Real-Time Pricing with Uncertain and Heterogeneous Consumer Preferences

Ceyhun Eksin, Hakan Deliç and Alejandro Ribeiro

Abstract—**Consumer demand profiles and fluctuating renewable power generation are two main sources of uncertainty in matching demand and supply. This paper proposes a model of the electricity market that captures the uncertainties on both, the operator and the user side. The system operator (SO) implements a temporal linear pricing strategy that depends on real-time demand and renewable generation in the considered period combining Real-Time Pricing with Time-of-Use Pricing. The announced pricing strategy sets up a Bayesian game among the users with unknown, heterogeneous and correlated consumption preferences. The explicit characterization of the optimal selfish user behavior using the Bayesian Nash equilibrium solution concept allows the SO to derive pricing policies that influence demand to serve practical objectives such as minimizing peak-to-average ratio or attaining a desired rate of return while at the same time hedging renewable generation uncertainty.**

I. Introduction

Matching power production to power consumption is a complex problem in conventional energy grids, exacerbated by the introduction of renewable sources, which by their very nature exhibit significant output fluctuations. This problem can be mitigated with a system of smart meters that control the power consumption of customers by managing the energy cycles of various devices while also enabling information exchange between customers and the system operator (SO) [1], [2]. The flow of information between meters and the SO can be combined with sophisticated pricing strategies so as to encourage a better match between power production and consumption [3]–[5]. The effort of operators to guide the consumption of end users through suitable pricing policies is referred to as demand response management [6].

To manage demand response we consider pricing mechanisms that combine *Real-Time Pricing (RTP)* with *Time-of-Use Pricing (TOU)*. Specifically, the price depends on total consumption at each time slot (RTP) and, in addition, the SO divides the operation cycle into periods (TOU). The use of TOU allows the SO to apply temporal policies based on its anticipation of consumption and renewable source generation in each period. The use of RTP transfers part of the risks and benefits to consumers and encourages their adaptation to power production. When producers use RTP, customers agree to a pricing function but actual prices are unknown a priori because they depend in the realized aggregate demand. In this context, customers must reason strategically about the consumption of others that will ultimately determine the realized price. Game-theoretic models of user behavior then arise naturally and various mechanisms and analyses have been proposed [3], [4], [6]–[9] – see also [10] for more comprehensive expositions. A common feature of these proposed schemes is that the SO and its customers run an iterative algorithm to solve a distributed optimization problem prior to the start of an operating cycle. The outcome of this optimization results in individual power targets that the customers agree to consume once the operating cycle starts.

An alternative perspective is to assume that total consumption and the amount of energy produced by renewable sources are unknown a priori and customers must decide their consumption based on uncertain estimates made public by the SO. Instead of running an iterative optimization problem prior to the start of the operating cycle, we can assume that this is all the information exchange that occurs between customers and the SO. This perspective yields a formulation in terms of games of incomplete information where user behavior is characterized by Bayesian Nash equilibria (BNE) [11]. In this paper, we provide an analysis of the temporal linear pricing policy using explicit expressions for the BNE. This analysis reveals desirable properties of the proposed RTP mechanism. For instance, the SO can adapt pricing policies to achieve a desired rate of return, or it can choose to minimize the power consumption peak-to-average ratio. In either case it can do so without any additional information exchange with customers other than the unidirectional broadcasting of the pricing policy parameters.

We begin with an introduction of the mathematical model for operator and consumer behavior (Sections II-A and II-B). The SO hedges its uncertainty in renewable sources on the price where predicted abundance of renewables creates consumption incentives through fixed discounts and the predicted scarcity discourages consumption at each period. The pricing policy and renewable generation prediction is broadcasted to the consumers at the beginning of the period. We model consumption behavior by designing payoffs for each customer that depends on self-preferences and price. The self-preference is private to the customer itself and is not known by the SO and other customers. The consumers act to maximize their myopic expected payoffs with respect to their belief on others' preferences and renewable power generation estimate (Section II-B). We present the explicit characterization of consumption behavior at each time with respect to self-preference by using BNE as the solution concept when the preferences come from a jointly

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normal distribution (Section III). In numerical experiments, we explore the behavior of aggregate utility, consumption, price, and operator's realized profit with respect to renewable prediction errors and price policy parameters (Section IV). Finally, we propose pricing policies that attain desired rate of return or minimize peak-to-average ratio of consumption leveraging the explicit characterization of consumer behavior. The proposed pricing policies are numerically compared with flat pricing or TOU pricing in which users are price takers (Section V).

II. SMART GRID MODEL

A system operator oversees a demand response management model with N customers. Customers, each equipped with a power consumption scheduler, are characterized by their individual load consumption l_{ih} defined as the power consumed by customer $i \in \mathcal{N}$ at time slot $h \in \mathcal{H} := \{1, \ldots, H\}$. Accordingly, we represent the total consumption of the population at time slot h with $L_h := \sum_{i \in \mathcal{N}} l_{ih}$. In order to be able to respond to changing conditions in the environment, e.g. resource prices or consumption preferences, the SO divides the day into K time zones $t_1, t_2, \ldots, t_K$. The time zone k is a batch of time slots starting at $h_k^s \in \mathcal{H}$ and ending at $h_k^e \in \mathcal{H}$, i.e., $t_k := [h_k^s, h_k^e)$. The time zones do not overlap and span the operation cycle $\mathcal{H}$, that is, $h_{k-1}^e = h_k^s$ for $k \in \{2, \ldots, K\}$ and $h_1^s = 1$ and $h_K^e = H$.

A. Power provider model

For a time slot $h \in t_k$ the total power consumption L_h results in the SO incurring a production cost of $C_k(L_h)$ units. Observe that the production cost function $C_k(L_h)$ depends on the time zone k and the total power produced L_h. When the generation cost per unit is constant, $C_k(L_h)$ is a linear function of L_h. More often, increasing the load L_h results in increasing unit costs as more expensive energy sources are brought online. This results in superlinear cost functions $C_k(L_h)$ with a customary model being the quadratic form[1]

$$C_k(L_h) = \frac{1}{2}\kappa_k L_h^2, \tag{1}$$

for given constants $\kappa_k > 0$ that depend on the day's time zone k. The cost in (1) has been experimentally validated for thermal generators [12] and is otherwise widely accepted as a reasonable approximation [4], [6], [7].

