

DISTRIBUTED DEMAND SIDE MANAGEMENT OF HETEROGENEOUS RATIONAL CONSUMERS IN SMART GRIDS WITH RENEWABLE SOURCES

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ABSTRACT

We consider a demand side management model in which the power provider adopts an adaptive pricing strategy that depends on fluctuations in renewable sources and consumption behavior of customers with heterogeneous marginal utilities in the smart grid. Given the adaptive pricing strategy, we formulate the power consumption behavior of customers as a repeated noncooperative game with incomplete information. We provide an explicit characterization of unique Bayesian Nash equilibrium strategy in terms of individual marginal utilities. The rational behavior is also characterized in a communication scheme where smart meters exchange consumption levels with neighboring meters. A local algorithm that computes equilibrium consumption and propagates beliefs is presented when the network is known. Simulation results show that communication is beneficial for welfare and that power provider can lower the peak-to-average ratio of total consumption by adjusting its target profit ratio.

Index Terms— Noncooperative game theory, smart grid, distributed demand side management, renewable energy.

1. INTRODUCTION

The matching of power production to power consumption is a complex problem in conventional energy grids. This problem is exacerbated by the introduction of renewable sources, which, by their very nature, exhibit significant output fluctuations. This problem can be mitigated with the introduction of a system of smart meters. Smart meters control the power consumption of customers by managing the energy cycles of various devices while also enabling information exchanges between customers and the power provider as well as between customers themselves [1, 2]. The web of information between customers' meters, and between meters and the power provider can be combined with sophisticated pricing strategies so as to encourage a better match between power production and consumption [3–9]. The effort of power providers to regulate the consumption of end users is referred to as demand side management (DSM) [10]. In this paper, we study the rational consumer behavior in a repeated non-cooperative game with incomplete information when the power provider employs an adaptive pricing policy. The adaptive price depends on renewable source output and total power consumption, and hence incentivizes customers with heterogeneous preferences to anticipate behavior of others and be aware of their influence on price.

The provider regulates consumption by determining price policy parameters based on its estimate of consumption and renewable power generation (Sections 2.1 and 3.1). The provider broadcasts its policy and estimate of renewable power generation to the consumers at the beginning of the period. Customers maximize expected payoff that depends on self preferences, total consumption and renewable power generation with respect to public information and their beliefs on others' preferences (Section 2.2). We assume that the customer's power control scheduler can adjust the load consumption between time slots according to his preferences. That is, we are interested in modeling consumption behavior for

shiftable appliances, e.g., electric vehicles, electronic devices, air conditioners etc. [5, 6]. We explicitly characterize individual consumption at each time with respect to self preference by using Bayesian Nash equilibrium (BNE) as the solution concept when the preferences come from jointly normal distribution (Section 3) – see [11, 12] for a survey of game theoretic models in DSM. The characterization shows the effects of expected renewable power output and pricing policy parameter on price.

In addition, we consider a communication scheme in which customers exchange their consumption levels after each time slot with their neighbors (Section 4). Similar to the cases in [7, 10], information exchange among entities is done via power control schedulers. For this model, we explicitly characterize the equilibrium behavior of customers and provide an algorithm to compute equilibrium and propagate beliefs based on local information using the results from [13]. Finally, we provide numerical experiments exploring the behavior of welfare, consumption, price, and provider's realized profit with respect to population's preference distribution and price policy parameters in settings with and without communication (Section 5). Numerical experiments show that communication among neighboring users is beneficial to welfare when the preferences are correlated. We further discuss how the pricing policy can be used to reduce peak-to-average ratio or total consumption.

2. DISTRIBUTED DSM WITH RENEWABLE ENERGY

A power provider oversees a DSM model with N customers. Customers each equipped with a power consumption scheduler are characterized by their individual load consumption l_{ih} defined as the power consumed by customer $i \in \mathcal{N}$ at time slot $h \in \mathcal{H} := \{1, \dots, H\}$. Accordingly, we represent the total consumption of the population at time slot h with $L_h := \sum_{i \in \mathcal{N}} l_{ih}$. In order to be able to be responsive to changing conditions in the environment, e.g. resource prices, consumption preferences, the provider divides the day into K time zones t_1, t_2, \dots, t_K . Specifically, the time zone k is a batch of time slots starting at $h_k^s \in \mathcal{H}$ and ending at $h_k^e \in \mathcal{H}$, i.e., $t_k := [h_k^s, h_k^e)$. The time zones do not overlap, that is, $h_{k-1}^e = h_k^s$ for $k \in \{2, \dots, K-1\}$ and $h_1^s = 1$ and $h_K^e = H$.

