Decentralized Channel Access for Wireless Control Systems ⋆

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Abstract: We consider a wireless control architecture where multiple sensors independently and randomly access a shared wireless medium to communicate with corresponding actuators, and we develop a decentralized channel access mechanism. Each sensor iteratively adapts the rate at which it accesses the medium in order to mitigate the effect of packet collisions from simultaneously transmitting sensors on its communication link. Control performance is abstracted as desired decrease rates of given Lyapunov functions for each loop, which translates to necessary packet success rates on each link. We provide theoretical conditions under which the decentralized mechanism converges to an operating point where performance of all control loops is met, and illustrate the approach in numerical simulations.

Keywords: Wireless sensor-actuator systems. Decentralized sensor coordination. Random access protocols.

1. INTRODUCTION

The emergence of numerous wireless sensing devices in smart homes or modern industrial environments poses new design challenges in the interface of control and communication. To guarantee desirable closed loop control performance, it is important to efficiently share the available wireless medium between such devices.

When networked control systems (Hespanha et al. (2007); Schenato et al. (2007)) share a wireless (or wired) communication medium, the prevalent approach is centralized scheduling. Scheduling can be static, for example sensors transmitting in a predefined periodic sequence designed to meet control objectives, see, e.g., Zhang et al. (2001); Hristu-Varsakelis (2001); Le Ny et al. (2011). Deriving optimal scheduling sequences is recognized as a hard combinatorial problem (Rehbinder and Sanfridson (2004)). Scheduling can also be dynamic, where a central authority decides which device accesses the medium at each time step. This dynamic decision can be stochastic (Gupta et al. (2006)), based on plant state information (Walsh et al. (2002); Donkers et al. (2011); Ramesh et al. (2013)), or on the wireless channel conditions (Gatsis et al. (2014a)).

Unlike centralized scheduling, decentralized channel access mechanisms are attractive as they decrease the need for coordination. In such a mechanism each sensor independently and randomly decides whether to access the channel to communicate to its controller. The drawback is that packet collisions can occur from simultaneously transmitting sensors, resulting in lost packets and control performance degradation. To the best of our knowledge, control under random access communication mechanisms has drawn limited attention. Comparisons between different medium access mechanisms for networked control systems and the impact of packet collisions have been considered in Liu and Goldsmith (2004); Blind and Allgöwer (2011); Rabi et al. (2010), including random access and related Aloha-like schemes (where after a packet collision the involved sensors wait for a random time interval and retransmit). Stability conditions under packet collisions were examined in Tabbara and Nesic (2008).

In contrast to previous work our goal is to design the medium access mechanism, i.e., the rate at which sensors access the shared wireless medium, so that desired control performance for all control systems is guaranteed. Related work by Zhang (2003) considers instead the Aloha-like scheme and characterizes what retransmission policies lead to stability. We have recently shown (see Gatsis et al. (2015)) that the optimal (minimum power) access rates follow a decoupled structure, whereby each sensor accesses the channel at a rate proportional to the desired control performance of its corresponding control loop, and inverse proportional to the aggregate collision effect it causes on all other control loops. Computation of these optimal access rates was made possible by a common access point. Similar decoupled structures are known in the context of random access wireless networks (Lee et al. (2007); Hu and Ribeiro (2011)), where the relevant quantity of interest are data rates on links or general utility objectives. Besides closed loop control, sensor transmission over collision channels for optimal remote estimation is considered recently by Vasconcelos and Martins (2014).

In this paper we consider the random access architecture with multiple control loops (Fig. 1). We develop an iterative decentralized mechanism by which the sensors can adapt their channel access rates without the need of centralized coordination, as is the case with the common access point of Gatsis et al. (2015). We employ a Lyapunov-like control performance requirement, motivated by our previous work (Gatsis et al. (2014a, 2015)). Each control system is abstracted via a given Lyapunov function which is desired to decrease at predefined rates, stochastically due to the random packet losses and collisions on the

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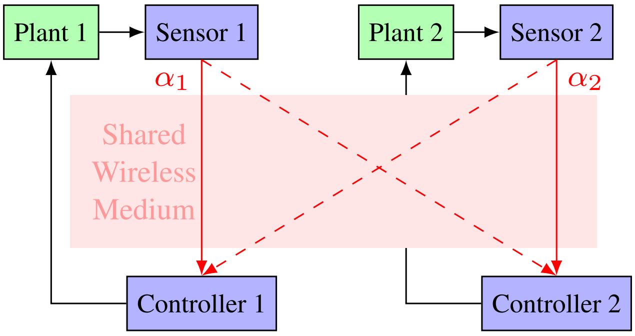