The SO utilizes an adaptive pricing strategy whereby customers are charged a slot-dependent price p_h that varies linearly with the total power consumption L_h. The SO owns renewable source plants such as wind farms and solar arrays, and incorporates renewable source generation into the pricing strategy by introducing a random variable $\omega_k \in \mathbb{R}$ that depends on the amount of renewable power produced. The per-unit power price at time slot $h \in t_k$ is set as

$$p_k(L_h; \omega_k) = \gamma_k (L_h + \omega_k), \tag{2}$$

[1]Our results hold when other linear and constant terms are added to the cost function. We exclude these terms to simplify notation.

where $\gamma_k > 0$ is a policy parameter to be determined by the SO based on its objectives. We present how the operator can pick its policy parameter $\gamma_k > 0$ to minimize consumption peak-to-average ratio or achieve a desired rate of return in Section V after modeling and analyzing consumption behavior. The random variable $\omega_k = 0$ when renewable sources operate at their nominal benchmark capacity $\bar{W}_k$; that is, the generation at time zone k W_k equals $\bar{W}_k$. If the realized production exceeds this benchmark, $W_k > \bar{W}_k$, the SO agrees to set $\omega_k < 0$ to discount the energy price and to share the windfall brought about by favorable weather conditions. If the realized production is below benchmark, i.e., $W_k < \bar{W}_k$, the SO sets $\omega_k > 0$ to reflect the additional charge on the customers. The specific dependence of ω_k with the realized energy production and the policy parameter γ_k, are part of the supply contract between the SO and the customers.

The operator's price function maps the amount of energy demanded to the market price. This is a standard model in pricing – see [13] for a similar formulation. A fundamental observation here is that the prices $p_k(L_h; \omega_k)$ in (2) become known *after* the end of the time zone t_k. This is because prices depend on the total demand L_h and the value of ω_k. Both of these quantities are unknown a priori as illustrated in Fig. 1.

We assume that the SO uses a model on the renewable power generation – see, e.g., [3] for the prediction of wind generation – to estimate the value of ω_k at the beginning of the time zone k. The corresponding probability distribution P_{ω_k} is made available to all customers at the beginning of the time zone. Henceforth, we use E_{ω_k} to denote expectation with respect to the belief P_{ω_k} and $\bar{\omega}_k := E_{\omega_k}[\omega_k]$ to denote the mean of the distribution P_{ω_k}. By including a term that depends on renewable generation in the price function, the SO aims to use the flexibility of consumption behavior to compensate for the uncertainties in renewables in real-time [3], [14].

Remark 1 The time zone-based adjustability of the pricing function in (2) allows the SO to be responsive to not only changes in predictions of renewable power but also to changes in predicted consumption behavior. Note that the SO can have $K = H$ time zones meaning that each time zone corresponds to a time slot. In this paper, we assume that the SO is able to correctly predict changes in consumption preferences within the operation cycle-based on past consumer data and determines time zones $t_1, \ldots, t_K$ accordingly.

B. Power consumer model

The consumption preferences of users are determined by random variables $g_{ik} > 0$ that are possibly different across customers and time zones. When user i consumes l_{ih} units of power at time slot h we assume that it receives the linear marginal utility $g_{ik} l_{ih}$. The user has a diminishing marginal utility from consumption which is captured by the introduction of a quadratic penalty $\alpha_k l_{ih}^2$. This quadratic

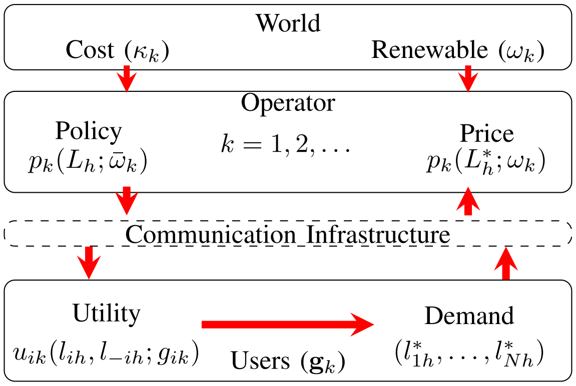

Fig. 1. Illustration of information flow between the system operator and the consumers. The SO determines the pricing policy (2) and broadcasts it to the users along with its prediction of renewable energy term P_{ω_k}. Selfish users respond optimally to realize demand $L_h^* = \sum_{i\in\mathcal{N}} l_{ih}^*$. The realized total demand L_h^* together with realized renewable generation term ω_k determines the price of time zone k.

penalty implies that even when the price charged by the SO is zero, e.g., when $\gamma_k = 0$, it is not in the user's interest to consume infinite amounts of energy. Note that the constant α_k may change across time zones k but is the same for all consumers. For each unit of power consumed, the SO charges the price $p_k(L_h;\omega_k)$, which results in user i incurring the total cost $l_{ih}p_k(L_h;\omega_k)$. The utility of user i is then given by the difference between the consumption return $g_{ik}l_{ih}$, the power cost $l_{ih}p_k(L_h;\omega_k)$ and the overconsumption penalty $\alpha_k l_{ih}^2$,

$$u_{ik}(l_{ih}, L_h; g_{ik}, \omega_k) = -l_{ih}p_k(L_h;\omega_k) + g_{ik}l_{ih} - \alpha_k l_{ih}^2. \tag{3}$$

Using the expressions for prices in (2) and L_h we express (3) as

$$u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k) = -l_{ih}\left[\gamma_k\left(\sum_{j\in\mathcal{N}} l_{jh} + \omega_k\right)\right] + g_{ik}l_{ih} - \alpha_k l_{ih}^2, \tag{4}$$

where we have also rewritten the utility of user i as $u_{ik}(l_{ih}, L_h; g_{ik}, \omega_k) = u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ to emphasize the fact that it depends on the consumption $l_{-ih} := \{l_{jh}\}_{j\neq i}$ of other users. Note that if the provider's policy parameter is set to $\gamma_k = 0$, the utility of user i is maximized by $l_{ih} = g_{ik}/2\alpha_k$ – see [7] for a similar formulation.

