Control of Social Processes

Asu Ozdaglar

Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

ARO MURI: Review Meeting
Social Processes

- Social processes underlying political and social change often have suboptimal properties.
- This raises questions related to control – how to influence, improve, or slow down certain dynamics (e.g., opinions, misinformation, purchasing decisions, or infectious diseases).

This Talk:

- Optimal closed-loop control to slow down and prevent a contagious social process.
  - e.g., how to prevent a false political rumor or spread of insurgency by intervening with new information at certain nodes.

Ongoing work: Step back and develop a framework of theoretical and empirical investigation for understanding which classes of contagious processes provide a good match for social dynamics.
Emerging literature on control of contagious processes over networks.

But focus is on open loop policies.

Interaction with social processes take place in real time, hence making more effective (and mathematically more challenging) closed-loop policies feasible.

Major questions:
- Characterization of networks that permit rapid containment of contagion.
- Design of efficient closed-loop control policies.
- When should costly intervention start?

(stochastic curing with probability $p$)
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Control of Contagion

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Related Work

- Maximizing spread of influence: static targeting
  - Approximation guarantees for efficient algorithms [Kempe, Kleinberg, Tardos 03].
  - Optimal targeting to limit impact of adversarial agents [Yildiz, Acemoglu, Ozdaglar, Saberi, Scaglione 11].
  - Optimal US efforts to affect support for insurgency and counter-insurgency in Afghanistan [Hung, Kolitz, Ozdaglar 11].

- Suppressing contagion: static curing
  - Optimal control of infection and curing rates [Preciado, Zargham, Jadbabaie et al. 13], [Sahneh, Scoglio 13].

- Suppressing contagion: dynamic curing
  - Degree based curing policy performs as well as any dynamic policy on expander graphs [Borgs, Chayes, Ganesh, Saberi 10].
Model

- Graph $G = (V, E)$ with maximum degree $\Delta$.
- Infected agents ($X_i(t) = 1$) infect healthy ($X_i(t) = 0$) neighbors according to a Poisson process with rate $\beta$.
- Infected agent $i$ gets cured according to Poisson process with rate $\rho_i$.
- We assume that $\rho_i$ is chosen by the network planner as a function of the entire infection history, i.e., $\rho_t^i(\{X(t')\}_{t' < t})$.
- The budget constraint for this problem is
  \[ \sum_{i \in V} \rho_t^i \leq R, \quad \text{for all } t. \]
  reflecting the fact that curing/intervention is costly.
Problem Setup

- When $\rho_i$ is a constant, $\rho_i^t = \rho$ then finite Markov chain with a unique absorbing state.
- Regardless of the initial infection, the epidemic is terminated in finite time: we denote the extinction time of the epidemic by $\tau$.

minimize \( E[\tau] \)

subject to \( \sum_{i \in V} \rho_i^t \leq R, \) for all $t$. 
The literature focuses on the case $\rho_i = \rho$ – no control [Pastor-Satorras, Vespignani 01], [Ganesh et al. 05], [Kephart and White 91].

An interesting transition is discovered: There exist two thresholds $\lambda_1(G)$ and $\lambda_2(G)$ such that

- if $\beta/\rho > \lambda_2(G)$ the expected extinction time is large (exponential in $n$).
- if $\beta/\rho < \lambda_1(G)$ the expected extinction time is small (logarithmic in $n$).

For a large class of graphs (star, preferential attachment,...) $\lim_{n \to \infty} \lambda_1(G) = 0$. 

\[
\frac{\beta}{\rho} \quad \lambda_1(G) \quad \lambda_2(G) \\
\text{fast} \quad \text{slow}
\]
Existing Results

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Our Results

- **Criterion for rapid containment:** Infection is terminated in sublinear extinction time using sublinear budget.
  
  Mathematically this means that as \( n \to \infty \), \( \frac{R(n)}{n} \to 0 \) and \( \frac{\tau(n)}{n} \to 0 \).

- We present a graph-theoretic measure, cutwidth \( CW(G) \) of a graph that provides a **sharp separation** between graphs that allow rapid containment and those that do not. [Drakopoulos, Ozdaglar, Tsitsiklis 14].

![Diagram with cutwidth measure](image-url)
A bag is a subset of the set of nodes $V$.

The cut of a bag $A$, $c(A)$ is the number of edges connecting $A$ and $V \setminus A$.

A monotone crusade $\omega : V \searrow \emptyset$, is a sequence $\omega_0, \omega_1, \ldots, \omega_{|\omega|-1}$ of bags, with the following properties:

(i) $\omega_0 = V$,  
(ii) $\omega_{|\omega|} = \emptyset$,  
(iii) $\omega_{i+1} \subseteq \omega_i$ and,  
(iv) $|\omega_i \setminus \omega_{i+1}| \leq 1$, for all $i \in \{1, \ldots, |\omega| - 2\}$.