Fig. 1. Random access architecture over a shared wireless medium for $m = 2$ control loops. Each i sensor randomly decides with probability α_i whether to transmit to a corresponding controller computing the plant inputs. Packet collisions occur when both sensors transmit at the same slot. The goal of a decentralized channel access mechanism is to guarantee performance for all control loops.

shared medium (Section 2). These control requirements are shown to be equivalent to a minimum packet success rate on each link.

In Section 3 we develop our decentralized mechanism whereby each sensor measures the discrepancy between desired and observed packet success rate locally on its link, and responds by appropriately adjusting its channel access rate. Our mechanism is motivated by game-theoretic formulations of random access protocols – see, e.g., Jin and Kesidis (2002); Chen et al. (2010). In Section 3 we provide technical conditions under which this mechanism converges locally to an operating point where control performance of all systems is met. These conditions are shown to hold for almost all cases with two control loops, as long as the problem is feasible. We conclude with numerical simulations and some remarks (Sections 4, 5).

2. PROBLEM FORMULATION

We consider a wireless control architecture where m independent plants are controlled over a shared wireless medium. Each sensor i ($i = 1, 2, ..., m$) measures and transmits the output of plant i to a corresponding controller i computing the plant control inputs. Packet collisions might arise on the shared medium between simultaneously transmitting sensors. The case for $m = 2$ control loops is shown in Fig. 1. We are interested in designing a decentralized mechanism for each sensor to decide whether to access the medium (random access) in a way that desirable control performance can be guaranteed for all control systems.

Communication takes place in time slots, and at every time k each sensor i randomly and independently decides to access the channel with some constant probability $\alpha_i \in [0, 1]$, which is our design variable. If only sensor i transmits at a time slot, the message is not always successfully decoded at the access point/controller because of noise added to the transmitted signal on the wireless channel – see Gatsis et al. (2014b). We assume that successful decoding occurs with some constant positive probability $q_i \in (0, 1]$.

To model the interference in the shared wireless medium, we suppose that if another sensor transmits at the same slot as sensor i, a collision occurs on sensor i's packet. This is a model usually considered in control literature (Zhang (2003); Tabbara and Nesic (2008)) and in wireless communication systems (Lee et al. (2007); Hu and Ribeiro (2011); Jin and Kesidis (2002); Chen et al. (2010)). Let us indicate with $\gamma_{i,k} \in \{0, 1\}$ the success of the transmission at time slot k for link/system i. This is a Bernoulli random variable with

$$\mathbb{P}(\gamma_{i,k} = 1) = \alpha_i \, q_i \prod_{j \neq i} \big(1 - \alpha_j\big). \tag{1}$$

This expression states that the probability of system i closing the loop at time k equals the probability that transmission i is successfully decoded at the receiver, multiplied by the probability that no other sensor $j \neq i$ is causing collisions on ith transmission.

Our goal is to design the communication aspects of the problem, hence we assume the dynamics for all m control systems are fixed, meaning that controllers have been already designed. We suppose the system evolution is described by a switched linear time invariant model,

$$x_{i,k+1} = \begin{cases} A_{c,i} \, x_{i,k} + w_{i,k}, & \text{if } \gamma_{i,k} = 1 \\ A_{o,i} \, x_{i,k} + w_{i,k}, & \text{if } \gamma_{i,k} = 0 \end{cases}. \tag{2}$$

Here $x_{i,k} \in \mathbb{R}^{n_i}$ denotes the state of control system i at each time k, which can in general include both plant and controller states. At a successful transmission the system dynamics are described by the matrix $A_{c,i} \in \mathbb{R}^{n_i \times n_i}$, where 'c' stands for closed-loop, and otherwise by $A_{o,i} \in \mathbb{R}^{n_i \times n_i}$, where 'o' stands for open-loop. We assume that $A_{c,i}$ is asymptotically stable, implying that if system i successfully transmits at each slot the state evolution of $x_{i,k}$ is stable. The open loop matrix $A_{o,i}$ may be unstable. The additive terms $w_{i,k}$ model an independent (both across time k for each system i, and across systems) identically distributed (i.i.d.) noise process with mean zero and covariance $W_i \succeq 0$. An example of such a networked control system model (2) is presented next.

