Cognitive Access Algorithms For Multiple Access Channels

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***Abstract*—This paper considers a multiple access fading channel where each terminal has a different belief about the channel states and adapts its transmission policy to the belief. In this setting, frequency division multiple access (FDMA) and channel aware random access (RA) are two special cases. To find solutions for general cases, we formulate the problem as a Bayesian game in which each terminal maximizes the expected utility based on its belief. We show that optimal solutions for both FDMA and RA are equilibrium points of the game. Therefore, the proposed game theoretic formulation can be regarded as general framework for multiple access channels. Furthermore, we develop a cognitive access algorithm that solves the problem approximately. Numerical results show that the proposed algorithm achieves good performance and is adaptable to different levels of channel beliefs.**

I. INTRODUCTION

Consider a multiple access fading channel in which terminals contend for communicating with a central access point. To exploit favorable channel conditions, terminals adapt their channel access as well as transmission power to the random states of the fading channel. The goal is to maximize the expected value of the sum transmission rates over all terminals subject to terminals' average power constraints. This problem has been studied extensively in the past and depending on the availability of the channel state information (CSI) the optimal solutions are different. When global CSI is available, i.e., each terminal knows the channel states of others, they can cooperate with each other to avoid collision. This is known as frequency-division multiple access (FDMA) in which the terminal with the largest channel state gets the opportunity to transmit, see e.g., [1], [2]. However, global CSI is usually not available in many practical scenarios and it is more reasonable to assume terminals only have access to local CSI. In this case, terminals make transmission decision and power allocation based on their local CSI without cooperating with each other. This is known as channel aware random access (RA), and it has been show the optimal solution is a threshold-based strategy, i.e., transmission is scheduled only when the local CSI exceeds certain threshold, see e.g., [3], [4].

FDMA and RA can be regarded as two extreme cases for multiple access channel where global CSI and local CSI are available for terminals. There are many other cases in between. For example, terminals may know their local channels perfectly and have some imperfect information about the channels of other terminals. In other words, terminals are cognitive in the sense that each terminal has a different belief about the channel states. In this setting, it is natural to formulate the problem as a Bayesian game in which each terminal is a self-interested but rational player that maximizes the expected utility based on its belief. Bayesian game has been used to study random access channels in which each terminal knows the prior distributions of other channels [5], [6]. However, these algorithms cannot be generalized to the cognitive setting where beliefs change over terminals and time. This motivates us to develop cognitive access algorithms that determine an optimal allocation of resources taking into account the fact that different terminals have different beliefs on the channel state.

The rest of the paper is organized as follows. Communciation model is introduced in Section II. In Section III, we define the Bayesian game and show that optimal solutions for FDMA and RA are Bayesian Nash Equilibrium (BNE) points of the game. In Section IV, we develop a cognitive access algorithm that finds solution for the Bayesian game approximately. Numerical results and conclusion are presented in Section V and VI, respectively.

II. COMMUNICATION MODEL

Consider a multiple access channel with n terminals contending to communicate with a common AP. The time varying channel $h_i(t) \in \mathbf{R}^+$ from user i to the AP is modeled as block fading and assumed independent for different times $t_1 \neq t_2$ and different terminals $i_1 \neq i_2$. The probability density function (pdf) of channel h_i is denoted as $f_{h_i(t)}(h) = f_{h_i}(h)$ and assumed to be time invariant and nonatomic. This latter condition is equivalent to having continuous cumulative distribution functions and holds true for practical models including Rayleigh, Rician, and Nakagami [7]. For future reference define the aggregate channel as $\mathbf{h}(t) := \{h_j(t)\}_{j=1}^n$ and the channel complement of terminal i as $\mathbf{h}_{-i}(t) := \{h_j(t)\}_{j=1,j\neq i}^n$. We also assume a backlogged system where all terminals have packets to transmit all the time.

Upon observing its own channel $h_i(t)$ node i makes a decision on whether to transmit or not in the current time slot and if it chooses to do so it selects a transmit power for the communication attempt. Transmission decisions are based on the attempt function $q_i : \mathbb{R} \rightarrow \{0, 1\}$ and the power allocation function $p_i : \mathbb{R}^+ \rightarrow [0, p_i^{\text{inst}}]$, where $p_i^{\text{inst}} > 0$ is a limit on the instantaneous power transmitted by terminal i. Given the channel value $h_i(t)$ terminal i makes a transmission attempt in time slot t if and only if $q_i(h_i(t)) = 1$ in which case it does so with power $p_i(h_i(t))$. The pair of functions $\mathbf{p}_i := (q_i, p_i)$ is termed the transmission strategy profile of terminal i. The joint strategy is defined as the grouping $\mathbf{p} := \{\mathbf{p}_j\}_{j=1}^n$ of all individual strategy profiles and the complementary strategy of terminal i as the grouping $\mathbf{p}_{-i} := \{\mathbf{p}_j\}_{j=1,j\neq i}^n$ of all strategies except the one of i. Observe that specifying the joint strategy $\mathbf{p}$ is equivalent to specifying the individual strategy $\mathbf{p}_i$ and the complementary strategy $\mathbf{p}_{-i}$.