The utility of user i depends on the powers l_{-ih} that are consumed by other users in the current slot. These l_{-ih} power consumptions depend partly on their respective preferences, i.e., marginal utilities $g_{-ik} := \{g_{jk}\}_{j\neq i}$, which are, in general, *unknown* to user i. We assume, however, that there is a probability distribution $P_{\mathbf{g}_k}(\mathbf{g}_k)$ on the vector of marginal utilities $\mathbf{g}_k := [g_{1k}, \ldots, g_{Nk}]^T$ from where these marginal utilities are drawn. We further assume that $P_{\mathbf{g}_k}$ is normal with mean $\bar{g}_k\mathbf{1}$ where $\bar{g}_k > 0$ and $\mathbf{1}$ is an $N\times 1$ vector with one in every element, and covariance matrix $\mathbf{\Sigma}_k$. We use the operator $E_{\mathbf{g}_k}$ to signify expectation with respect to the distribution $P_{\mathbf{g}_k}$ and $\sigma_{ij}^k := [[\mathbf{\Sigma}_k]]_{ij}$ to denote the (i,j)th entry of the covariance matrix $\mathbf{\Sigma}_k$. Having mean $\bar{g}_k\mathbf{1}$ implies that all customers have equal average preferences in that $E_{\mathbf{g}_k}(g_{ik}) = \bar{g}_k$ for all i. If $\sigma_{ij}^k = 0$ for some pair $i\neq j$, it means that the marginal utilities of these customers are uncorrelated. In general, $\sigma_{ij}^k \neq 0$ to account for correlated preferences due to, e.g., common weather. It is assumed that marginal utilities $\mathbf{g}_k$ and $\mathbf{g}_l$ for different time zones $k\neq l$ are independent, e.g., the jump in consumption preference from off-peak to peak zone is independent.

The probability distributions P_{ω_k} and $P_{\mathbf{g}_k}$ and the parameters α_k and γ_k are common knowledge among the SO and its customers. That is, the probability distribution of $P_{\mathbf{g}_k}$ is correctly predicted by the SO based on past data by assumption and is announced to the customers. The pricing parameter γ_k and the operator's belief on the renewable energy parameter ω_k, P_{ω_k} is also announced. In addition, customer i knows its private value of consumption preference g_{ik}.

A selfish customer's goal is to maximize the utility $u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$ in (4) given its partial knowledge of the others' consumptions l_{-ih}. This maximization requires a model of behavior for other users that comes in the form of a BNE that we introduce next.

III. Customers' Bayesian Game

User i's load consumption at time $h\in t_k$ is determined by its *belief* q_{ih} and *strategy* s_{ih}. The belief of i is a conditional probability distribution on $\mathbf{g}_k$ given g_{ik}, $q_{ih}(\cdot) := P_{\mathbf{g}_k}(\cdot|g_{ik})$. We use $E_{ih}[\cdot] := E_{\mathbf{g}_k}[\cdot|g_{ik}]$ to indicate conditional expectation with respect to belief of user i at time h. In order to second-guess the consumption of other customers, user i forms beliefs on their preferences given the common prior $P_{\mathbf{g}_k}$ and self-preferences up to time zone k $\{g_{im}\}_{m=1,\ldots,k}$. Observe that self-preferences of previous time zones $\{g_{im}\}_{m<k}$ are not relevant to belief at time zone k as they are independent from the preferences at time zone k. Note further that user i's belief is static over the time horizon as it receives no other information about the preferences of others. User i's load consumption at time $h\in t_k$ is determined by its strategy which is a complete contingency plan that maps any possible local observation that it may have to its consumption, that is, $s_{ih}: g_{ik}\mapsto\mathbb{R}$ for any g_{ik}. In particular, for user i, its best response strategy is to maximize expected utility with respect to its belief q_{ih} given the strategies of other customers $\mathbf{s}_{-ih} := \{s_{jh}\}_{j\neq i}$,

$$BR(g_{ik};\mathbf{s}_{-ih}) = \arg\max_{l_{ih}} E_{\omega_k}\left[E_{ih}\left[u_{ik}(l_{ih}, \mathbf{s}_{-ih}; g_{ik}, \omega_k)\right]\right]. \tag{5}$$

A BNE strategy profile $\mathbf{s}^* := \{s_{ih}\}_{i\in\mathcal{N}, h\in\mathcal{H}}$ at time $h\in t_k$ is a strategy in which each customer maximizes expected utility with respect to its own belief given that other customers play with respect to BNE strategy.

Definition 1 *A Bayesian Nash equilibrium (BNE) strategy* $\mathbf{s}^*$ *is such that for all* $i\in\mathcal{N}$, $k=1,\ldots,K$, $h\in t_k$, *and*

$\{q_{ih}\}_{i\in\mathcal{N},h\in\mathcal{H}}$,

$$E_{\omega_k}\big[E_{ih}\big[u_{ik}(s^*_{ih},\mathbf{s}^*_{-ih};g_{ik},\omega_k)\big]\big] \geq E_{\omega_k}\big[E_{ih}\big[u_{ik}(s_{ih},\mathbf{s}^*_{-ih};g_{ik},\omega_k)\big]\big]. \quad (6)$$

A BNE strategy (6) is computed using beliefs formed according to Bayes' rule. Note that the BNE strategy profile is defined for all time slots, that is, no user at any given point in time has a profitable deviation to another strategy. Equivalently, a BNE strategy is one in which users play best response strategy given their individual beliefs as per (5) to best response strategies of other users – see [13], [15], [16] for a detailed explanation. As a result, the BNE strategy is defined with the following fixed point equations

$$s^*_{ih}(g_{ik}) = BR(g_{ik};\mathbf{s}^*_{-ih}) \quad (7)$$

for all $i\in\mathcal{N}$, $h\in t_k$, and g_{ik}. Using the definition in (7), the following result characterizes the unique linear BNE strategy.