Example. Suppose each closed loop i consists of a scalar linear plant and an output of the form

$$\begin{aligned} x_{i,k+1} &= \lambda_{o,i} x_{i,k} + u_{i,k} + w_{i,k}, \\ y_{i,k} &= x_{i,k} + v_{i,k}, \end{aligned} \tag{3}$$

where $w_{i,k}$ and $v_{i,k}$ are i.i.d. Gaussian disturbance and measurement noise respectively. Each wireless sensor i transmits the output measurement $y_{i,k}$ to the controller. Consider a simple control law which applies a zero input $u_{i,k} = 0$ when no information is received, and upon receiving a measurement it applies an output feedback $u_{i,k} = f_i y_{i,k}$ leading to a stable closed loop mode $\lambda_{c,i} = \lambda_{o,i} + f_i$. The overall networked system dynamics are expressed as

$$x_{i,k+1} = \begin{cases} \lambda_{c,i} \, x_{i,k} + w_{i,k} + f_i v_{i,k}, & \text{if } \gamma_{i,k} = 1 \\ \lambda_{o,i} \, x_{i,k} + w_{i,k}, & \text{if } \gamma_{i,k} = 0 \end{cases}. \tag{4}$$

which is of the form (2). Dynamic control laws with local estimates of the plant state at the controller (e.g. Hespanha et al. (2007)) can be expressed similarly.

The random packet success on link i modeled by (1) causes each control system i in (2) to switch in a random fashion between the two modes of operation (open and closed loop). As a result the access rate vector α to be designed affects the performance of all control systems. The following result characterizes, via a Lyapunov-like abstraction, a connection between control performance and the packet success rate.

Theorem 1. Consider a switched linear system i described by (2) with $\gamma_{i,k}$ being an i.i.d. sequence of Bernoulli random variables, and a quadratic function $V_i(x_i) = x_i^T P_i x_i,\ x_i \in \mathbb{R}^{n_i}$ with a positive definite matrix $P_i \in S_{++}^{n_i}$. Then the function decreases with an expected rate $\rho_i < 1$ at each step, i.e., we have

$$\mathbb{E}\left[V_i(x_{i,k+1}) \,\middle|\, x_{i,k}\right] \le \rho_i\, V_i(x_{i,k}) + Tr(P_i W_i) \tag{5}$$

for all $x_{i,k} \in \mathbb{R}^{n_i}$, if and only if

$$\mathbb{P}(\gamma_{i,k} = 1) \ \ge\ c_i, \tag{6}$$

where $c_i \ge 0$ is computed by the semidefinite program

$$c_i = \min\{\theta \ge 0 :\ \theta A_{c,i}^T P_i A_{c,i} + (1-\theta) A_{o,i}^T P_i A_{o,i} \preceq \rho_i P_i\} \tag{7}$$

Proof. The expectation over the next system state $x_{i,k+1}$ on the left hand side of (5) accounts via (2) for the randomness introduced by the process noise $w_{i,k}$ as well as the random success $\gamma_{i,k}$. In particular we have that

$$\begin{aligned}\mathbb{E}\left[V_i(x_{i,k+1}) \,\middle|\, x_{i,k}\right] &= \mathbb{P}(\gamma_{i,k} = 1)\ x_{i,k}^T A_{c,i}^T P_i A_{c,i} x_{i,k} \\ &+ \mathbb{P}(\gamma_{i,k} = 0)\ x_{i,k}^T A_{o,i}^T P_i A_{o,i} x_{i,k} + Tr(P_i W_i),\end{aligned} \tag{8}$$

Here we used the fact that the random variable $\gamma_{i,k}$ is independent of the system state $x_{i,k}$. Plugging (8) at the left hand side of (5) we get for $x_{i,k} \ne 0$

$$\mathbb{P}(\gamma_{i,k} = 1) \ \ge\ \frac{x_{i,k}^T (A_{o,i}^T P_i A_{o,i} - \rho_i P_i) x_{i,k}}{x_{i,k}^T (A_{o,i}^T P_i A_{o,i} - A_{c,i}^T P_i A_{c,i}) x_{i,k}}. \tag{9}$$

