum_{i,f,j\in\mathcal{N}(i)} \left[\lambda_{ij}(t)\ln\left(1+\gamma_{ij}^f\right) - \mu_i(t)p_{ij}^f(\mathbf{h})\right]. \tag{11e}$$

Eq. (11e) is obtained by noting that the part of (7) that involves the $\mathbb{E}_{\mathbf{h}}[.]$ operator can be maximized if the term inside the expectation is maximized for each fading state $\mathbf{h}$. Equations (11a)–(11e) can be interpreted as separating the solution of the wireless networking problem into conventional layers [4]. In particular, (11a) solves the flow control problem at the transport layer; (11b) performs routing at the network layer; (11c) and (11d) address the link rate control and (average) power control at the data link layer; and (11e) solves the power allocation at the physical layer.

Subgradient iterations (10b) can also be written explicitly as

$$\nu_i^k(t+1) = \left[\nu_i^k(t)+\epsilon_t\left(a_i^k(t) - \sum_{j\in\mathcal{N}(i)} r_{ij}^k(t) + \sum_{\substack{j\in\mathcal{N}(i)\\ j\neq k}} r_{ji}^k(t)\right)\right]^+ \tag{12a}$$

$$\xi_{ij}(t+1) = \left[\xi_{ij}(t) + \epsilon_t\left(\sum_{k\in\mathcal{K}_i} r_{ij}^k(t) - c_{ij}(t)\right)\right]^+ \tag{12b}$$

$$\lambda_{ij}(t+1) = \left[\lambda_{ij}(t)+\epsilon_t\left(c_{ij}(t) - \mathbb{E}_{\mathbf{h}}\left[\sum_f \ln\left(1+\gamma_{ij}^f(t)\right)\right]\right)\right]^+ \tag{12c}$$

$$\mu_i(t+1) = \left[\mu_i(t)+\epsilon_t\left(\mathbb{E}_{\mathbf{h}}\left[\sum_{j\in\mathcal{N}(i),f} p_{ij}^f(\mathbf{h};t)\right] - p_i(t)\right)\right]^+. \tag{12d}$$

Iterations (12) will converge to the optimal dual variables, whenever the stepsize sequence (ϵ_t) is non-summable diminishing, i.e., it satisfies $\sum_{t=0}^{\infty}\epsilon_t = \infty$, $\lim_{t\to\infty}\epsilon_t = 0$ [7, Sec. 2.2].

In order to perform iterations (12), the solution of (11) is needed. The solution of (11a)–(11d) (as a function of the Lagrange multipliers) is straightforward given the concavity of the respective objectives and the box constraints. But the solution of (11e) is considerably more challenging. In particular, (11e) bears resemblance with the problem of power control in deterministic DSL channels, see e.g., [5] and references therein. This problem is known to be NP-hard [8]. The solution of (11e) will be undertaken in the ensuing section, via a suitable approximation algorithm.

After the solution of (11) is obtained, the subgradients in (12a), (12b) are readily evaluated. On the other hand, (12c) and (12d) involve the expectation $\mathbb{E}_{\mathbf{h}}[.]$. This can be evaluated via Monte Carlo whenever the cdf of $\mathbf{h}$ is known, by drawing independent realizations of $\mathbf{h}$, and solving (11e) for each of them.

Remark 1 *Apart from the optimal dual variables, it is important to obtain the optimal (primal) solution of* (6)*, e.g., rates* a_i^k*. It is possible to recover optimal primal variables from the sequence* $(\mathbf{X}(t), \mathbf{p}(\mathbf{h}, t))$ *obtained as a byproduct of the subgradient method (cf.* (10a)*), even though* (6) *is nonconvex. This is omitted here due to space limitations; see [9] for details.*

4. APPROXIMATION ALGORITHM

An approximate solution to (11e) is pursued here, based on the SCALE algorithm [5]. In particular, instead of solving (11e) directly, the solution of a sequence of successive convex approximations of (11e) is sought. Such an approximation is

$$\max_{0\le p_{ij}^f(\mathbf{h})\le p_{ij\,\max}^f} \sum_{i,f,j\in\mathcal{N}(i)} \left[\lambda_{ij}(t)\alpha_{ij}^f \ln\gamma_{ij}^f - \mu_i(t)p_{ij}^f(\mathbf{h})\right] \tag{13}$$

where α_{ij}^f are properly selected weights and γ_{ij}^f is the SINR given by (3). Each approximation (13) is solved exactly (for fixed weights); the weights are then updated; and subsequently the new approximation is solved. The powers obtained as the solution of the last problem in this procedure are approximate solutions of (11e). In what follows, the algorithm for solving (13) is described, and then the method for updating the weights is given.