At every step, only removal of nodes allowed. Moreover, at most one node can be removed.

For a given graph $G$, the CutWidth $CW(G)$ is given by

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At every step, any number of nodes can be added, but at most one node can be removed.

For a given bag $A$, the resistance $\delta(A)$ is given by

$$\delta(A) = \min_{\omega : A \to \emptyset} \max_{i \in \{1, \ldots, |\omega|\}} c(\omega_i).$$
Useful Properties

Theorem (Bienstock and Seymour, 1994)

**Initial Condition:** The resistance of the entire set of nodes is equal to the CutWidth of the graph:

\[ \delta(V) = CW(G). \]

Theorem

For any graph \( G \),

- **[Monotonicity]** If \( A \subset B \) then \( \delta(A) \leq \delta(B) \).
- **[Smoothness]** If \( B = A + v \), then \( \delta(A) \geq \delta(B) - \Delta \) (where \( \Delta \) is the maximum degree).

These properties imply that we can use \( \delta(I_t) \) as a Lyapunov function to study the evolution of the system.
Useful Properties

- Infection process is a stochastic process whose upward drift is related to the $c(l_t)$. Therefore, it is necessary to relate $\delta(l_t)$ and $c(l_t)$.

**Theorem**

[Bottlenecks] Whenever the resistance drops, the cut of the corresponding bag is lower bounded by resistance minus, i.e. if there exists $v \in A$ such that $\delta(A - v) < \delta(A)$ then,

$$c(A) \geq \delta(A) - \Delta.$$
We first prove that for graphs with linear cutwidth, sublinear extinction time is impossible with sublinear budget.

**Theorem**

Consider a graph $G$ and let $V$ be the initially infected set. Then, if $CW(G) \gg R$, we have

$$\mathbb{E}[\tau] \geq C \frac{CW(G)^3}{R^2}$$

**Proof sketch:**

- Initially $\delta(V) = CW(G)$ and eventually $\delta(\emptyset) = 0$.
- In order for $\delta(I_t)$ to drop from $CW(G)$ to 0, it has to drop to $CW(G)/2$ in the meantime.
Therefore, it has to drop at least \( CW(G)/2\Delta \) times.

At every such instant
\[ c(I_t) \geq CW(G)/2 - \Delta > R. \]

Rate at which number of infected nodes increases is higher than rate at which it decreases.

For a typical sample path of the process the size increases enough for \( \delta(I_t) \) to grow (e.g. when the whole graph is infected, \( \delta(I_t) \) returns to \( CW(G) \)).

"Typically" \( \delta(I_t) \) increases many times before dropping to 0, thus providing the desirable lower bound.
Implications

- When $CW(G) = \alpha n$ and $R = o(n)$,
  \[ \mathbb{E}[\tau] \geq n. \]

- For graphs with linear CutWidth, it is impossible to achieve sublinear extinction time with sublinear budget for any policy.

- For any set $I_0$ of initially infected nodes, we obtain a similar lower bound with $CW(G)$ replaced with resistance of $I_0$, $\delta(I_0)$:
  \[ \mathbb{E}_{I_0}[\tau] \geq C \frac{\delta(I_0)^3}{R^2}. \]

- This bound gives insight into when intervention should take place:
  - If $\delta(I_0) \gg R^{2/3}$, then fast extinction is impossible. Therefore, intervention should take place before $\delta(I_t) > R^{2/3}$. 
Timing of Intervention
**The DOT policy**

**Theorem**

Assume \( R = 4\beta CW(G) + 4\frac{n}{\log(n)} \). Then, there exists a closed-loop policy such that \( E[\tau] \leq C \log(n) \) for some constant \( C \).

**Policy:**

- Find optimal path (optimal monotone crusade)
- On the optimal path: Allocate budget to the node that leads to the next step.
- Excursion: Starts with an infection. Could be short (return to optimal path) or long (use optimal policy).
- On excursion, allocate budget to any node on deviation.

**Analysis:**

- \( R > CW(G) \) and therefore probability of long excursions and duration of excursions small.
- Process mostly on optimal path which takes \( n/(n/\log(n)) = \log(n) \) time!
Implications

- For graphs with sublinear CutWidth, there exists a policy that achieves sublinear extinction time with sublinear budget.
- The preceding is an existence result (involves using the “optimal policy” along some paths).
- In recent work, we explicitly constructed an implementable policy with similar performance [Drakopoulos, Ozdaglar, Tsitsiklis 14].
  - It involves calculation of $\delta(I_t)$ many times, hard to compute.
  - Combinatorial optimization tools guarantee $\log^2 n$ approximation [Vazirani 01].
Simulations

- Gnutella peer to peer network - 1000 nodes.