Since conditions (5) needs to hold at any value of $x_{i,k} \in \mathbb{R}^{n_i}$, we can rewrite (9) as $\mathbb{P}(\gamma_{i,k} = 1) \ge c_i$ where

$$c_i = \sup_{y \in \mathbb{R}^{n_i}, y \ne 0} \frac{y^T (A_{o,i}^T P_i A_{o,i} - \rho_i P_i) y}{y^T (A_{o,i}^T P_i A_{o,i} - A_{c,i}^T P_i A_{c,i}) y}. \tag{10}$$

This is equivalent to the semidefinite program (7).

The interpretation of the quadratic function $V_i(x_i)$ in this proposition is that it acts as a Lyapunov function for the control system. When the loop closes the Lyapunov function of the system state decreases, while in open loop it increases, and (5) describes an overall decrease in expectation over the packet success. Condition (5) guarantees control performance, in the sense that the variance of the plant state decreases exponentially at a rate ρ_i. In this paper we assume that quadratic Lyapunov functions $V_i(x_i)$ and desired expected decrease rates ρ_i are given for each control system. They present a control interface for communication design over a shared wireless medium. The equivalent desired packet success rates c_i for each system i abstract the system dynamics, the Lyapunov function, and the desired control performance, as can be seen from (7). An example is shown next.

Example (continued). Consider the wireless control system given in (4). For this scalar system we can without loss of generality consider the quadratic function $V_i(x_i) = x_i^2$. Then for any desired decrease rate ρ_i we can compute an equivalent packet success rate using (7) as

$$c_i = \frac{\lambda_{i,o}^2 - \rho_i}{\lambda_{o,i}^2 - \lambda_{i,c}^2}. \tag{11}$$

This illustrates the relationship of the necessary packet success rate to the system dynamics (open and closed loop eigenvalues) and the control performance requirement.

We design a decentralized mechanism for the sensors to select their access rates α so that the Lyapunov functions *for all control loops* i decrease *in expectation* at the desired rates $\rho_i < 1$ at any time k. By the above theorem, these control performance requirements can be transformed to necessary and sufficient packet success rates (6) for each link i, computed by (7). Hence we need to ensure that (6) holds for all links i.

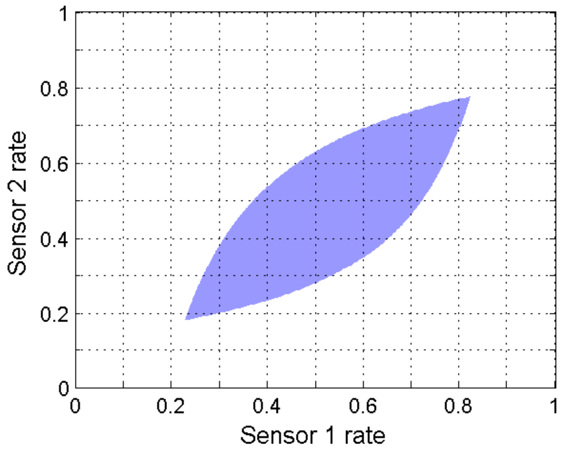

Fig. 2. An example of the feasible set of access rates for $m = 2$ sensors. This is the set or rates α_1, α_2 satisfying inequalities of the form (12), and equivalently meeting the desired closed loop control performance of the two systems. The sensors cannot select neither very low rates, otherwise performance is not met, nor very high rates, otherwise too many collisions occur.

Without loss of generality in the rest of the paper we consider the decoding probabilities on each link i are $q_i = 1$. So the control requirement that each sensor needs to satisfy is

$$c_i \le \alpha_i \prod_{j \ne i} \big(1 - \alpha_j\big). \tag{12}$$

If $q_i < 1$ it is immediate that that the requirement (6) can be expressed as in (12) with the left hand side modified to c_i/q_i. An example of the set of feasible sensor access rates is shown in Fig. 2.

In the following section we present our decentralized mechanism. It is based on the fact that each sensor can measure in a decentralized manner the effect that other transmitting sensors have on the control performance of its system. That is, by packet acknowledgments sent from each controller to the corresponding sensor, the latter can infer the rate at which the other sensors cause collisions on its corresponding link/loop. Given these decentralized observations the sensors can respond by adapting the rate at which they access the channel without a need for coordination.