A communication attempt with power $p_i(h_i(t))$ when the channel is $h_i(t)$ proceeds at a rate $C[h_i(t), p_i(h_i(t))]$, where $C : \mathbb{R}^+ \times [0, p_i^{\text{inst}}] \rightarrow \mathbb{R}^+$ is a function mapping channels and powers to transmission rates. If more than one terminal attempts transmission in the same time slot a collision occurs. Thus, user i is able to reach the AP at time t if and only if $q_i(h_i(t)) = 1$ and $q_j(h_j(t)) = 0$ for all $j \neq i$. Therefore, the instantaneous transmission rate for terminal i at time t is

$$r_i(\mathbf{h}(t), \mathbf{p}(\mathbf{h}(t))) \ = C\big[h_i(t), p_i(h_i(t))\big] \times q_i(h_i(t)) \prod_{j=1, j\neq i}^{n} [1 - q_j(h_j(t))], \tag{1}$$

For joint transmission strategy $\mathbf{p}$, the average rate experienced by terminal i is the expectation $\mathbb{E}_{\mathbf{h}}\left[r_i(\mathbf{h}(t), \mathbf{p}(\mathbf{h}(t)))\right]$ with respect to $\mathbf{h}$ of the instantaneous rate in (1). Adopting the sum rate as our utility function, it then follows that the utility associated with policy $\mathbf{p}$ is given by

$$\bar{U}(\mathbf{p}) = \mathbb{E}_{\mathbf{h}}\Big[U(\mathbf{p}(\mathbf{h}))\Big] = \mathbb{E}_{\mathbf{h}}\left[\sum_{i=1}^{n} r_i(\mathbf{h}, \mathbf{p}(\mathbf{h}))\right]. \tag{2}$$

where we dropped the time index because the channel distribution is assumed stationary and defined the instantaneous utility $U(\mathbf{p}(\mathbf{h}))$. Further note that each communication attempt, successful or not, incurs a power cost $p_i(h_i)$. Therefore, the average power consumption of terminal i is the expectation $\mathbb{E}_{h_i}\Big[q_i(h_i)p_i(h_i)\Big]$ and in order to satisfy an average power budget p_i^{avg} we must have

$$\mathbb{E}_{h_i}\Big[q_i(h_i)p_i(h_i)\Big] \leq p_i^{\text{avg}}. \tag{3}$$

The goal of each terminal is to select the policy $\mathbf{p}_i$ that maximizes the average sum rate utility $\bar{U}(\mathbf{p})$ in (2) while satisfying the average power constraint in (3). However, terminal i lacks the information to do so. The rate $r_i(\mathbf{h}, \mathbf{p}(\mathbf{h}))$ attained by terminal i is dependent upon the scheduling $q_i(h_i)$ of all terminals [cf. (1)]. For terminal i to be able to solve this problem, it requires policies $\mathbf{p}_{-i}$ and channels $\mathbf{h}_{-i}$ of all other terminals. Since neither $\mathbf{p}_{-i}$ nor $\mathbf{h}_{-i}$ is available to terminal i, necessary assumptions need to be made in order to solve the problem. E.g., assume all channels have the same distributions and all terminals employ the same policies, i.e., $f_{h_j}(h) = f_{h_i}(h)$ and $\mathbf{p}_j = \mathbf{p}_i$ for all $j \neq i$, the problem can be formulated as optimal random access (RA) [3], [4]. In this paper, we would like to study more general cases in which terminals have partial knowledge about others' channels and apply different policies.

Assume terminal i has access to channel estimations of other terminals $\tilde{\mathbf{h}}_{-i} := \{\tilde{h}_j\}_{j=1, j\neq i}^{n}$ where $\tilde{h}_j$ is terminal i's estimation of terminal j's channel. We assume $\tilde{h}_j$ has the same distribution as h_j and the accuracy of $\tilde{h}_j$ is indicated by conditional probability distributions of h_j given $\tilde{h}_j$, i.e., $f_{h_j|\tilde{h}_j}(h)$ (see Remark 1). When the channel estimation is perfect, i.e., $\tilde{h}_j = h_j$, it implies that $f_{h_j|\tilde{h}_j}(h) = \delta(h - \tilde{h}_j)$. When $\tilde{h}_j$ does not reveal any information about h_j, the conditional pdf $f_{h_j|\tilde{h}_j}(h)$ is the prior distribution of h_j, i.e. $f_{h_j|\tilde{h}_j}(h) = f_{h_j}(h)$. The conditional pdf $f_{h_j|\tilde{h}_j}(h)$ is called terminal i's belief about terminal j's channel. We remark that the belief changes over time.