2.1. Power provider model

For a time slot $h \in t_k$ the total power consumption L_h results in the power provider incurring a production cost of $C_k(L_h)$ units. Observe that the production cost function $C_k(L_h)$ depends on the time zone k and the total power produced L_h . When the generation cost per unit is constant, $C_k(L_h)$ is a linear function of L_h . More often, increasing loads L_h result in increasing unit costs as more expensive energy sources are brought online. This results in superlinear cost functions $C_k(L_h)$ with a customary model being the quadratic form

$$C_k(L_h) = \kappa_k L_h^2, \quad (1)$$

for a given constant $\kappa_k > 0$ that depends on the day's time zone k . The cost in (1) has been experimentally validated for thermal generators [14] and is otherwise widely accepted as a reasonable approximation [4, 10, 15]. This paper adopts the model in (1).

The provider utilizes an adaptive pricing strategy whereby customers

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are charged a slot-dependent price p_h that varies linearly with the total power consumption L_h . To incorporate renewables into the pricing strategy we further introduce a random variable $\omega_k \in \mathbb{R}$ that depends on the amount of power produced by renewable sources and set the per unit power price at time slot $h \in t_k$ as

$$p_h(L_h; \omega_k) = \beta_k(L_h + \omega_k), \quad (2)$$

where $\beta_k > 0$ is a policy parameter to be determined by the provider. The parameter β_k may depend on the day's time zone k . The random variable ω_k is such that $\omega_k = 0$ when renewable sources operate at their nominal benchmark capacity. If realized production exceeds this benchmark the provider agrees to set $\omega_k < 0$ to discount the energy price and share the windfall brought about by favorable weather conditions. If realized production is below benchmark the provider sets $\omega_k > 0$ to reflect the additional charge on the customers. The specific dependence of ω_k with the realized energy production and the policy parameter β_k , are part of the supply contract between the provider and its customers.

The provider's realized revenue at time slot $h \in t_k$ are obtained by multiplying the total consumption L_h by the unit price in (2),

$$R_k(L_h) = L_h p_h(L_h; \omega_k). \quad (3)$$

Further observe that the provider's rate of return for the time slot is given by the ratio $R_k(L_h)/C_k(L_h)$.

A fundamental observation here is that the prices $p_h(L_h; \omega_k)$ in (2) become known *after* the end of the time slot h . This is because prices depend on the total power consumption L_h and the value of ω_k which is determined by the amount of renewable energy produced in the time zone to which the slot belongs. Both of these quantities are unknown a priori. Regarding the parameter ω_k we assume that the provider uses a model on the renewable power generation – see, e.g., [3] for the prediction of wind generation – to estimate the value of ω_k at the beginning of the time zone k . The corresponding probability distribution P_{ω_k} is made available to all customers at the beginning of the time zone. The provider's goal is to rely on this belief and on a model of consumer behavior to obtain a target expected rate of return – see Section 3.1. Henceforth, we use E_{ω_k} to denote expectations with respect to the belief P_{ω_k} and $\bar{\omega}_k := E_{\omega_k}[\omega_k]$ to denote the mean of the distribution P_{ω_k} .

2.2. Power consumer model

The consumption preferences of users are determined by constants $g_{ik} > 0$ that are possibly different across customers and time zones. When user i consumes l_{ih} units of power at time slot h we assume that it receives the linear utility $g_{ik}l_{ih}$. For each unit of power consumed the provider charges the price $p_h(L_h; \omega_k)$ which results in user i incurring the total cost $l_{ih}p_h(L_h; \omega_k)$. It also charges a quadratic penalty $\alpha_k l_{ih}^2$ to discourage excessive consumption. Note that the constant α_k may change across time zones k but is the same for all consumers. The utility of user i is then given by the difference between the consumption return $g_{ik}l_{ih}$, the energy cost $l_{ih}p_h(L_h; \omega_k)$, and the overconsumption penalty $\alpha_k l_{ih}^2$,

$$u_{ih}(l_{ih}, L_h; g_{ik}, \omega_k) = -l_{ih}p_h(L_h; \omega_k) + g_{ik}l_{ih} - \alpha_k l_{ih}^2. \quad (4)$$