3. DECENTRALIZED CHANNEL ACCESS MECHANISM

The proposed mechanism is iterative. Suppose at iteration t each sensor selects an access rate $\alpha_i(t)$. Sensor i then observes in a decentralized manner on link i a discrepancy between the current packet success rate and the minimum desired one for control performance as

$$d_i(t) = c_i - \alpha_i(t) \prod_{j \ne i} \big(1 - \alpha_j(t)\big). \tag{13}$$

If this discrepancy is positive sensor i intuitively needs to increase its access rate to meet the control performance, and decrease it otherwise. Consider then a simple decentralized update rule that imitates this intuition according to

$$\alpha_i(t+1) = \Big[\alpha_i(t) + \frac{\varepsilon(t)}{\alpha_i(t)} d_i(t)\Big]_{\mathcal{A}}. \qquad (14)$$

Here $\varepsilon(t) > 0$ is a suitably small positive step size, and the term $\alpha_i(t)$ at the denominator is chosen to simplify the analysis. Since the access rate always needs to be in the unit interval, here $[\ ]_{\mathcal{A}}$ denotes the projection on a subset $\mathcal{A} = [\alpha_{\min}, 1]$ of the unit interval, where $\alpha_{\min} > 0$ is a small positive value to guarantee the denominator in (14) is well-behaved.

In the following theorem we analyze the convergence of this decentralized mechanism. It follows the arguments of Jin and Kesidis (2002); Chen et al. (2010) where a similar random access mechanism is developed – see also the remark at the end of this section.

Theorem 2. Consider the random access architecture with m sensors, where communication is modeled by (1), and a given desired packet success rate c_i for each link $i = 1, \dots, m$. Then:

(1) An access rate vector $\alpha^* \in [0,1]^m$ that satisfies all control performance constraints with equality is an equilibrium of the decentralized access mechanism (14).
(2) Additionally if $M(\alpha^*) \prec 0$, where $M(\alpha^*)$ is the square matrix defined for $1 \le i \neq j \le m$ as
$$M_{ii} = -\frac{c_i}{(\alpha_i^*)^2}, \quad M_{ij} = \prod_{\ell \neq i,j} (1 - \alpha_\ell^*), \qquad (15)$$
then the access rate vector α^* is locally stable for mechanism (14) for a sufficiently small step $\varepsilon(t) > 0$.

Proof. Proof of the first part is straightforward. Define the functions $f_i(\alpha) = \alpha_i \prod_{j\neq i}(1 - \alpha_j)$, so that the feasible set of access rates is defined as $\{\alpha \in [0,1]^m : f(\alpha) \ge c\}$. A point α^* such that $f(\alpha^*) = c$ also satisfies that if $\alpha_t = \alpha^*$, then $d_i(t) = 0$ in (13), so that $\alpha_{t+1} = \alpha_t = \alpha^*$.

To prove the second part, we begin by defining the function
$$H(\alpha) = \sum_{i=1}^m c_i \log(\alpha_i) + \prod_{j=1}^m (1 - \alpha_j). \qquad (16)$$
Note that the partial derivatives equal
$$\frac{\partial H}{\partial \alpha_i} = \frac{c_i}{\alpha_i} - \prod_{j \neq i} (1 - \alpha_j), \qquad (17)$$
from which we conclude that mechanism (13)-(14) is equivalently written as
$$\alpha_{t+1} = \big[\alpha_t + \varepsilon_t \nabla H(\alpha_t)\big]_{\mathcal{A}} \qquad (18)$$
where the projection $[\ ]_{\mathcal{A}}$ is element-wise. So the update mechanism moves along the gradients of the function $H(\alpha)$.