Remark 1 The probability distribution $f_{h_j|\tilde{h}_j}(h)$ depends on the channel estimation method. A typical way is to assume that $\tilde{h}_j$ is an outdated version of h_j modeled by an order-1 autoregressive (AR) process. For example, suppose h_j is complex Gaussian with pdf $\mathcal{CN}(0, 2)$, then the estimation can be modeled by $h_j = \rho\tilde{h}_j + e_j$ where ρ is the correlation coefficient between h_j and $\tilde{h}_j$ and e_j is complex Gaussian random noise with pdf $\mathcal{CN}(0, 1 - \rho^2)$. In this case, $f_{h_j|\tilde{h}_j}(h)$ is a noncentral chi-square distribution [8]

$$f_{h_j|\tilde{h}_j}(h) = \frac{1}{2(1-\rho^2)} \exp\left(-\frac{h + \rho^2\tilde{h}_j}{2(1-\rho^2)}\right) I_0\left(\frac{\rho^2\sqrt{h\tilde{h}_j}}{(1-\rho^2)}\right), \tag{4}$$

where $I_0(x) = \sum_{i=0}^{\infty} (x^2/4)^i/(i!)^2$ is the zeroth order modified Bessel function of the first kind. This particular form for the conditional pdf $f_{h_j|\tilde{h}_j}(h)$ is used to provide numerical results in Section V. The rest of the development in the paper holds independently of the particular form of this pdf.

III. Bayesian Nash Equilibrium

Since terminals are not allowed to cooperate with each other, each terminal has to make transmission decisions independently. On the other hand, the system utility depends on the actions of all terminals [cf. (2)]. This situation can be modeled as a game with incomplete information where each terminal makes decisions on its own channel and beliefs about channels of others while receives a global utility.

From terminal i's perspective, the utility $U(\mathbf{p})$ is a function of its own action $\mathbf{p}_i$ and actions of other terminals $\mathbf{p}_{-i}$. As a result, we can write $U(\mathbf{p})$ as $U(\mathbf{p}_i, \mathbf{p}_{-i})$. In each time slot, terminal i observes its own channel h_i and estimations about channels of other terminals $\tilde{\mathbf{h}}_{-i}$. Based on $\tilde{\mathbf{h}}_{-i}$, terminal i has a belief about $\mathbf{h}_{-i}$ which is represented by the conditional pdf $f_{h_j|\tilde{h}_j}(h)$. Suppose other terminals' policies $\mathbf{p}_{-i}$ are given, the *best response* terminal i can take is to maximize the expected value of the sum rate utility based on its belief subject to the average power constraint

$$\begin{aligned} \mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}) := &\operatorname{argmax} \mathbb{E}_{h_i, \tilde{\mathbf{h}}_{-i}}\Big[\mathbb{E}_{\mathbf{h}_{-i}|\tilde{\mathbf{h}}_{-i}} U(\mathbf{p}_i, \mathbf{p}_{-i})\Big] \\ &\text{s.t. } \mathbb{E}_{h_i, \tilde{\mathbf{h}}_{-i}}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i \end{aligned} \tag{5}$$

where $\mathcal{P}_i = \{\mathbf{p}_i | q_i(h_i) \in \{0, 1\}, p_i(h_i) \in [0, p_i^{\text{inst}}]\}$ is the set of values that $\mathbf{p}_i$ can take. Note that the double expectation $\mathbb{E}_{h_i, \tilde{\mathbf{h}}_{-i}}\Big[\mathbb{E}_{\mathbf{h}_{-i}|\tilde{\mathbf{h}}_{-i}} U(\mathbf{p}_i, \mathbf{p}_{-i})\Big]$ in (5) is the same as the expected utility in (2): the outer expectation in (5) is taken over terminal i's observations h_i and $\tilde{\mathbf{h}}_{-i}$ while the inner expectation in (5) is with respect to terminal i's belief about channels of other terminals, i.e., $\mathbf{h}_{-i}$ given $\tilde{\mathbf{h}}_{-i}$. In this game, each terminal solves an optimization problem like (5). The equilibrium point of the game is defined by the following:

Definition 1 $\mathbf{p}_i^{\text{BNE}}$ *is Bayesian Nash Equilibrium (BNE) if for all i the following holds true*

$$\begin{aligned} \mathbf{p}_i^{\text{BNE}} = &\operatorname{argmax} \mathbb{E}_{h_i, \tilde{\mathbf{h}}_{-i}}\Big[\mathbb{E}_{\mathbf{h}_{-i}|\tilde{\mathbf{h}}_{-i}} U(\mathbf{p}_i, \mathbf{p}_{-i}^{\text{BNE}})\Big] \\ &\text{s.t. } \mathbb{E}_{h_i, \tilde{\mathbf{h}}_{-i}}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i \end{aligned} \tag{6}$$

At BNE, every terminal plays *best response* to the policies of other terminals. A natural question arises is how to obtain solutions for BNE. As we will see next, two conventional formulations of optimal wireless access that follow from (2) and (3), namely optimal FDMA and optimal RA, are BNE for two special cases when terminals have perfectly correlated beliefs ($f_{h_j|\tilde{h}_j}(h) = \delta(h - \tilde{h}_j)$ for all $j \neq i$) and uncorrelated beliefs ($f_{h_j|\tilde{h}_j}(h) = f_{h_j}(h)$ for all $j \neq i$).