Using the expressions for prices in (2) and L_h we can write the payoff in (4) as

$$\begin{aligned} & u_{ih}(l_{ih}, l_{-ih}; g_{ik}, \omega_k) \\ &= -l_{ih} \left[\beta_k \left(\sum_{j \in \mathcal{N}} l_{jh} + \omega_k \right) \right] + g_{ik}l_{ih} - \alpha_k l_{ih}^2, \end{aligned} \quad (5)$$

where we have also rewritten the utility of user i as $u_{ih}(l_{ih}, L_h; g_{ik}, \omega_k) = u_{ih}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ to emphasize the fact that it depends on the consumption $l_{-ih} := \{l_{jh}\}_{j \neq i}$ of other agents. Note that if the provider's policy price is set to $\beta_k = 0$, the utility of agent i is maximized by $l_{ih} = g_{ik}/2\alpha_k$. Thus, the overconsumption penalty results in users

consuming a finite amount of power even when power is free of charge – see [15] for a similar formulation.

The utility of user i depends on the powers l_{-ih} that are consumed by other users in the current slot. These powers are unknown to user i . Further note that the power consumptions l_{-ih} of other users depend partly on their marginal utilities $g_{-ik} := \{g_{jk}\}_{j \neq i}$. These are assumed to be also *unknown* to user i . Rather, we assume there is a probability distribution $P_{\mathbf{g}_k}(\mathbf{g}_k)$ on the vector of marginal utilities $\mathbf{g}_k := [g_{1k}, \dots, g_{Nk}]^T$ from where self preferences are drawn. This probability distribution is known to all agents. We further assume that $P_{\mathbf{g}_k}$ is normal with mean $\bar{g}_k \mathbf{1}$ where $\bar{g}_k > 0$ and covariance matrix Σ_k ,

$$P_{\mathbf{g}_k}(\mathbf{g}_k) = \mathcal{N}(\mathbf{g}_k; \bar{g}_k \mathbf{1}, \Sigma_k) \quad (6)$$

Let the operator $E_{\mathbf{g}_k}$ signify expectation with respect to the distribution $P_{\mathbf{g}_k}$ and $\sigma_{ij}^k := ((\Sigma_k))_{ij}$ denote the (i, j) th entry of the covariance matrix Σ_k . Having mean $\bar{g}_k \mathbf{1}$ means that all customers have equal average preferences in that $E_{\mathbf{g}_k}(g_{ik}) = \bar{g}_k$ for all i . If $\sigma_{ij}^k = 0$ for some pair $i \neq j$, it means that the marginal utilities of these customers are uncorrelated. In general, $\sigma_{ij}^k \neq 0$ to account for correlated preferences due to, e.g., common weather. We further assume that marginal utilities \mathbf{g}_k and \mathbf{g}_l for different time zones $k \neq l$ are independent.

The probability distributions P_{ω_k} and $P_{\mathbf{g}_k}$ and the pricing parameters α_k and β_k are common knowledge among users. The pricing parameters α_k and β_k are announced by the provider at the beginning of the time zone. The renewable energy parameter ω_k is unknown until the end of the time zone but the provider's belief P_{ω_k} on this parameter is also announced. The probability distribution $P_{\mathbf{g}_k}$ in (6) is known to all agents by assumption. Customer i also knows the realized value of his utility marginal yield g_{ik} . His goal is to maximize the utility $u_{ih}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ in (5) given its partial knowledge of the renewable energy parameter ω_k and the power consumptions l_{-ih} of other agents. This maximization requires a model of behavior for other agents that comes in the form of a BNE that we introduce in the following section.