Then we argue that for a sufficiently small value ε_t the function $H(\alpha)$ increases along the trajectory. In particular consider the Taylor expansion at the point α_{t+1}
$$\begin{aligned} H(\alpha_{t+1}) =& H(\alpha_t) + \nabla H(\alpha_t)^T (\alpha_{t+1} - \alpha_t) \\ &+ 1/2 (\alpha_{t+1} - \alpha_t)^T \nabla^2 H(\tilde{\alpha})(\alpha_{t+1} - \alpha_t) \qquad (19) \end{aligned}$$
for some $\tilde{\alpha}$ that is a convex combination of α_t, α_{t+1}. We can bound the term involving the gradient in this expression as follows. By the projection theorem (see Prop. 2.2.1 by Bertsekas et al. (2003)) we have that for any point $w \in \mathcal{A}^m$ and any point z it holds that $(z - [z])^T(w - [z]) \le 0$. Applying this inequality for $z = \alpha_t + \varepsilon_t \nabla H(\alpha_t)$ and $w = \alpha_t$ we get that
$$\|\alpha_{t+1} - \alpha_t\|^2 - \varepsilon_t \nabla H(\alpha_t)(\alpha_{t+1} - \alpha_t) \le 0 \qquad (20)$$
Substituting (20) in (19) we have
$$\begin{aligned} &H(\alpha_{t+1}) \ge H(\alpha_t) + \\ &+ 1/2 (\alpha_{t+1} - \alpha_t)^T \Big[1/\varepsilon_t I + \nabla^2 H(\tilde{\alpha})\Big] (\alpha_{t+1} - \alpha_t). \qquad (21) \end{aligned}$$
For a sufficiently small ε_t the matrix $1/\varepsilon_t I$ dominates the Hessian $\nabla^2 H(\tilde{\alpha})$, and the matrix in the brackets above becomes positive definite. So we have $H(\alpha_{t+1}) > H(\alpha_t)$.

We have thus shown that for a sufficiently small ε_t the mechanism is essentially a gradient ascent. This will converge to some local maximum. A sufficient condition for the point α^* to be a local maximum is $\nabla^2 H(\alpha^*) \prec 0$. By differentiating (17) we have that
$$\frac{\partial^2 H}{\partial \alpha_i^2} = -\frac{c_i}{(\alpha_i^*)^2}, \quad \frac{\partial^2 H}{\partial \alpha_i \partial \alpha_j} = \prod_{\ell \neq i,j} (1 - \alpha_\ell^*), \qquad (22)$$
which shows that $\nabla^2 H(\alpha^*)$ equals the matrix $M(\alpha^*)$ defined in the statement of the theorem and completes the proof. ■

The theorem states that the iterative decentralized channel access mechanism will converge to an operating point where all control specifications are exactly met, as long as the starting point is close enough. It is possible however that the sensors do not converge to a feasible access rate. An example will be given in the simulations (Section 4). Hence global convergence is not guaranteed unless some other mechanism is employed. On the other hand, the theorem provides some explicit conditions under which the mechanism converges locally to a desirable operating point meeting all control performance requirements. These conditions indeed hold true for cases of practical interest, as we explicitly show next for $m = 2$ systems.

By the iterative decentralized mechanism each sensor can adapt to respond to the other sensors' access rates, however this adaptation may also be employed in other scenarios, for example when the control performance specifications of the systems are varying over time. Following the abstraction of Theorem 1, a sensor i can adapt to a varying control performance by adapting to an equivalently varying packet success rate $c_i(t)$ in (13)-(14). Moreover we show next in simulations how the mechanism adapts when new control systems are introduced in the shared wireless medium.

Remark. The decentralized mechanism has a game theoretic interpretation according to Jin and Kesidis (2002); Chen et al. (2010). At each round t of the game, each sensor/agent i selects an action $\alpha_i(t)$ and all agents observe corresponding outcomes, which here can be thought as the observed packet success rates. At the next round each agent i adjusts its action to a new value $\alpha_i(t+1)$ by responding to the actions of the other agents according to (14). Agent i selects an action to improve her utility assuming that all other agents will retain their previous actions. Such a policy is called better response, or gradient play (Chen et al. (2010)). Utilities are not explicitly formulated in our paper, but intuitively each sensor is satisfied with the smallest access rate that meets its control performance. The proof of Theorem 2 is based on a common potential function, as in the game-theoretic framework.