A. Perfectly correlated beliefs

When terminals' belief about others is perfect, this implies that each terminal has access to the global channel state. By using global CSI, terminals can cooperate with each other so that collision can be avoided. To do so, we introduce a constraint $\sum_{i=1}^{n} q_i(h_i) \leq 1$ which allows only at most one terminal to transmit in each time slot. Since collision is avoided, the instantaneous rate r_i can be rewritten as $q_i(h_i)C(h_i, p_i(h_i))$. As a result, each terminal can solve the following global optimization problem locally

$$\begin{aligned} \mathbf{p}_i^{\text{FDMA}} = & \operatorname{argmax} \sum_{i=1}^{n} \mathbb{E}_{h_i}\left[q_i(h_i)C(h_i, p_i(h_i))\right] \\ & \text{s.t. } \mathbb{E}_{h_i}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i \quad \forall i \\ & \sum_{i=1}^{n} q_i(h_i) \leq 1 \end{aligned} \tag{7}$$

Notice that problem (7) is the optimal FDMA with single frequency [2]. It can be shown that problems like (7) have null duality gap [9], [10], and the optimal solution is uniquely determined by the optimal solution for its dual problem. Let λ_i^{FDMA} be the optimal dual variable for the dual problem of (7), the optimal solution $\mathbf{p}_i^{\text{FDMA}}$ for (7) is then given by

$$p_i^{\text{FDMA}}(h_i) = \max_{p_i \in \mathcal{P}_i} C(h_i, p_i) - \lambda_i^{\text{FDMA}} p_i, \tag{8}$$

$$q_i^{\text{FDMA}}(h_i) = H\left(g_i^{\text{FDMA}}(h_i) > \max\left\{\max_{j \neq i} g_j^{\text{FDMA}}(h_j), 0\right\}\right) \tag{9}$$

where $H(\cdot)$ is the Heaviside function and $g_i^{\text{FDMA}}(h_i)$ is defined by

$$g_i^{\text{FDMA}}(h_i) = C(h_i, p_i^{\text{FDMA}}(h_i)) - \lambda_i^{\text{FDMA}} p_i^{\text{FDMA}}(h_i). \tag{10}$$

$g_i^{\text{FDMA}}(h_i)$ can be regarded as a local utility function that only depends on terminal i's local CSI h_i. In each time slot, terminals compute their local utilities $g_i^{\text{FDMA}}(h_i)$ and the one with the largest nonnegative utility gets the opportunity to transmit. We show this solution has the following property.

Proposition 1 *Suppose terminals have perfect beliefs about other terminals, i.e., $f_{h_j|\tilde{h}_j}(h) = \delta(h - \tilde{h}_j)$, then $\mathbf{p}_i^{FDMA}$ obtained by* (8) *and* (9) *that solves problem* (7) *is BNE of the game as is defined by* (6).

Proof: When terminal i has perfect beliefs about other terminals, we have $\tilde{\mathbf{h}}_{-i} = \mathbf{h}_{-i}$. As a result, we can write the maximization problem in (5) as

$$\begin{aligned} \mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}) = & \operatorname{argmax} \mathbb{E}_{\mathbf{h}}\left[U(\mathbf{p}_i, \mathbf{p}_{-i})\right] \\ & \text{s.t. } \mathbb{E}_{\mathbf{h}}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i. \end{aligned} \tag{11}$$

To show $\mathbf{p}_i^{\text{FDMA}}$ is BNE, we need to show that given $\mathbf{p}_{-i} = \mathbf{p}_{-i}^{\text{FDMA}}$ the optimal solution for (11) is $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{FDMA}}) = \mathbf{p}_i^{\text{FDMA}}$. We prove this by contradiction. Suppose given $\mathbf{p}_{-i} = \mathbf{p}_{-i}^{\text{FDMA}}$ the optimal solution for (11) is $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{FDMA}}) \neq \mathbf{p}_i^{\text{FDMA}}$. This implies

$$\mathbb{E}_{\mathbf{h}}\left[U(\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{FDMA}}), \mathbf{p}_{-i}^{\text{FDMA}})\right] > \mathbb{E}_{\mathbf{h}}[U(\mathbf{p}_i^{\text{FDMA}}, \mathbf{p}_{-i}^{\text{FDMA}})]. \tag{12}$$

Moreover, the constraint in (11) implies that $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{FDMA}})$ is feasible for the FDMA problem. This contradicts with the fact that $\mathbf{p}_i^{\text{FDMA}}$ is the global maximizer for the FDMA problem (7). Therefore, it must be $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{FDMA}}) = \mathbf{p}_i^{\text{FDMA}}$. ■