Problem (13) is decomposable per tone, i.e., one can solve

$$\max_{0\le p_{ij}^f(\mathbf{h})\le p_{ij\,\max}^f} \sum_{i,j\in\mathcal{N}(i)} \left[\lambda_{ij}(t)\alpha_{ij}^f \ln\gamma_{ij}^f - \mu_i(t)p_{ij}^f(\mathbf{h})\right]. \tag{14}$$

Using the method in [5], (14) may be recast as a convex problem. For its solution the following iteration is used, indexed by τ (note that t is constant here, and the dependence of p_{ij}^f on $\mathbf{h}$ is dropped):

$$p_{ij}^f(\tau+1) = \left[\frac{\lambda_{ij}(t)\alpha_{ij}^f}{\mu_i(t) + \sum_{(m,n):(i,j)\in\mathcal{I}_{mn}} \frac{\lambda_{mn}(t)\alpha_{mn}^f h_{in}^f}{\sigma_n^f + \sum_{(k,l)\in\mathcal{I}_{mn}} h_{kn}^f p_{kl}^f(\tau)}}\right]_0^{p_{ij\,\max}^f} \tag{15}$$

where $[.]_a^b$ denotes projection onto $[a, b]$. The following can be shown[1]:

Lemma 1 *Iterations* (15) *converge to the global optimum of* (14) *from any initialization* $0 \le p_{ij}^f(0) \le p_{ij\,\max}^f$.

Note that although (13) or equivalently the per tone problems (14) can be solved each time optimally, the powers obtained as successive solutions of (13) converge in general to local optimizers of (11e).

Now attention is turned to the weights α_{ij}^f. Specifically, each time problem (14) is solved (i.e., upon convergence of (15)), the weights are updated according to $\alpha_{ij}^f = \gamma_{ij}^f/(1+\gamma_{ij}^f)$; see [5, Sec. III] for details. Here the SINRs γ_{ij}^f are evaluated at the powers obtained upon convergence of (15). Then the weights are kept fixed, and a new problem (13) is solved. This procedure ends when the sequence of powers obtained as solutions of (13) converges (or practically, after a prespecified number of approximations). The weights are initialized with $\alpha_{ij}^f = 1$ (high-SINR approximation).

5. NUMERICAL RESULTS

Consider the wireless network in Fig. 1, placed on a 300m $\times$ 100m regular grid. Transmissions occur on five bands centered around five tones over which the channels h_{ij}^f are Rayleigh. Average channel power $\bar{h}_{ij}^f$ is determined by a path loss model $\bar{h}_{ij}^f := K_{\mathrm{PL}}(d_{ij}/d_0)^{-\delta}$ with $K_{\mathrm{PL}} = 0.1$, $\delta = 2$, $d_0 = 20$m, and d_{ij} is the distance between nodes i and j. The self-interference gain h_{jj}^f is set to 10 dB. Channels are independent across links, tones and time, while the noise power at terminal j over tone f is set to $\sigma_j^f = \min_{i\in\mathcal{N}(j)} \bar{h}_{ij}^f$ (W/Hz). Each terminal has an average power constraint $p_{i\,\max} =$ 5 W/Hz and uses a flat spectral mask $p_{ij\,\max}^f = 5$ W/Hz.

The utilities for rates a_i^k (rate rewards) are logarithmic, i.e., $U_i^k(a_i^k) = \ln a_i^k$, while the power cost functions $V_i(p_i)$ are quadratic, i.e., $V_i(p_i) = p_i^2$ for all i. The arrival rate requirements are set to $a_{i\,\max}^k = 14$ bps/Hz and $a_{i\,\min}^k = 1.4 \cdot 10^{-4}$ bps/Hz. The upper bound $c_{ij\,\max}$ on the link capacities is simply the largest among the maximum link capacities when there is no interference, while the upper bound $r_{ij\,\max}$ on flows r_{ij}^k is set to $r_{ij\,\max} = c_{ij\,\max}$. The stepsize is $\epsilon_t = .75/(1+.002t)$; the total number of convex approximations is 7 (including the initial approximation); and the number of iterations (15) per approximation is 50. The expected value $\mathbb{E}_{\mathbf{h}}[.]$ was approximated via Monte Carlo using 10 realizations.

[1] The proof is omitted due to space limitations; it is adapted from [5].