Proposition 1 [2] *Consider the game defined by the payoff in (4) at time $h\in t_k$ for $k=1,\ldots,K$. Let the information given to customer i be its preference g_{ik}, the common normal prior on preferences $P_{\mathbf{g}_k}$ and the prior on renewable generation P_{ω_k}. Then, the unique BNE strategy of customer i is linear in $\bar{\omega}_k,\bar{g}_k,g_{ik}$ for all $k=1,\ldots,K$ such that*

$$s^*_{ih}(g_{ik}) = a_{ih}(\bar{g}_k-\bar{\omega}_k\gamma_k)+b_{ih}(g_{ik}-\bar{g}_k) \quad (8)$$

where the constants a_{ih} and b_{ih} are entries of the vectors $\mathbf{a}_h=[a_{1h},\ldots,a_{Nh}]^T$ and $\mathbf{b}_h=[b_{1h},\ldots,b_{Nh}]^T$ which are given by

$$\mathbf{a}_h = ((N+1)\gamma_k+2\alpha_k)^{-1}\mathbf{1}, \quad \mathbf{b}_h=\rho_k\mathbf{d}(\mathbf{\Sigma}_k), \quad (9)$$

with constant $\rho_k=(2(\gamma_k+\alpha_k))^{-1}$ and inference vector

$$\mathbf{d}(\mathbf{\Sigma}_k) = (\mathbf{I}+\rho_k\gamma_k\mathbf{S}(\mathbf{\Sigma}_k))^{-1}\mathbf{1}. \quad (10)$$

obtained from the pairwise inference matrix $\mathbf{S}(\mathbf{\Sigma}_k)$ defined as

$$[[\mathbf{S}(\mathbf{\Sigma}_k)]]_{ii}=0, [[\mathbf{S}(\mathbf{\Sigma}_k)]]_{ij}=\sigma^k_{ij}/\sigma^k_{ii} \quad \forall i\in\mathcal{N}, j\in\mathcal{N}\setminus i. \quad (11)$$

Proposition 1 shows that there exists a unique BNE strategy that is linear in self-preference g_{ik} at each time slot. This is a direct consequence of the fact that the utility in (4) has quadratic form and the prior on preferences is normal. From the linear strategy in (8), we observe that increase in mean preference $\bar{g}_k$ causes an increase in consumption when $a_{ih}>b_{ih}$. From the first set of strategy coefficients in (9), $\mathbf{a}_h$, we observe that the estimated effect of renewable power $\bar{\omega}_k$ has a decreasing effect on individual consumption. This is expected since increasing $\bar{\omega}_k$ implies an expected increase in the price which lowers the incentive to consume. Observe that the BNE strategy (8) does not contain any time slot h dependent parameter hence the consumption level of an individual is fixed for all $h\in t_k$. This is due to the fact that users do not receive any new information within a time zone. This is supported by the finding that significant changes to consumption behavior are few within an operation cycle [18].

Further observe that the strategy coefficients a_{ih} and b_{ih} do not depend on information specific to customer i. A consequence of this observation is that the SO knows the strategy functions of all the users via the action coefficient equations in (9). However, the realized load consumption l_{ih} is a function of realized preference g_{ik}, i.e., $l^*_{ih}=s^*_{ih}(g_{ik})$, which is private. Hence, knowing the strategy function does not imply that the SO knows the consumption level of the users. Nevertheless, the SO can use the BNE strategies of users to estimate the expected total consumption in order to achieve its policy design objectives defined in Section V.

The strategy coefficients $\mathbf{a}_h$ and $\mathbf{b}_h$ in (9) depend on the inference vector $\mathbf{d}(\mathbf{\Sigma}_k)$ which is driven by the covariance matrix $\mathbf{\Sigma}_k$. In order to identify the effect of correlation among preferences on user behavior, we define the notion of σ-correlated preferences.

Definition 2 *The preferences of users are σ-correlated at time zone k if $\sigma^k_{ij}=\sigma$ for all $i\in\mathcal{N}$ and $j\in\mathcal{N}\setminus i$ and $\sigma^k_{ii}=1$ for all $i\in\mathcal{N}$ where $0\leq\sigma\leq 1$.*

In σ-correlated preferences, the correlation among all users vary according to the parameter σ. Hence, the definition does not allow heterogeneous correlation among pairs. When the parameter σ is varied, the preference correlation change is ubiquitous. The inference vector $\mathbf{d}(\mathbf{\Sigma}_k)$ is well-defined for σ-correlated preferences where $0\leq\sigma\leq 1$. We interpret the effect of correlation on the BNE strategies of users with respect to varying σ in the next result.

Proposition 2 *Denote the BNE strategy weights by $\mathbf{a}^\sigma_h$, $\mathbf{b}^\sigma_h$ when preferences are σ-correlated at time zone k. Then, when $\sigma''>\sigma'$, we have the following relationship in time zone k,*

$$a^{\sigma'}_{ih}=a^{\sigma''}_{ih} \text{ and } b^{\sigma'}_{ih}>b^{\sigma''}_{ih} \quad \forall i\in\mathcal{N}. \quad (12)$$

Proof: When the preferences are σ-correlated, the off-diagonal elements of the inference matrix $\mathbf{S}(\mathbf{\Sigma}_k)$ in (11) are equal to σ. As a result, it can be expressed as $\mathbf{S}(\mathbf{\Sigma}_k)=\sigma(\mathbf{1}\mathbf{1}^T-\mathbf{I})$ which allows us to express the inference vector as $\mathbf{d}(\mathbf{\Sigma}_k)=(\mathbf{I}+\rho_k\gamma_k\sigma(\mathbf{1}\mathbf{1}^T-\mathbf{I}))^{-1}\mathbf{1}$. Use the relationship that $(\mathbf{I}+c(\mathbf{1}\mathbf{1}^T-\mathbf{I}))^{-1}\mathbf{1}=((N-1)c+1)^{-1}$ for a constant c to obtain the following weights for $\mathbf{a}^\sigma_h$ and $\mathbf{b}^\sigma_h$ in (9),

$$\mathbf{a}^\sigma_h=((N+1)\gamma_k+2\alpha_k)^{-1}\mathbf{1},$$
$$\mathbf{b}^\sigma_h=\rho_k((N-1)\gamma_k\rho_k\sigma+1)^{-1}\mathbf{1}. \quad (13)$$