3. CUSTOMERS' BAYESIAN GAME

User i 's load consumption at time $h \in t_k$ is determined by his *belief* q_{ih} and *strategy* s_{ih} . The belief of i is a conditional probability distribution on \mathbf{g}_k given $g_{ik}, q_{ih}(\cdot) := P_{\mathbf{g}_k}(\cdot | g_{ik})$. In order to second guess the consumption of other users, user i forms beliefs on their marginal utilities given the common prior $P_{\mathbf{g}_k}$ and self marginal utilities up to time zone k $\{g_{im}\}_{m=1, \dots, k}$. Observe that self marginal utilities of previous time zones $\{g_{im}\}_{m < k}$ are not relevant to belief at time zone k as they are independent from the marginal utilities at time zone k . Further note that user i 's belief is static over the time horizon as he receives no other information about the marginal utilities of others. User i 's load consumption at time $h \in t_k$ is determined by his strategy which is a complete contingency plan that maps any possible local observation that he may have to his consumption. In particular, user i 's best response strategy is to maximize expected utility with respect to his belief q_{ih} given strategies of other customers $\mathbf{s}_{-ih} := \{s_{jh}\}_{j \neq i}$,

$$BR(g_{ik}; \mathbf{s}_{-ih}) = \operatorname{argmax}_{l_{ih}} E_{\omega_k} \left[E_{\mathbf{g}_k} [u_{ih}(l_{ih}, \mathbf{s}_{-ih}; g_{ik}, \omega_k) | g_{ik}] \right]. \quad (7)$$

Since the utility of customer i is strictly concave quadratic function of l_{ih} as per (5), the same is true for his conditional expected utility which we maximize in (7). Hence, we can rewrite the best response strategy in (7) by taking the derivative of the conditional expected utility with respect to l_{ih} , equate the resultant to zero and solve for l_{ih} ,

$$BR(g_{ik}; \mathbf{s}_{-ih}) = \frac{1}{2(\beta_k + \alpha_k)} \left(g_{ik} - \beta_k (\bar{\omega}_k + \sum_{j \neq i} E_{\omega_k} [E_{\mathbf{g}_k} [s_{jh} | g_{ik}]]]) \right) \quad (8)$$

A BNE strategy profile $\mathbf{s}^* = \{s_{ih}\}_{i \in \mathcal{N}, h \in \mathcal{H}}$ at time $h \in t_k$ is a strategy in which each customer maximizes expected utility with respect to his own belief given that other customers are playing with respect to BNE strategy. Equivalently, BNE strategy is one in which users play best response strategy as per (8) to best response strategies of other users – see [13, 16] for a detailed explanation. As a result, the BNE strategy is defined with the following fixed point equations

$$s_{ih}^*(g_{ik}) = BR(g_{ik}; \mathbf{s}_{-ih}^*) \quad (9)$$

for all $i \in \mathcal{N}$, $h \in t_k$, and g_{ik} . Using the definition in (9), the following result characterizes the unique linear BNE strategy at $h \in t_k$.

First we define the pairwise inference matrix $S(\Sigma_k)$ as,

$$[[S(\Sigma_k)]]_{ii} = 0, \quad [[S(\Sigma_k)]]_{ij} = \sigma_{ji}^k / \sigma_{jj}^k \quad \forall i \in \mathcal{N}, j \in \mathcal{N} \setminus i \quad (10)$$

Theorem 1¹ Consider the game defined by the payoff in (5) at time $h \in t_k$. Let the information given to customer i be his marginal utility g_{ik} , the common normal prior on marginal utilities $P_{\mathbf{g}_k}$ as per (6) and prior on renewable power generation P_{ω_k} . Then the unique BNE strategy of customer i is linear in signals $\bar{\omega}_k, \bar{g}_k, g_{ik}$ and is given by,

$$s_{ih}^*(g_{ik}) = a_{ik} + b_{ik} g_{ik} \quad (11)$$

where $\mathbf{a}_k = [a_{1k}, \dots, a_{Nk}]^T$ and $\mathbf{b}_k = [b_{1k}, \dots, b_{Nk}]^T$ are given by

$$\mathbf{a}_k = \frac{\bar{g}_k - \bar{\omega}_k \beta_k}{2(\alpha_k + \beta_k)((N-1)\rho_k + 1)} \mathbf{1} - \bar{g}_k \frac{\rho_k}{\beta_k} \mathbf{d}(\Sigma_k), \quad \mathbf{b}_k = \frac{\rho_k}{\beta_k} \mathbf{d}(\Sigma_k), \quad (12)$$

and

$$\rho_k = \frac{\beta_k}{2(\beta_k + \alpha_k)}, \quad \mathbf{d}(\Sigma_k) = (\mathbf{I} + \rho_k \mathbf{S}(\Sigma_k))^{-1} \mathbf{1} \quad (13)$$