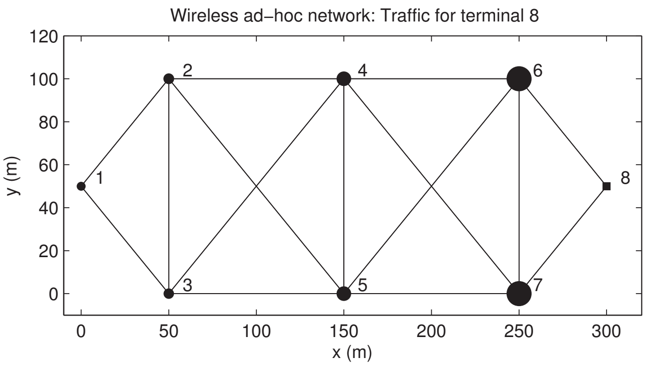

Fig. 1. A wireless ad-hoc network with its physical dimensions indicated; and incoming rates with node 8 as destination.

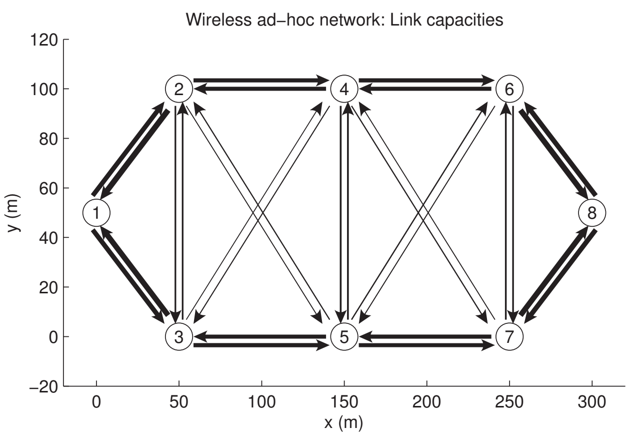

Fig. 2. Link capacities.

Routes are specified by the routing variables r_{ij}^k. Consider the flow $k = 8$ consisting of packets transmitted to terminal 8. Terminal 8 is depicted in Fig. 1 as a square while the remaining terminals are shown as circles with area proportional to the total (endogenous plus exogenous) rate that arrives to that terminal and has terminal 8 as destination, i.e., $a_i^8 + \sum_{j \in \mathcal{N}(i)}^{j \neq 8} r_{ji}^8$. This represents the amount of packets for terminal 8 that each node handles. Observe that packets accumulate as we move from terminal 1 closer to terminals 6 and 7.

Link capacities are shown in Fig. 2. The width of each arrow is proportional to the average capacity c_{ij} of the corresponding link. Notice that certain links have larger capacity than others, with the links (2,1), (3,1), (6,8) and (7,8) having the largest capacities. The reason is that these links experience the least interference, as the receiving terminals 1 and 8 are the "furthest" among all other nodes in the network. Corroborating intuition, the aforementioned links with the transmitting and receiving ends reversed, such as (1,2), have smaller capacities because the receiving ends, which are now 2, 3, 6, and 7, have more interfering neighbors than terminals 1 and 2. Also observe that the 'diagonal' links such as (3,4) have smaller capacities than the links which correspond to the edges of the 'squares', e.g., (2,4) and (5,4). This is expected, because the distance between terminals 3 and 4 is larger than the distance between terminals 2 and 4 (100m), or, between 5 and 4 (100m).

Fig. 3 depicts the average power p_i consumed per terminal i. Notice that all terminals use approximately the same average power; however, the 'middle nodes', i.e., 4 and 5, require slightly higher power, consistent with the fact that nodes in the middle have more neighbors and are likely to handle a larger share of the overall traffic. The fact that all terminals use approximately the same average power

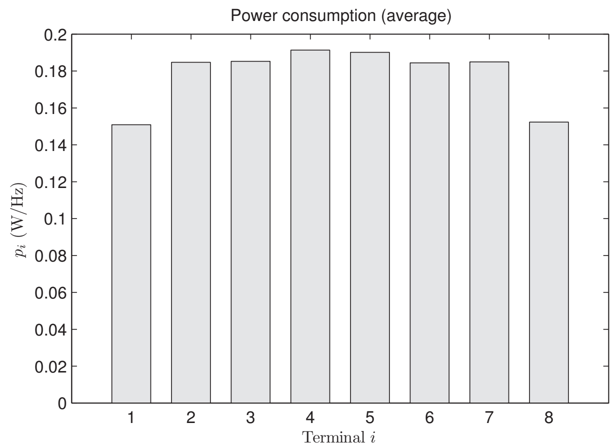

Fig. 3. Average power consumption at the network terminals.

is partly due to the symmetry of the network, as well as the chosen utilities $V_i(p_i) = p_i^2$. In particular, utility $V_i(p_i)$ was selected with relatively high weights for all terminals so as to render the average power consumption approximately uniform across terminals, which may be desirable in an ad-hoc network. Recall that the designer is flexible to control the average power expenditure through appropriate selection of power costs $V_i(p_i)$.[2]

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