B. Uncorrelated beliefs

In this case, terminals only have prior knowledge about channels of others and solves the following optimization problem

$$\begin{aligned} \mathbf{p}_i^{\text{RA}} = & \operatorname{argmax} \mathbb{E}_{\mathbf{h}}\left[U(\mathbf{p})\right] \\ & \text{s.t. } \mathbb{E}_{h_i}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i \quad \forall i \end{aligned} \tag{13}$$

This is known as the optimal random access problem in which terminals operate independently without cooperating with each other and the optimal solution can be found by [3], [4]. We show that $\mathbf{p}_i^{\text{RA}}$ has the following property:

Proposition 2 *Suppose terminals have uncorrelated beliefs about other terminals, i.e., $f_{h_j|\tilde{h}_j}(h) = f_{h_j}(h)$, then $\mathbf{p}_i^{RA}$ that solves problem* (13) *is BNE of the game as is defined by* (6).

Proof: When terminal i has uncorrelated beliefs about other terminals, we have $f_{\mathbf{h}_{-i}|\tilde{\mathbf{h}}_{-i}}(\mathbf{h}_{-i}) = f_{\mathbf{h}_{-i}}(\mathbf{h}_{-i})$. As a result, we can rewrite the maximization problem in (5) as

$$\begin{aligned} \mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}) = & \operatorname{argmax} \mathbb{E}_{\mathbf{h}}\left[U(\mathbf{p}_i, \mathbf{p}_{-i})\right] \\ & \text{s.t. } \mathbb{E}_{h_i}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i. \end{aligned} \tag{14}$$

To show $\mathbf{p}_i^{\text{RA}}$ is BNE, we need to show that given $\mathbf{p}_{-i} = \mathbf{p}_{-i}^{\text{RA}}$ the solution for (14) is $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{RA}}) = \mathbf{p}_i^{\text{RA}}$. To see this is true, note that given $\mathbf{p}_{-i} = \mathbf{p}_{-i}^{\text{RA}}$ problem (13) and (14) are identical. In this case, optimal solution for (13) is the optimal solution for (14), i.e., $\mathbf{p}_i^{\text{BR}}(\mathbf{p}_{-i}^{\text{RA}}) = \mathbf{p}_i^{\text{RA}}$. ■

In summary, when terminals have perfectly correlated and uncorrelated beliefs about other terminals it is possible to formulate the problem as FDMA or RA and find corresponding optimal solutions. Interestingly, optimal solutions for these two different problems both coincide with the BNE defined by (6). In other words, BNE can be used as a unified framework to model multiple access channels. Indeed, from an individual terminal's point of view the only difference between FDMA and RA is the knowledge about other channels which is captured by the belief in BNE. However, for intermediate cases where beliefs are neither perfectly correlated nor uncorrelated finding the BNE solution is not an easy task because the objective in (6) evolves policies of other terminals which is unknown to terminal i. Next, we develop algorithms that solve (6) approximately.

IV. Cognitive Access Algorithms

The key in designing an algorithm for solving (6) is to model the actions of other terminals. Let $\tilde{q}_j(\cdot)$ be the modeling of terminal j's action. The easiest way of modeling terminal j is to assume that $\tilde{h}_j$ is the true channel gain and terminal j makes decision based on it, i.e., $\tilde{q}_j(\tilde{h}_j)$. The intuition behind this modeling is that each terminal finds a strategy that is optimal when their belief about other terminals are perfect as in the case of optimal FDMA (7). As a result, terminal i solves the following problem locally

$$\begin{aligned} \{\mathbf{p}_i^{\text{CA}}, \tilde{\mathbf{p}}_{-i}^{\text{CA}}\} = & \max \mathbb{E}_{h_i}\left[q_i(h_i)C(h_i, p_i(h_i))\right] \\ & + \sum_{j=1, j \neq i}^{n} \mathbb{E}_{\tilde{h}_j}\left[\tilde{q}_j(\tilde{h}_j)C(\tilde{h}_j, \tilde{p}_j(\tilde{h}_j))\right] \\ & \text{s.t. } \mathbb{E}_{h_i}\left[q_i(h_i)p_i(h_i)\right] \leq p_i^{\text{avg}}, \mathbf{p}_i \in \mathcal{P}_i \\ & \mathbb{E}_{\tilde{h}_j}\left[\tilde{q}_j(\tilde{h}_j)\tilde{p}_j(\tilde{h}_j)\right] \leq p_j^{\text{avg}}, \tilde{\mathbf{p}}_j \in \mathcal{P}_j \quad \forall j \neq i \\ & q_i(h_i) + \sum_{j=1, j \neq i}^{n} \tilde{q}_j(\tilde{h}_j) \leq 1. \end{aligned} \tag{15}$$