Theorem 1 shows that there exists a unique BNE strategy that is linear in self marginal utility g_{ik} . This is a direct consequence of the fact that the best response strategy (8) is a linear function of strategies of other users and the normal prior on marginal utilities in (6). From equilibrium action in (11), we observe that the estimated effect of renewable power $\bar{\omega}_k$ has a decreasing effect on individual consumption. This is expected since increasing $\bar{\omega}_k$ implies an expected increase in the price which lowers the incentive to consume. Observe that the BNE strategy (11) does not contain any time slot dependent parameter hence the consumption level of an individual is fixed for all $h \in t_k$. This is due to the fact that users do not receive any new information within a time zone.

Further observe that the strategy coefficients a_{ik} and b_{ik} do not depend on information specific to customer i . A consequence of this observation is that the power provider knows the strategy functions of all the users via the action coefficient equations in (12). On the other hand, the realized load consumption l_{ih} is a function of realized marginal utility g_{ik} , i.e., $l_{ih} = s_{ih}^*(g_{ik})$. Hence, knowing the strategy function does not imply that the provider knows the consumption level of each other. Nevertheless, the provider can use the BNE strategies of users to estimate the expected total consumption in order to compute its expected rate of return as we present in the following section.

3.1. Producer adaptive pricing

The provider determines its policy parameter β_k so that it expects to achieve a target rate of return r_k^* . The expected rate of return ratio is

obtained by dividing expected revenue with expected cost where the cost and the revenue are defined as in (1) and (3), respectively. Given that customers consume with respect to the BNE strategy $\{\sigma_{ih}^*\}_{i \in \mathcal{N}, h \in t_k}$, the provider solves the following equation for β_k to attain the desired profit ratio at $h \in t_k$,

$$\frac{E[R_h(L_h^*(\beta_k))]}{E[C_h(L_h^*(\beta_k))]} = r_k^* \quad (14)$$

where $L_h^*(\beta_k) := \sum_{i \in \mathcal{N}} s_{ih}^*(g_{ik}; \beta_k)$ is the total load when customers use BNE strategy. We include β_k as a parameter at customer i 's BNE strategy to indicate that from the perspective of the provider, the BNE strategy of agent i is a function β_k . Using (1) and (3), the above equation can be equivalently be written as follows

$$(\kappa_k r_k^* - \beta_k) E[(L_h^*(\beta_k))^2] - \beta_k \bar{\omega}_k E[L_h^*(\beta_k)] = 0 \quad (15)$$

Given that customers follow BNE strategy in (11), the provider can compute the expectations of total load and total load squared in (15) – see [17]. The expected rate of return is the same at all time slots within a time zone due to the fact that BNE strategy profile is fixed within a time zone.

4. REPEATED GAME FOR COOPERATING CUSTOMERS

We consider a communication scheme where power control schedulers are interconnected via network represented by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$ and customer i observes consumption levels of his neighbors in the network $\mathcal{N}_i := \{j \in \mathcal{N} : (j, i) \in \mathcal{E}\}$ after each time slot. Given the communication setup, the information of customer i at time slot $h \in t_k$ contains his preferences $g_{i,1:k} := \{g_{im}\}_{m=1,\dots,k}$ and the consumption of his neighbors up to time $h-1$, $l_{\mathcal{N}_i,1:h-1} := \{l_{jt}\}_{j \in \mathcal{N}_i, t=1,\dots,h-1}$, that is, $I_{ih} = \{g_{i,1:k}, l_{\mathcal{N}_i,1:h-1}\}$. We assume that the information exchange is among power consumption schedulers which keeps the information private. We further assume that the schedulers know the network structure.