3.1 Special case: Convergence of decentralized channel access mechanism for two systems

Consider the setup with $m = 2$ sensors trying to achieve control performance captured by two packet success rates c_1, c_2. A pair of sensor access rates α_1, α_2 that meet exactly the requirements satisfy

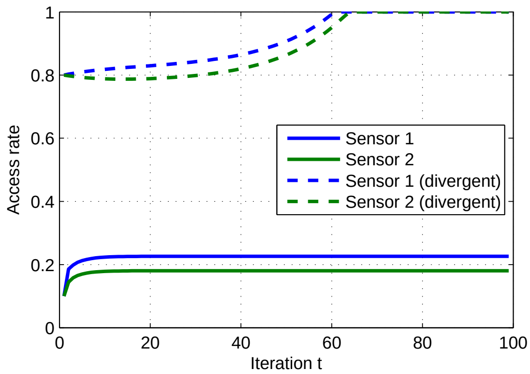

Fig. 3. Evolution of the decentralized channel access evolution for different starting points. Both convergence to a desired operating point, and divergence to the non-feasible region can occur.

$$\alpha_1 (1 - \alpha_2) = c_1, \qquad \alpha_2 (1 - \alpha_1) = c_2. \tag{23}$$

A solution α^* of these equations will be a locally stable operating point of the decentralized channel access mechanism (14) if it satisfies the condition of Theorem 2. That is the negative definiteness of the matrix $M(\alpha^*)$ given by

$$\begin{bmatrix} -\frac{c_1}{(\alpha_1^*)^2} & 1 \\ 1 & -\frac{c_2}{(\alpha_2^*)^2} \end{bmatrix} = \begin{bmatrix} -\frac{1-\alpha_2^*}{\alpha_1^*} & 1 \\ 1 & -\frac{1-\alpha_1^*}{\alpha_2^*} \end{bmatrix} \tag{24}$$

where the equality follows by substituting (23). Examining the characteristic polynomial, it follows that $M(\alpha^*) \prec 0$ holds if and only if $\alpha_1^* + \alpha_2^* < 1$.

We then directly solve (23) to check this condition. From the first equation we get $\alpha_1 = c_1/(1-\alpha_2)$, which if we plug in the second equation yields a quadratic equation

$$(\alpha_2)^2 - (1 + c_2 - c_1)\alpha_2 + c_2 = 0 \tag{25}$$

This equation has a (real) solution if and only if the discriminant is non-negative, i.e.,

$$\Delta = (1 - c_1 - c_2)^2 - 4c_1c_2 \geq 0. \tag{26}$$

We note in passing that given the relationship between packet success rate c_i and control performance of system i (cf. Theorem 1) condition (26) describes the control performance specifications that can be supported by our random access architecture. In general (25) might have two solutions, as seen e.g., in Fig. 2. The minimum solution is of the form $\alpha_2^* = (1 + c_2 - c_1 - \sqrt{\Delta})/2$. A symmetric argument shows that $\alpha_1^* = (1 + c_1 - c_2 - \sqrt{\Delta})/2$. From these two expressions we verify that the minimum equilibrium point satisfies $\alpha_1^* + \alpha_2^* = 1 - 2\sqrt{\Delta}$, which is always less than unity except for the corner case $\Delta = 0$. To sum up, for $m = 2$ sensors in almost all cases the access rates meeting the control performance requirements are a locally stable equilibrium of the decentralized channel access mechanism.

4. NUMERICAL SIMULATIONS

We first consider the decentralized channel access mechanism for $m = 2$ scalar systems. Using the notation of the example of Section 2, we assume both systems have identical dynamics, unstable open loop $\lambda_{o,1} = \lambda_{o,2} = 1.05$ and stable closed loop $\lambda_{c,1} = \lambda_{c,2} = 0.1$. We suppose that the two control systems have different control performance requirements given by the desired Lyapunov decrease rates $\rho_1 = 0.9$ and $\rho_2 = 0.95$, corresponding to different packet success rates on the two links according to Theorem 1. The set of feasible sensor access rates α_1, α_2 that meet both control performances is shown in Fig. 2.

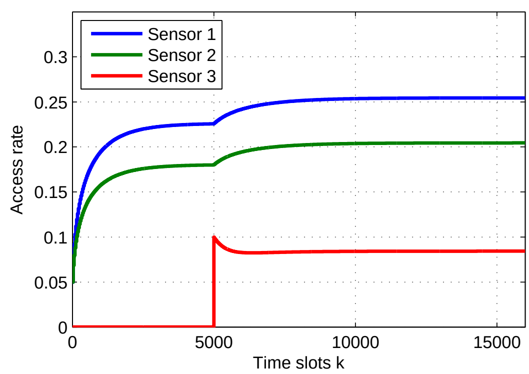

Fig. 4. Evolution of the access rates after an introduction of a third control system in the wireless medium. The sensors need to increase their access rates to mitigate the increased collisions and meet control performance.