The result is obtained by comparing individual entries of (13). ■

Proposition 2 shows that user i's strategy is to place less weight on self-preference g_{ik} when the correlation between the users increases. If the user i's preference is higher than the mean, $g_{ik}>\bar{g}_k$, then increasing correlation coefficient decreases consumption of user i. When $g_{ik}<\bar{g}_k$, user i's consumption increases as σ is increased. The intuition is as follows. Consider the case when $g_{ik}>\bar{g}_k$. As the correlation

[2]Proof omitted due to page limit. It can be found in the extended version of this paper in preparation – see proof of Proposition 1 in [17].

coefficient increases, it is more likely that others' preferences are also above the mean. For instance, others' preferences are certainly above the mean when $\sigma = 1$, given $g_{ik} > \bar{g}_k$. This implies that consumption willingness of others is similar to i, which then means the price will be higher than what is expected when the population's preference is at the mean. As a result, user i decreases its consumption. An identical reasoning follows when $g_{ik} < \bar{g}_k$.

The increase in correlation coefficient enhances the ability of individuals to predict each other's preference. Alternatively, this increase in prediction ability can be achieved via communication among individuals. Hence, Proposition 2 states that if communication is such that the predictive ability of all the individuals increase, then users place less weight on self-preferences and more on the mean estimate $\bar{g}_k$. In [11], we considered a scheme where individuals communicate with their neighbors and learn the aggregate load profile for a given time horizon.

We note that the strategy coefficients of all users are the same when the preferences are σ-correlated; that is, $a^{\sigma}_{ih} = a^{\sigma}_{jh}$ and $b^{\sigma}_{ih} = b^{\sigma}_{jh}$ for all $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus i$. Furthermore, the effect of γ_k on strategy coefficients is readily identified from (13). BNE strategy coefficients $\mathbf{a}^{\sigma}_h$ and $\mathbf{b}^{\sigma}_h$ decrease with respect to increasing γ_k – see equations in (13). The decreasing trend on consumption is conceivable since increasing γ_k means increasing the elasticity of price with respect to total consumption.

We remark that similar analysis as in Proposition 2 follows when σ_{ii} is equal to some constant $c > \sigma$ for all $i \in \mathcal{N}$, that is, it suffices that the diagonals of $\mathbf{\Sigma}_k$ are equal.

IV. Numerical Analyses

We numerically evaluate the effects of the policy parameter γ_k (Section IV-A) and prediction errors of renewable power term ω_k (Section IV-B) on aggregate utility ($U := \sum_h \sum_{i \in \mathcal{N}} u_{ik}(l_{ih}, l_{-ih}; g_{ik}, \omega_k)$), total consumption L_h, price (2), operator's realized rate of return defined as its revenue divided by its cost. In our simulations, there are three time zones $K = 3$ in a $H = 24$ hour day where each time slot is an hour. The start and end times of the time zones are given by $t_1 = [1, 8]$, $t_2 = [9, 17]$ and $t_3 = [18, 24]$. While the first and last time zones are off-peak time zones with mean marginal utilities equal to $\bar{g}_1 = 30$ and $\bar{g}_3 = 35$, the second time zone is the peak zone with mean preference $\bar{g}_2 = 50$. We choose the variance of the preferences to be identical for all time zones, that is, $\sigma_{ii} = 4$. The covariances σ_{ij} are set to 2 for all agents at all time zones unless otherwise stated. That is, we consider σ-correlated preferences but use the variable σ_{ij} to refer to off-diagonal elements of $\mathbf{\Sigma}_k$ for $k = 1, 2, 3$. There are $N = 10$ users. We consider selfish users with payoffs (4) and the decay parameter chosen as $\alpha_k = 1.5$. The cost function of the SO is as given in (1) with the cost parameter $\kappa_k = 1$ for $k = 1, 2, 3$. For the baseline results, the policy parameter is set to $\gamma_k = 1.2$ for $k = 1, 2, 3$. Unless stated otherwise, we let the renewable power term ω_k come from normal distribution with mean $\bar{\omega}_k = 0$ and variance $\sigma_{\omega_k} = 2$ for $k = 1, 2, 3$.

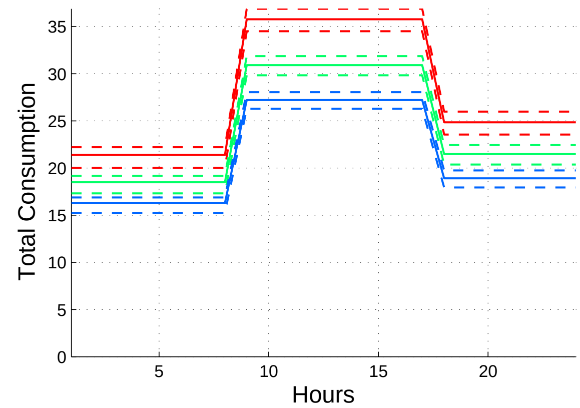

Fig. 2. Effect of policy parameter γ_k on total consumption L_h. Each solid line represents the average value (over 20 realizations) of total consumption with respect to three values of $\gamma \in \{1, 1.2, 1.4\}$ where $\gamma_k = \gamma$ for $k = 1, 2, 3$ as color coded in the legend of Fig. 3. Dashed lines mark minimum and maximum values over all scenarios. Total consumption decreases with increasing γ.