Note that problem (15) is the same as the one for FDMA (7) except that $\mathbf{p}_{-i}$ is replaced by $\tilde{\mathbf{p}}_{-i}$. Suppose the optimal dual variable for the dual problem of (15) is λ_i^{CA}, then the optimal solution for (15) is given by

$$p_i^{\text{CA}}(h_i) = \max_{p_i \in \mathcal{P}_i} C(h_i, p_i) - \lambda_i^{\text{CA}} p_i, \tag{16}$$

$$q_i^{\text{CA}}(h_i) = H\left(g_i^{\text{CA}}(h_i) > \max\left\{\max_{j \neq i} g_j^{\text{CA}}(h_j), 0\right\}\right), \tag{17}$$

where $g_i^{\text{CA}}(h_i)$ is given by

$$g_i^{\text{CA}}(h_i) = C(h_i, p_i^{\text{CA}}(h_i)) - \lambda_i^{\text{CA}} p_i^{\text{CA}}(h_i) \tag{18}$$

In practice, each terminal solves (15) offline locally to find the optimal multiplier λ_i^{CA}. Based on λ_i^{CA}, terminal i makes transmission and power allocation decisions according to (16)-(18). When terminal i is scheduled for transmission, it assumes other terminals are silent, i.e., $\tilde{q}_j = 0$ for all $j \neq i$, and the attained instantaneous transmission rate is $C(h_i, p_i^{\text{CA}}(h_i))$. However, this may not be true since the modeled action $\tilde{q}_j$ may be different from the real action q_j of terminal j which is obtained by terminal j solving another maximization problem like (15). Therefore, collision may happen when all terminals operate by following (16)-(18). Before proceeding to show the performance of the proposed algorithm, we first prove the next properties.

Proposition 3 *Let λ_i^{CA} and λ_i^{FDMA} be optimal multipliers associated with terminal i's average power constraints in* (15) *and* (7)*, then $\lambda_i^{CA} = \lambda_i^{FDMA}$. Let $p_i^{CA}(h_i)$ and $p_i^{FDMA}(h_i)$ be the optimal power allocations for terminal i that solve problems* (15) *and* (7)*, then $p_i^{CA}(h_i) = p_i^{FDMA}(h_i)$.*

Proof: Since $\tilde{h}_j$ and h_j have the same pdf, the dual problems of (15) and (7) are the same. Therefore, the optimal dual variables for their dual problems are the same, i.e., $\lambda_i^{\text{CA}} = \lambda_i^{\text{FDMA}}$.

To see $p_i^{\text{CA}}(h_i) = p_i^{\text{FDMA}}(h_i)$, observe that $p_i^{\text{CA}}(h_i)$ and $p_i^{\text{FDMA}}(h_i)$ are functions of λ_i^{CA} and λ_i^{FDMA}, respectively [cf. (16) and (8)]. Since we have shown $\lambda_i^{\text{CA}} = \lambda_i^{\text{FDMA}}$, it follows that $p_i^{\text{CA}}(h_i) = p_i^{\text{FDMA}}(h_i)$. This completes the proof. ■

Let the expected utilities achieved by $\mathbf{p}_i^{\text{CA}}$ and $\mathbf{p}_i^{\text{FDMA}}$ are $\bar{U}^{\text{CA}}$ and $\bar{U}^{\text{FDMA}}$, respectively. We show that the performance of the proposed algorithm for the following two cases.

Proposition 4 *If terminals have perfectly correlated belief, i.e., $f_{h_j|\tilde{h}_j}(h) = \delta(h - \tilde{h}_j)$, the expected utility achieved by $\mathbf{p}_i^{CA}$ is the same as that of FDMA, i.e., $\bar{U}^{CA} = \bar{U}^{FDMA}$.*

Proof: When terminals have perfectly correlated belief, $\tilde{h}_j = h_j$. This implies that the problem (15) solved by the proposed algorithm is the same as the problem (7) solved by FDMA. Therefore, the performance achieved by both algorithms are identical, i.e., $\bar{U}^{\text{CA}} = \bar{U}^{\text{FDMA}}$. ■

Proposition 5 *If terminals have uncorrelated belief, i.e., $f_{h_j|\tilde{h}_j}(h) = f_{h_j}(h)$, the expected utility achieved by $\mathbf{p}_i^{CA}$ is a fraction of that of FDMA, i.e., $\bar{U}^{CA} = \beta\bar{U}^{FDMA}$, where $\beta \in [0, 1]$ is a constant. In particular, when channels are symmetric, $\beta = 1/e$ as the number of terminals goes to infinity.*