We employ the decentralized update rule (14) for each sensor to attempt to reach a global operating point. In Fig. 3 we show the evolution of the two sensor access rates for different initial points. On one case the feasible point where both control performances are exactly satisfied is reached after some iterations. Intuitively, sensor 1 access the medium more often because it has a stricter desired control performance captured by $\rho_1 < \rho_2$. On the other case, both sensors transmit initially at a very high rate, and no feasible operating point is reached. Intuitively each sensor tries to respond to the packet collisions inflicted by the other by increasing its access rate higher and higher. The iteration reaches a deadlock where both sensors transmit at full rate $\alpha_1 = \alpha_2 = 1$. In practice one would have to enforce some arbitration rule to overcome such deadlocks, but this is not explored further in this paper.

Next we turn our attention to how the mechanism can adapt to other changes in the environment, in particular to an introduction of a new control system in the shared wireless medium. After the two systems are in a steady state, a third system with parameters $\lambda_{o,3} = 1$, $\lambda_{c,3} = 0$, $\rho_3 = 0.95$ is introduced. We assume that an iteration of the sensor access update rule is performed every 50 time slots. Each sensor i measures the discrepancy $d_i(t)$ in (13) during this period, for example using acknowledgments from the corresponding receiver/controller, and adjusts its access rate. In practice this measurement of the discrepancy has some error which is however neglected in our simulations. In Fig. 4 we plot the trajectory of the sensor access rates. We observe that after the introduction of the new system there is a period of adaptation till the new operating point is found. The access rates are increased to mitigate the counteract the increased collisions on the medium due to the introduction of the new system.

We are also interested in how the plant states of each control system evolve. As an indication of the latter we plot the average square error of the state $1/N \sum_{k=1}^{N} x_{i,k}^2$ over time for all systems in Fig. 5. After some transients the mean square errors reach a steady state corresponding to desired control performance. Even as the third system is introduced no sig-

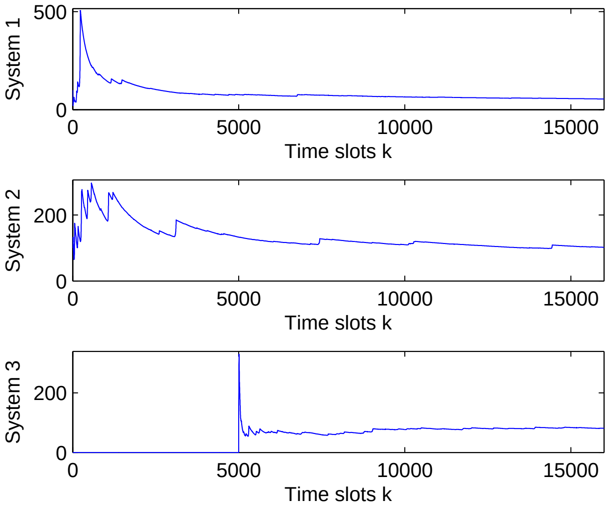

Fig. 5. Evolution of the mean square error $1/N \sum_{k=1}^{N} x_{i,k}^2$ of the plant states during the introduction of a third system. All systems remain stable.

nificant deviations are observed, since convergence to the new necessary access rates is relatively fast (Fig. 4).

5. CONCLUSION

We consider a random access architecture for multiple control loops sharing a common wireless channel. Each sensor randomly decides whether to access the channel, at a rate that needs to be appropriately selected to meet control performance by mitigating the effect of packet collisions from simultaneous transmissions. We develop a decentralized mechanism that reaches a desired operating point without the need to coordinate among the sensors.

Our work is a starting point for exploring efficient, easily implementable, and distributed mechanisms for control systems over shared wireless channels. Future work includes theoretical analysis of the decentralized mechanism under varying or asymmetric channel conditions. Further exploration is also required for mechanisms adapted to varying plant conditions, similar to the state-based approaches of e.g., Gatsis et al. (2014b); Donkers et al. (2011); Ramesh et al. (2013), and random access protocols for estimation and linear quadratic control (Schenato et al. (2007)).

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