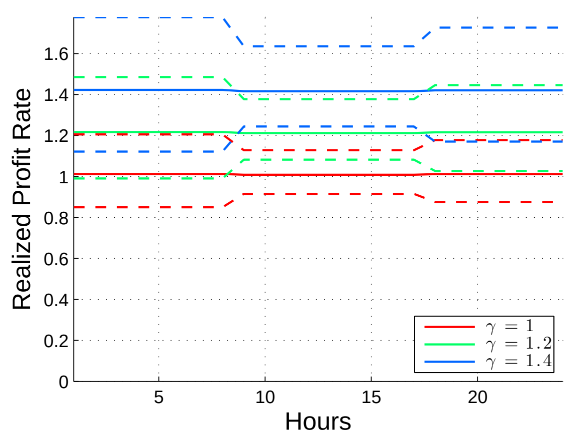

Fig. 3. Effect of policy parameter γ_k on realized profit rate $R_k(L_h)/C_k(L_h)$ where revenue is defined as $R_k(L_h) := L_h p_k(L_h; \omega_k)$ and cost is defined as in (1). Each solid line represents the average value (over 20 realizations) of the realized profit rate with respect to three values of $\gamma \in \{1, 1.2, 1.4\}$ where $\gamma_k = \gamma$ for $k = 1, 2, 3$ as color coded in the legend. Dashed lines mark minimum and maximum values over all scenarios. Average realized profit rate is proportional to the policy parameter value.

Our findings can be summarized as follows. Policy parameter γ_k has a decreasing effect on total consumption. Based on this effect, we observe that increasing γ_k during peak time zones can reduce consumption peak-to-average ratio. Prediction error of renewable generation $\omega_k - \bar{\omega}_k$ is beneficial to the SO if it is positive; otherwise, it is beneficial to the user. Furthermore, we observe that given the same amount of prediction error in renewable generation a predicted discount $\bar{\omega}_k < 0$ is always preferable by the consumers.

A. *Effect of policy parameter*

Figs. 2 and 3 illustrate the effect of policy parameter γ_k on total consumption and realized profit, respectively. We fix the policy parameter across time zones $k = 1, 2, 3$, that is,

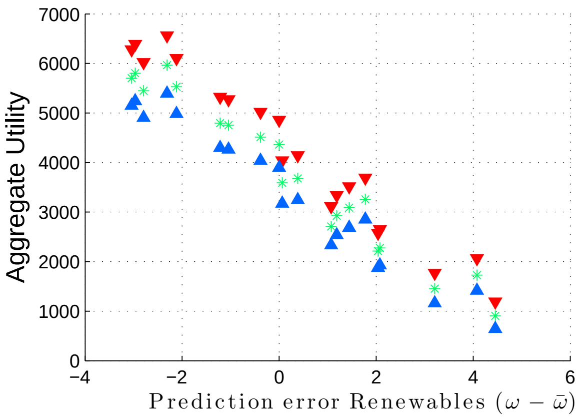

Fig. 4. Effect of prediction error of renewable power uncertainty ω_k on aggregate utility. Horizontal axis shows the prediction error for the renewable term in price, that is, $\omega_k = \omega$ and $\bar{\omega}_k = \bar{\omega}$ and it shows $\omega - \bar{\omega}$. Color code represents different $\bar{\omega}$ and is presented in Fig. 5. Each point corresponds to the value of the aggregate utility at a single run. There are 20 runs for a given $\bar{\omega} \in \{-2, 0, 2\}$. When the realized renewable term ω is larger, the aggregate utility decreases.

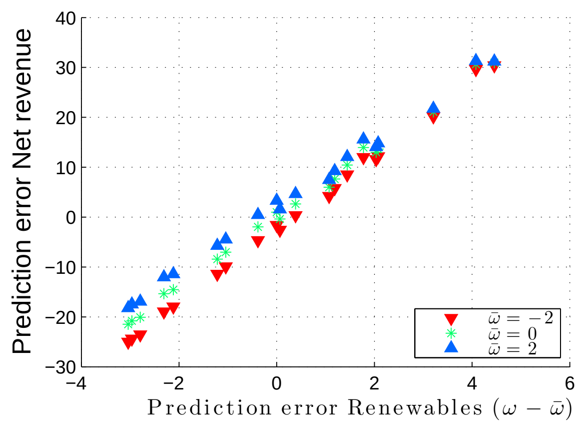

Fig. 5. Effect of prediction error of renewable power uncertainty ω_k on prediction error of net revenue. Horizontal axis shows the prediction error for the renewable term in price, that is, $\omega_k = \omega$ and $\bar{\omega}_k = \bar{\omega}$ and it shows $\omega - \bar{\omega}$. Vertical axis value of each point shows the value of the difference between net revenue and estimated net revenue at a single run. There are 20 runs for a given $\bar{\omega} \in \{-2, 0, 2\}$. When the realized renewable term ω is larger, the net revenue increases.

$\gamma_k = \gamma \in \{1, 1.2, 1.4\}$ for all $k = 1, 2, 3$. Solid lines indicate average value over 20 instantiations of $\mathbf{g}_k$ and ω_k and dashed lines indicate minimum and maximum values over the 20 runs. The legend in Fig. 3 color codes each line according to the policy parameter. The same coding is used in Fig. 2.

Total consumption decreases as γ increases in Fig. 2 as noted in the discussion following Proposition 2. We observe that the mean realized profit ratio overlap with the policy parameter γ in Fig. 3. Furthermore, peak-to-average ratio of total consumption is not altered when γ is fixed over the time horizon in Fig. 3. As a policy to reduce peak-to-average ratio, the SO might choose high γ in the peak time zone and low γ when demand is low. Based on this observation we propose a formal peak-to-average ratio minimizing policy in Section V and compare it with other commonly used pricing schemes.