Proof: In both FDMA and the proposed algorithm, transmission decision for each terminal is made by comparing its local utility with maximum of the utilities of others [cf. (9) and (17)]. Therefore, given local channel h_i terminal i transmits with certain probability under both policies. Let $\alpha_i^{\text{FDMA}}(h_i)$ and $\alpha_i^{\text{CA}}(h_i)$ be the probability that terminal i transmit in the proposed algorithm and FDMA, respectively. According to (9) and (17), $\alpha_i^{\text{FDMA}}(h_i)$ and $\alpha_i^{\text{CA}}(h_i)$ are given by

$$\alpha_i^{\text{FDMA}}(h_i) = \Pr\left(g_i^{\text{FDMA}}(h_i) > \max\left\{\max_{j \neq i} g_j^{\text{FDMA}}(h_j), 0\right\}\right) \tag{19}$$

$$\alpha_i^{\text{CA}}(h_i) = \Pr\left(g_i^{\text{CA}}(h_i) > \max\left\{\max_{j \neq i} g_j^{\text{CA}}(\tilde{h}_j), 0\right\}\right). \tag{20}$$

By definition, g_j^{FDMA} and g_j^{CA} are functions of the local channel h_i, corresponding optimal multipliers and optimal power allocations [cf. (10) and (18)]. Since $\lambda_i^{\text{CA}} = \lambda_i^{\text{FDMA}}$ and $p_i^{\text{CA}}(h_i) = p_i^{\text{FDMA}}(h_i)$ by Proposition 3, for the same channel we have $g_j^{\text{CA}}(h_j) = g_j^{\text{FDMA}}(h_j)$. Moreover, since $\tilde{h}_j$, h_j have the same distribution, we conclude that $\alpha_i^{\text{CA}}(h_i) = \alpha_i^{\text{FDMA}}(h_i) = \alpha_i(h_i)$. As a result, the expected utility achieved by $\mathbf{p}_i^{\text{FDMA}}$ is

$$\bar{U}^{\text{FDMA}} = \sum_i \mathbb{E}_{h_i}\left[C(h_i), p_i^{\text{FDMA}}(h_i))\alpha_i(h_i)\right], \tag{21}$$

and the expected utility achieved by $\mathbf{p}_i^{\text{CA}}$ is

$$\bar{U}^{\text{CA}} = \sum_i \mathbb{E}_{h_i}[C(h_i), p_i^{\text{CA}}(h_i))\alpha_i(h_i)] \prod_{j \neq i}\left[1 - \mathbb{E}_{h_j}[\alpha_j(h_j)]\right], \tag{22}$$

where the product $\prod_{j \neq i}\left[1 - \mathbb{E}_{h_j}[\alpha_j(h_j)]\right]$ in (22) represents the probability that all terminals other than terminal i are silient. Define $\beta_i := \prod_{j \neq i}\left[1 - \mathbb{E}_{h_j}[\alpha_j(h_j)]\right]$ and rewrite (22) as

$$\bar{U}^{\text{CA}} = \sum_i \beta_i \mathbb{E}_{h_i}[C(h_i), p_i^{\text{CA}}(h_i))\alpha_i(h_i)]. \tag{23}$$

Let $\beta_{\min} = \min_i \beta_i$ and $\beta_{\max} = \max_i \beta_i$, then there must exist $\beta \in [\beta_{\min}, \beta_{\max}]$ such that

$$\begin{aligned}\bar{U}^{\text{CA}} &= \beta \sum_i \mathbb{E}_{h_i}[C(h_i), p_i^{\text{CA}}(h_i))\alpha_i(h_i)] \\ &= \beta \sum_i \mathbb{E}_{h_i}[C(h_i), p_i^{\text{FDMA}}(h_i))\alpha_i(h_i)],\end{aligned} \tag{24}$$

where the second equality follows from the fact that $p_i^{\text{CA}}(h_i) = p_i^{\text{FDMA}}(h_i)$. Substitute (21) into (24) yields

$$\bar{U}^{\text{CA}} = \beta \bar{U}^{\text{FDMA}}. \tag{25}$$

When the channels are symmetric, it can be show that $\beta = (1 - 1/n)^{n-1}$. Since $\lim_{n \to \infty}(1 - 1/n)^{n-1} = 1/e$, $\bar{U}^{\text{CA}}$ goes to $1/e \cdot \bar{U}^{\text{FDMA}}$ as n goes to infinity. ■

V. Numerical Results

Numerical tests are conducted to evaluate performance of the proposed algorithm. We assume local channel h_i follows a complex Gaussian distribution $\mathcal{CN}(0, 2)$ and the imperfect channel estimation $\tilde{h}_j$ is modeled by (4). Assume capacity achieving codes are used for transmission and the capacity function takes the form of $C(h_i, p_i(h_i)) = \log(1 + h_i p_i(h_i)/N_0)$ where N_0 is normalized noise power. Without loss of generality, we assume $N_0 = 1$. The average power budget is 1 for all terminals, i.e. $p_i^{\text{avg}} = 1$ for all i. We conducted simulations for different total number of terminals $n \in \{10, 20, 30, 40, 50\}$ and different correlation coefficient $\rho \in \{0, 0.1, 0.2, \cdots, 1\}$. In the simulation,