Upon observing actions of his neighbors' consumption, i learns about the consumption preferences of other users which he can use to better estimate the total consumption in future steps. For this customer i keeps an estimate of the marginal utilities of all the customers. First, define the marginal utility vector augmented with mean \bar{g}_k , $\tilde{\mathbf{g}}_k := [\mathbf{g}_k^T, \bar{g}_k]^T$. The mean and error covariance matrix of i 's belief q_{ih} at time $h \in t_k$ is denoted by $E[\tilde{\mathbf{g}}_k | I_{ih}]$ and $\mathbf{M}_{\tilde{\mathbf{g}}_k, \tilde{\mathbf{g}}_k}^i(h) := E[(\tilde{\mathbf{g}}_k - E[\tilde{\mathbf{g}}_k | I_{ih}])(\tilde{\mathbf{g}}_k - E[\tilde{\mathbf{g}}_k | I_{ih}])^T]$, respectively. Next result shows that at each time slot of the game, strategies of users are linear weighting of their mean estimate of $\tilde{\mathbf{g}}_k$ where the weights are obtained by solving a set of linear equations.

Theorem 2 Consider the repeated Bayesian game defined by the payoffs in (5). Let the information of customer i at time $h \in t_k$ be $I_{ih} = \{g_{i,1:k}, l_{\mathcal{N}_i,1:h-1}\}$. Given the normal prior on marginal utilities in (6), the mean estimate of user i at time $h \in t_k$ can be written as a linear combination of $\tilde{\mathbf{g}}_k$, that is, $E[\tilde{\mathbf{g}}_k | I_{ih}] = \mathbf{T}_{ih} \tilde{\mathbf{g}}_k$ where $\mathbf{T}_{ih} \in \mathbb{R}^{N+1 \times N+1}$ for all $h \in t_k$, and the unique equilibrium strategy for i is linear in his estimate of the marginal utilities,

$$s_{ih}^* = \mathbf{v}_{ih}^T E[\tilde{\mathbf{g}}_k | I_{ih}] + r_{ih} \quad (16)$$

where $\mathbf{v}_{ih} \in \mathbb{R}^{N+1 \times 1}$ and $r_{ih} \in \mathbb{R}$ are strategy coefficients. The strategy coefficients are calculated by solving the following set of equations

$$\mathbf{v}_{ih}^T \mathbf{T}_{ih}^T + \rho_k \sum_{j \in \mathcal{N} \setminus i} \mathbf{v}_{jh}^T \mathbf{T}_{jh}^T = \frac{\rho_k}{\beta_k} \mathbf{e}_i \quad \forall i \in \mathcal{N}, \quad (17)$$

$$r_{ih} + \rho_k \sum_{j \in \mathcal{N} \setminus i} r_{jh} = -\rho_k \bar{\omega}_k \quad \forall i \in \mathcal{N} \quad (18)$$

where $\rho_k := \beta_k / (2(\beta_k + \alpha_k))$ and $\mathbf{e}_i \in \mathbb{R}^{N+1 \times 1}$ vector has all zeros except one in the i th element.

¹Proofs of results in this paper are available in [17].

Algorithm 1 Sequential Smart Grid Game Filter for customer i

Require: Posterior distribution on $\tilde{\mathbf{g}}_k$ at time slot $h_k^s \in t_k$ and $\{\mathbf{T}_{j,h_k^s}, \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h_k^s)\}_{j \in \mathcal{N}}$ according to distribution in (6).
for $h \in t_k$ **do**

- [1] *Equilibrium strategy:* Solve $\{\mathbf{v}_{jh}, r_{jh}\}_{j \in \mathcal{N}}$ in (17) and (18).
- [2] *Play and observe:* Compute $l_{ih} = \mathbf{v}_{ih}^T E[\tilde{\mathbf{g}}_k | I_{ih}] + r_{ih}$ and observe $l_{\mathcal{N}_i, h}$.
- [3] *Observation matrices:* Construct $\{\mathbf{H}_{j,h}\}_{j \in \mathcal{N}}$

$$\mathbf{H}_{j,h}^T := [\mathbf{v}_{m_{j1},h}^T \mathbf{T}_{m_{j1},h}; \dots; \mathbf{v}_{m_{jd(j)},h}^T \mathbf{T}_{m_{jd(j)},h}].$$