B. *Effect of uncertainty in renewable power*

From the BNE strategy of customers in (8), we observe that the announced expectation of $\omega_k = \omega$ for $k = 1, 2, 3$, $\bar{\omega}_k = \bar{\omega}$ affects the load of the customers linearly. Hence, the SO can use the response of its customers to mitigate the effects of fluctuations in renewable source generation. However, the contract between the operator and the users is such that the users are charged based on the realization of the random variable ω. We analyze the effect of prediction errors of ω on aggregate utility and the operator's net revenue in Figs. 4 and 5 where we plot the two metrics with respect to realized prediction error $\omega - \bar{\omega}$. Note that both figures use the legend in Fig. 5. Fig. 4 shows that aggregate utility decreases as the realized value grows. Fig. 5 shows that the realized net revenue is most likely larger than the estimated net revenue when the realized value of $\omega - \bar{\omega}$ is positive. Furthermore, observe that a decrease in the announced estimate, $\bar{\omega}$ is always beneficial to the aggregate utility of users when the amount of prediction error is fixed. On the other hand, an expected discount decreases the net revenue of the SO.

V. Comparison among pricing policy mechanisms

We propose desired rate of return and consumption peak-to-average ratio minimization as the two objectives that the SO may determine the pricing policy parameter with respect to. Below we first explain these two objectives and then compare them with flat and TOU pricing schemes.

Desired Rate of Return RTP. The rate of return is defined as the revenue divided by the cost. The SO's revenue at time slot $h \in t_k$ is obtained by multiplying the total consumption L_h by the price in (2), $R_k(L_h) := L_h p_k(L_h; \omega_k)$. The operator's rate of return for the time slot is given by the ratio $R_k(L_h)/C_k(L_h)$. Given its uncertainties on users' marginal utilities $\mathbf{g}_k$, the SO relies on the consumer behavior determined by the BNE (8) to obtain a target expected rate of return $r_k^* = E[R_k(L_h(\gamma_k))/C_k(L_h(\gamma_k)]$ at time zone k by adjusting its policy parameter γ_k. The term $L_h(\gamma_k)$ makes the operator's possible influence on consumption behavior through the adjustment of γ_k explicit. In a budget balancing scheme, the SO would set the desired rate of return to $r_k^* = 1$. Otherwise, it is customary that the desired profit rate is larger than one – see [6], [8] for similar pricing policies. Solving the desired rate of return $r_k^* = E[R_k(L_h(\gamma_k))/C_k(L_h(\gamma_k))]$ with respect to price yields that the policy parameter is equal to $\gamma_k = r_k^* \kappa_k$ when we neglect the renewable generation term, that is, we let $\bar{\omega}_k = 0$. This explains the relation between γ_k and the realized mean rate of return observed in Fig. 3. In our comparisons, we choose the desired rate of return to be $r_k^* = 1.2$ which yields $\gamma_k = 1.2$ when $\kappa_k = 1$ and $\bar{\omega}_k = 0$.

Peak-to-average ratio minimizing Price (PAR). The peak-to-average ratio of load profile $\{L_h\}_{h=1,\ldots,H}$ is defined as the ratio of the maximum load over the operation cycle to the average load profile. The SO can pick the policy parameter $\{\gamma_k\}_{k=1,\ldots,K}$ to minimize the expected peak-to-average ratio of consumption behavior which is formulated as follows

$$\min_{\{\gamma_k\}_{k=1,\dots,K}} E\Big[\frac{H \max_{h=1,\dots,H} L_h(\gamma_k)}{\sum_{h=1}^{H} L_h(\gamma_k)}\Big]. \tag{14}$$

In computing its expected peak-to-average ratio, the SO relies on the model of user optimal behavior as defined by the BNE in (8). The closed form solution to the above optimization problem does not exist. For this reason, we use an evolutionary optimization algorithm to find the minimizing set of policy parameters $\{\gamma_k^*\}_{k=1,\dots,K}$ within the range $[1, 1.5]$ and compute the expected peak-to-average ratio using Monte Carlo sampling [19]. The optimal policy parameters are at the boundaries of the parameter range $[1, 1.5]$, that is, $\gamma_1^* = 1$, $\gamma_2^* = 1.5$, and $\gamma_3^* = 1$. The peak-to-average ratio minimizing choices of high policy parameter in the peak time zone $k = 2$ and low γ_k during off-peak $k = 1, 3$ supports the intuition developed from Fig. 3.

We compare the above pricing schemes with commonly used flat and TOU pricing schemes which we explain below.

Flat Price (FLAT). Customers are charged with a flat price p across the horizon that is determined as the average of realized RTP prices in (2), that is, $p = \sum_{h\in\mathcal{H}} p_k(L_h^*, \omega_k)/H$. Customers respond by optimizing their utility in (3) with price replaced by flat price p, that is, they are price-takers. The user response is obtained from the first order conditions as $l_{ih}^* = (-p + g_{ik})/2\alpha_k$.

TOU Price. Customers are charged with hourly prices p_h that are determined by maximizing hourly expected net revenue, that is, $p_h = \operatorname{argmax}_p E[pL_h - C_k(L_h)]$. Customers optimally respond to hourly prices by $l_{ih}^* = (-p_h + g_{ik})/2\alpha_k$. The SO solves for the p_h^* that maximizes its net revenue given the price-taker behavior of users, which yields the following close form for the maximizing price $p_h = (\alpha_k + \kappa_k N)\bar{g}_k/(2\alpha_k + \kappa_k N)$ for $h \in t_k$.

Figs. 6-9 compare the aforementioned pricing schemes with respect to their influence on customer utility, load behavior, operator's net revenue, and price, respectively. We observe that the flat pricing scheme results in high peak-to-average ratio of consumption Fig. 7, negative revenue Fig. 8 and high variation in all performance metrics across scenarios. We note that while it is possible to increase the net revenue by raising the flat price, this lowers aggregate utility below the levels observed in RTP and PAR pricing causing customer dissatisfaction. The TOU scheme performs comparable to RTP in terms of net revenue and also has a comparable mean peak-to-average-ratio of total consumption. However, in TOU, the aggregate utility is considerably lower than other pricing schemes and there is higher variation in peak-to-average ratio of total consumption when compared with RTP scheme. The RTP and PAR schemes achieve identical net revenues as shown in Fig. 8. This is due to the fact that the policy parameter in RTP $\gamma_k^* = 1.2$ is equal to the time weighted average of the policy parameters in PAR policy. The dashed lines in Fig. 9 illustrating the