\[ \rho_i = \frac{R}{n} \]
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\]

\[
\rho_i = \frac{R}{\sum_{X_j(t)} X_i(t) = 1}
\]

\[
\rho_i = R \frac{\text{deg}(i)}{\sum_{X_j(t) = 1} \text{deg}(j)} \cdot X_i(t) = 1
\]

\[
\rho_i = R \frac{\sum_{j \sim i} X_j(t)}{\sum_{X_k(t) = 1} \sum_{j \sim k} X_j(t)} \cdot X_i(t) = 1
\]
Simulations

- Gnutella peer to peer network - 1000 nodes.

\[ \rho_i = \frac{R}{n} \]

\[ \rho_i = R \frac{\text{deg}(i)}{\sum_{i \in V} \text{deg}(i)} \]

\[ \rho_i = \frac{R}{\sum_{X_j(t), X_i(t) = 1} \text{deg}(j)} \]

\[ \rho_i = R \sum_{j \sim i} \frac{X_j(t)}{\sum_{X_k(t) = 1} \sum_{j \sim k} X_j(t)}, X_i(t) = 1 \]

\[ \rho_i - \text{CW-optimal (Approximate DP)} \]
Ongoing Work: Empirical Validation of Social Contagion Models

- Different stochastic models of social processes lead to drastically different dynamics and limiting patterns of influence.
- How do we distinguish between these models?
- Our ongoing work develops properties of two different classes of social contagion models: stochastic absolute and relative linear threshold model.
- We present an outline of empirical strategy for distinction between these models.
**Models**

- **Absolute** Stochastic Linear Threshold Model (ASLTM).

Consider a social network and a seed set of initial adopters of innovation. Each agent $i$ has a threshold $0 \leq \phi_i \leq \text{degree}_i$.

- At each step...
  - The agent considers adoption as soon as the number of his neighbours, who have adopted the innovation, reaches $\phi_i$.
  - The agent adopts the innovation with probability $p$.
  - If the agent rejects adoption, he does not consider it again.

- **Examples of innovations**: Information diffusion, supply-chain failure, social product adoption.
Motivation

- **Relative** Stochastic Linear Threshold Model (**RSLTM**).

Consider a social network and a seed set of initial adopters of innovation. Each agent $i$ has a threshold $0 \leq \phi_i \leq 1$.

- At each step...
- The agent considers adoption as soon as the **ratio** of his neighbours, who have adopted the innovation, reaches $\phi_i$.
- The agent adopts the innovation with probability $p$.
- If the agent rejects adoption, he does not consider it again.

- **Examples of innovations**: Mobile operator churn; platform or technology adoption.

- **Comparative statics with respect to network structure** differ in these two models.
Initial Results

- **Final adopters:**
  - Characterization of final adopter set for LTM (i.e. when $p = 1$) [Acemoglu, Ozdaglar, Yildiz 11].
  - Bounds on total number of adopters for SLTM in any social network [Acemoglu, Ozdaglar, Teytelboym, Yildiz 14].

- **Effects of clustering (holding total degree constant):** [AOTY 14]
  - Higher clustering $\Rightarrow$ lower expected adoption in RSLTM.
  - Higher clustering $\Rightarrow$ higher expected adoption in ASLTM.

- **Goal:** Examine effects of other network properties (e.g., diameter) on expected adoption.
Empirical Plan

- Use two different large-scale data sets, one for ASLTM, one for RSLTM.
- Verify that in each case, the distinctive implications of underlying process hold for the limiting configurations.
  - Study effects of clustering on different models.
  - There is already empirical evidence for clustering helping adoption in absolute threshold models as predicted in our results [Centola, 10].
- Data for ASLTM: adoption of a social app.
  - Data from a social media start-up that developed an app requiring users to add all their online social networks.
  - Very rich user data; average degree $\approx 1000$; high clustering.
  - Many adopters and non-adopters in the network: we can test whether app spreads faster through very dense clusters.
There is empirical evidence of threshold effects in mobile operator churning. Big jump in probability of switching operator when $\phi_i \approx \frac{1}{2}$ [Zhang et al. 10].

Data for RSLTM: mobile operator churning (collaboration with MIT Media Lab).

- We have data for all voice calls over a six-month period in a small country.
- Unique cross-operator data set: we can estimate the social network and full network properties.
- We can identify which clusters may be resistant to churning.
- We can use these insights for more effective targeted advertising and contract pricing.
Conclusions and Ongoing Work

- We presented efficient closed loop control policies in the context of social contagion.

- In ongoing work, we are investigating how different contagion processes affect dynamics and limiting properties of the contagion.

- Our next steps are to estimate these models using large data sets from social media and mobile phone networks with different structures leading to distinct social dynamics.

- We would also like to study control of contagion using different models, though this is a very challenging problem mathematically.

- Finally, in ongoing work, we are investigating detection of cascades from incomplete data.