- [4] *Gain matrices:* Compute $\{K_{\tilde{\mathbf{g}}_k}^j(h)\}_{j \in \mathcal{N}}$

$$K_{\tilde{\mathbf{g}}_k}^j(h) := \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h) \mathbf{H}_{j,h} (\mathbf{H}_{j,h}^T \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h) \mathbf{H}_{j,h})^{-1}$$

- [5] *Estimation weights:* Update $\{\mathbf{T}_{j,h+1}, \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h+1)\}_{j \in \mathcal{N}}$

$$\mathbf{T}_{j,h+1} = \mathbf{T}_{j,h} + K_{\tilde{\mathbf{g}}_k}^j(h) (\mathbf{H}_{j,h}^T - \mathbf{H}_{j,h}^T \mathbf{T}_{j,h})$$

$$\mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h+1) = \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h) - K_{\tilde{\mathbf{g}}_k}^j(h) \mathbf{H}_{j,h}^T \mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h).$$

- [6] *Bayesian estimates:* Calculate $E[\tilde{\mathbf{g}}_k | I_{ih+1}]$

$$E[\tilde{\mathbf{g}}_k | I_{ih+1}] = E[\tilde{\mathbf{g}}_k | I_{ih}] + K_{\tilde{\mathbf{g}}_k}^i(h) (l_{\mathcal{N}_i, h} - E[l_{\mathcal{N}_i, h} | I_{ih}]).$$

end for

Theorem 2 presents how i computes his BNE strategy at each time slot which is integrated with belief propagation. Beliefs are propagated using sequential LMMSE estimates and hence the beliefs remain Gaussian and the mean estimates are linear combinations of private signals at all times. In order to compute the BNE strategy, user i also needs to keep track of beliefs of others as we show in Algorithm 1.

In Algorithm 1, we provide a sequential local algorithm for i to compute its consumption level and propagate his belief. User i initializes his belief on $\tilde{\mathbf{g}}_k$ at the beginning of the time zone according to the preference distribution in (6). It also determines the estimation weights \mathbf{T}_{j,h_k^s} and error covariance matrix $\mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h_k^s)$ at the beginning of time zone for $j \in \mathcal{N}$. Note that user i does not need any local information from other users in this initialization. Using the estimation weights $\{\mathbf{T}_{j,h_k^s}\}_{j \in \mathcal{N}}$, it can locally construct the equations in (17) and (18), and solve for the strategy coefficients $\{\mathbf{v}_{jh}, r_{jh}\}_{j \in \mathcal{N}}$. In Step 2, i consumes the amount based on his local estimate of the augmented preferences – see (16). Once the consumption occurs, it is transmitted between neighbors. At this point, if the upcoming time slot belongs to the same time zone as the current time slot, i propagates his belief on the marginal utilities – see Remark 1. Propagation of his belief starts by computing observation matrices of all the users in Step 3 with the use of its knowledge of estimation weights, strategy coefficients and network. Next, these observation matrices are used in computing the gain matrices in Step 4 for all the users. In Step 5, i propagates the estimation weights $\mathbf{T}_{j,h+1}$ and error covariance matrix $\mathbf{M}_{\tilde{\mathbf{g}}_k \tilde{\mathbf{g}}_k}^j(h+1)$ for all the users in the network. These computations do not require the local observations of other users given the knowledge of the network topology G . Finally in Step 6, i propagates his mean estimate $E[\tilde{\mathbf{g}}_k | I_{ih+1}]$ by the use of his local observation $l_{\mathcal{N}_i, h}$.

Remark 1 *The belief propagation steps in Algorithm 1 are only valid for sequential time slots that are in the same time zone. If $h+1 \in t_{k+1}$ then there is new prior $P_{\tilde{\mathbf{g}}_{k+1}}$ on the preferences.*

5. NUMERICAL EXPERIMENTS

We evaluate the performance of the smart-grid model with and without communication and effects of desired profit ratio r^* through numerical