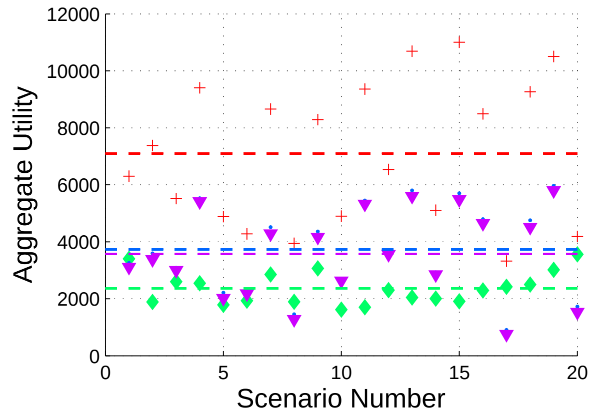

Fig. 6. Comparison of different pricing schemes with respect to aggregate utility $U := \sum_h \sum_{i\in\mathcal{N}} u_{ik}(l_{ih}, l_{-ih})$. Each point corresponds to the value of the aggregate utility for that scenario and dashed lines correspond to the average value of these points over the 20 scenarios. The associated legend is given in Fig. 9. Flat pricing scheme yields the highest expected aggregate utility however the variance of aggregate utility is large.

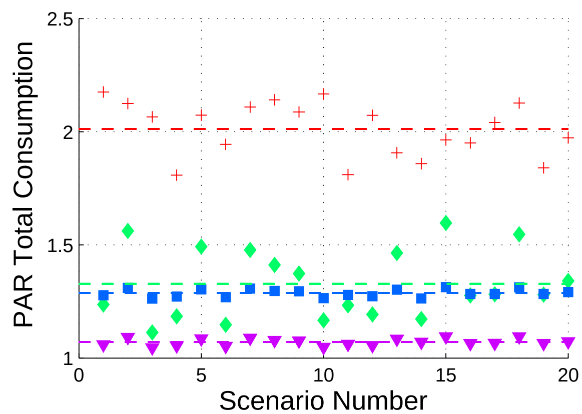

Fig. 7. Comparison of different pricing schemes with respect to peak-to-average ratio of total consumption. Each point corresponds to the value of the peak-to-average ratio of consumption for that scenario and dashed lines correspond to the average value of these points over the 20 scenarios. The associated legend is given in Fig. 9. The PAR policy performs better than others in minimizing peak-to-average ratio of consumption.

minimum and maximum prices observed in 20 runs of RTP show that the variation in price for different scenarios is low. We note that some of the variation observed in metrics for RTP and PAR are due to the uncertainty introduced by the renewable energy term ω_k in (2). In comparison to RTP, the PAR scheme improves on total consumption peak-to-average ratio. When we compare the total consumption over the whole horizon for the two cases we observe no difference; that is, the average aggregate consumption over 20 runs is equal to 576 kWh for both RTP and PAR. This implies that users are shifting their consumption from peak time zone to off-peak time zones based on the policy parameter in PAR pricing. Furthermore, the mean of aggregate utility of PAR is close to the mean in RTP as observed from Fig. 6. This means customer satisfaction is not significantly hurt by PAR pricing.

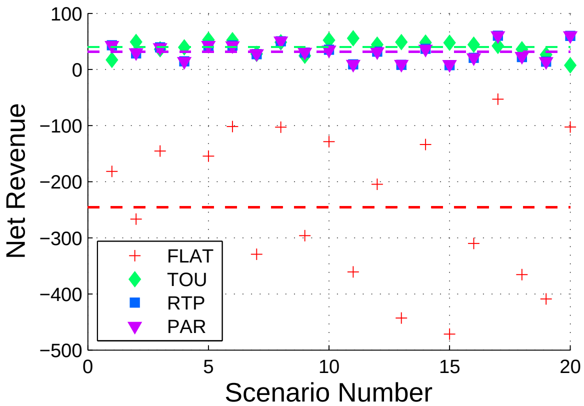

Fig. 8. Comparison of different pricing schemes with respect to realized net revenue $\sum_{h\in\mathcal{H}} pL_h - C_k(L_h)$. Each point corresponds to the net revenue for that scenario and dashed lines correspond to the average value of these points over the 20 scenarios. The associated legend is given in Fig. 9. PAR policy and TOU pricing achieves high net revenue for the SO.

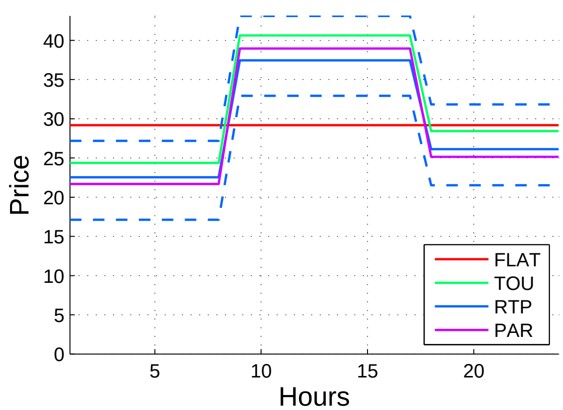

Fig. 9. Comparison of different pricing schemes with respect to realized price. Average price over all scenarios is depicted for each hour with solid lines and dashed lines mark minimum and maximum values over 20 scenarios.

VI. Conclusion

We considered a demand response management model where customers with unknown and heterogeneous marginal utilities respond to RTP announced by the SO ahead of each time zone in the operation cycle. The pricing mechanism incorporated a renewable energy term that allows the provider to incentivize consumption when there is estimated abundance of renewable source within a time zone. Given the pricing mechanism, we discussed the effects of changes in price policy parameters on the customer satisfaction, total consumption and net revenue of the provider. Based on the characterized user behavior and pricing strategy, we proposed a consumption peak-to-average ratio minimizing pricing scheme which can be implemented without any prior communication with the users. Numerical comparisons proved that the proposed peak-to-average ratio minimizing scheme is effective in reducing peak-to-average ratio while its performances in customer satisfaction and net revenue are comparable to other existing pricing schemes.

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