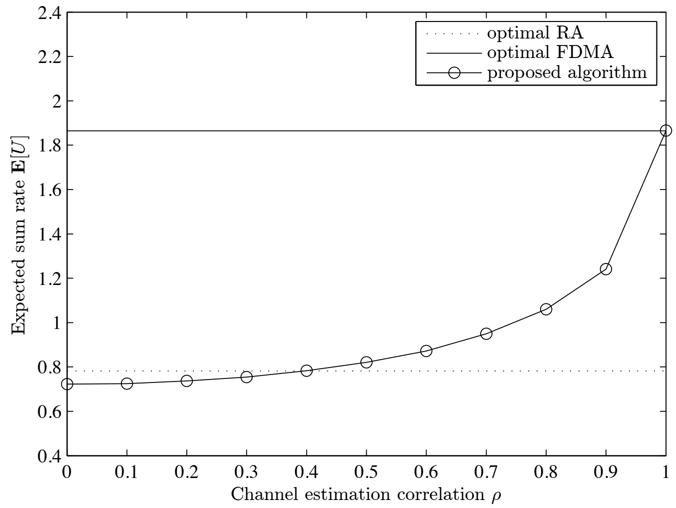

Fig. 1. Comparison of the expected sum rate utility achieved by the optimal FDMA ($\rho = 1$), the optimal RA ($\rho = 0$) and the proposed algorithm ($\rho \in \{0, 0.1, 0.2, \cdots, 1\}$). The total number of terminals is $n = 10$.

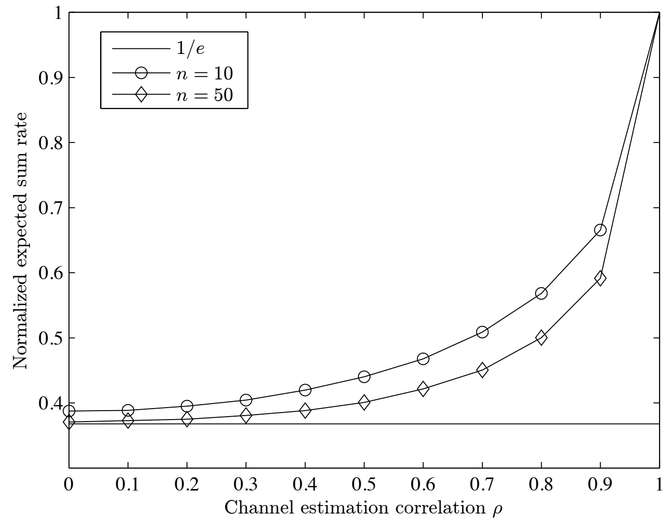

Fig. 2. The expected sum rate utility achieved by the proposed algorithm normalized by that achieved by the optimal FDMA for $n = 10$ and $n = 50$. The horizontal line is $1/e \approx 0.368$.

stochastic subgradient descent algorithm [10] is used to iteratively compute the primal and dual variables. Optimal solutions for FDMA (when $\rho = 1$) and RA (when $\rho = 0$) are also computed.

Fig. 1 compares the expected sum rate achieved by optimal FDMA (when $\rho = 1$), optimal RA (when $\rho = 0$) and the proposed algorithm (when $\rho \in \{0, 0.1, 0.2, \cdots, 1\}$) for $n = 10$. When $\rho = 1$ the expected utility achieved by the proposed algorithm is 1.87 which is equal to that achieved by the optimal FDMA. This corroborates the results in Proposition 4. As the correlation ρ decreases, the performance of the proposed algorithm degrades gracefully and achieves an expected utility of 0.72 when $\rho = 0$. This is very close to the expected utility achieved by the optimal RA (0.78).

In Proposition 5, it is shown for symmetric channel the expected utility achieved by the proposed algorithm for $\rho = 0$ is about $1/e$ of the utility achieved by optimal FDMA as n goes to infinity. To show this is true, we normalized the expected utility achieved by the proposed algorithm by the utility achieved by the optimal FDMA. Fig. 2 shows the normalized expected utility achieved by the proposed algorithm for $n = 10$ and $n = 50$. The horizontal line is $1/e \approx 0.368$. Indeed, for $n = 50$ the normalized utility converges to $1/e$ when $\rho = 0$. Moreover, notice that the normalized utility decreases as n increases. This is because when n increases the imperfect channel estimation is more likely to cause collisions.

VI. Conclusion

We considered algorithms that adapts transmission policy to the random channel states in multiple access fading channels where each terminal has a different belief about the channel states. In this setting, we formulated the problem as a Bayesian game in which each terminal maximizes the expected utility based on its belief subject to an average power constraint. We showed that optimal solutions for both FDMA and RA are Bayesian Nash Equilibrium (BNE) points of the formulated game. Therefore, the proposed game theoretic formulation can be regarded as general framework for multiple access channels. Moreover, a cognitive algorithm is developed to solve the problem approximately. Numerical results show that the proposed algorithm achieves performance equal to as the optimal FDMA when the channel estimation is perfectly correlated and performance very close to the optimal RA when channel estimation is uncorrelated.[1].

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