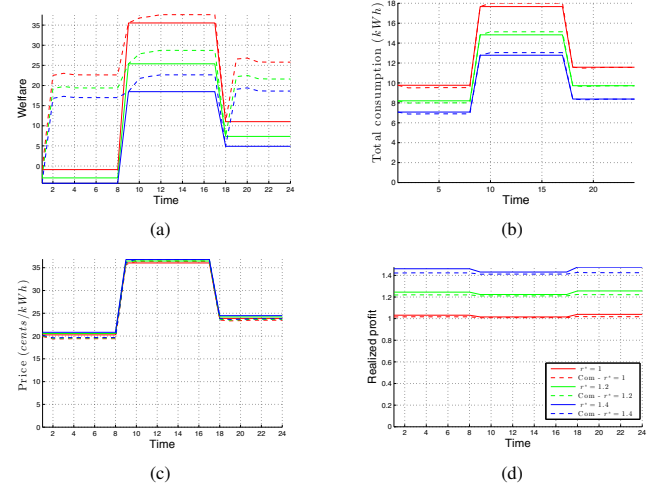


Fig. 1. Effect of target profit ratio on performance metrics welfare (a), total consumption L_h (b), price $p_h(L_h^*; \omega_k)$ (c), and realized profit $R_k(L_h^*)/C_k(L_h^*)$ (d). L_h^* decreases significantly with increasing r^* .

experiments. We let welfare defined as the sum of individual utilities, total consumption L_h^* , price in (2) and realized profit ratio defined as $r_h := R_k(L_h)/C_k(L_h)$ be the performance metrics. There are three time zones $K = 3$ in a $H = 24$ hour day. The time zones are as given by $t_1 = [1, 8]$, $t_2 = [9, 17]$ and $t_3 = [18, 24]$ with corresponding mean preferences $\bar{g}_1 = 20$, $\bar{g}_2 = 40$ and $\bar{g}_3 = 25$. We choose the covariance matrix Σ_k to be identical for all time zones with $\sigma_{ii} = 8$ and $\sigma_{ij} = 4$ for all $i \in \mathcal{N}$, $j \in \mathcal{N} \setminus i$. There are $N = 30$ users on a geometric network in 5 mile \times 3 mile area with a connection threshold of 1.5 miles. Each user has the payoff in (5) with the decay parameter $\alpha_k = 1.5$ for $k = 1, 2, 3$. The cost function of the power provider is as given in (1) with the parameter $\kappa_k = 2$ for $k = 1, 2, 3$. We let the renewable power effect on price be normal with mean $\bar{\omega}_k = 0$ and variance $\sigma_{\omega_k} = 1$ for $k = 1, 2, 3$, respectively. Figs. 1(a)-(d) illustrate the effect of desired profit ratio r_k^* on welfare, total consumption, price and realized profit, respectively. Each line in Figs. 1(a)-(d) corresponds to an average of five runs with a target profit ratio fixed across time zones, i.e., $r_k^* = r^*$ for all k . Each line is color coded with respect to r^* as shown by the legend in Fig. 1(d). Solid lines indicate performance without any communication, and dashed lines are performance with communication where users utilize Algorithm 1.

Observe that in Figs. 1(a)-(d) at the beginning of each time zone, the behavior is identical with or without communication as it should. The welfare is positively affected when there is communication in all the scenarios in Fig. 1(a). Note that the effect of communication differs based on the value of \bar{g}_k . Mean preference has a significant effect on all of the performance metrics. We observe that in the peak time zone total consumption and price is higher in Figs. 1(b)-(c), respectively. The increase in price is expected in peak hours with a jump in L_h^* . Increase in price does not automatically translate to an increase in realized profit ratio in Fig. 1(d) since the cost also grows with total consumption.

In Fig. 1(c), we observe that price across different values of desired profit ratio does not change significantly. This is because users adjust by decreasing consumption in Fig. 1(b). In Fig. 1(a), we observe that welfare decreases when r^* . Finally, we observe that realized profit ratio align with the target profits in Fig. 1(d). The peak-to-average ratio in total consumption is not altered when desired profit ratio is fixed over all time horizons in Fig. 1(b) for different r^* . As a policy to reduce peak-to-average ratio, power provider might choose high profit ratio in the peak time zone $k = 2$ and low profit ratio when demand is low. Specifically, from Fig. 1(b), we can see that peak-to-average ratio is lowest when $r_1^* = 1$, $r_2^* = 1.4$ and $r_3^* = 1.2$.

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