Distributed Network Optimization

- Network optimization
  - Agents strive to minimize global cost by selecting local variables.
- Distributed network optimization
  - Network restricts information access.
  - Solution method for estimation and detection problems in WSNs.

Rational Optimization

- Network: \( G = (V, E) \)
  - Vertices \( i \) denote agents.
  - Edges \((i, j)\) denote connections between agents.
- Variables: \( x = (x_1, x_2, x_3, x_4) \) where \( V = N \) and \( x_i \in \mathbb{R}^n \).
- Individual cost functions: \( f_i(x) = \sum_j (x_i - x_j)^2 \)
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- Optimal value \( y \) yields optimal cost, \( p = \arg \min_f \)
- Distributed Optimization:
  - Observe neighborhood information \( (x_{N(i)}); select k such that \kappa \rightarrow \argmin_k \)
- Selecting rational action \( k \) is too cunning = heuristic rationality

Heuristic Rational Optimization

- Consider network agent \( i \) associated with variable \( x_i \) and denote as \( x_{N(i)}(t) \) the values of neighboring variables at time \( t \). We say that a probabilistic rule \( x_i(t) \) is heuristic rational and only if it expects a rational action.
  \[ E(x_i(t) | x_{N(i)}(t)) - x_i(t) \]
- Model heuristic actions, and modeling and perception errors.

Random Activation Rule

- All agents have positive activation probability.
- Agent that activates at \( A(t) \) at time \( t = i \).

Opinion Propagation in Social Networks

- \( x \in \{1, -1\} \) is the opinion of a social agent.
- Stubborn agents \( y \) have fixed extreme opinions \( x \in \{1, -1\} \).
- Compliant agents pay penalty for disagreement \( \xi(x, y) = (x^2 - y^2)^2 \).
- Individual cost function
  \[ f_i(x) = \sum_j (x_i - x_j)^2 \]
- Rational action \( k \)
- Averaging is not done exactly but rather guessed
- Agents only consider opinions of a random subset \( \mathcal{X}(t) \)
- \( \mathcal{X} = N = \{1, 2, \ldots, N\} \)
- Voter models

Field Estimation with WSN

- WSN deployed to estimate spatially varying field
- \( x \) is sensor's estimate value for its own location
- \( y \) independent local noisy observation for sensor \( i \) location
- \( P(Y_i = x) = \prod_i P(Y_i = x_i) \)
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Optimality

- The goal is to compare \( x_{opt} \) with the optimal configuration \( x^* \).
- Stochastic process \( F_t \) of optimality gaps \( F_t = t(x_{opt}) - t(x) \).
- Heuristic updates are akin to stochastic block coordinate descent.

Convergence to Near Optimality

- Theorem (Contraction to Near Optimality): If assumptions (A1)-(A4) hold, the optimality gaps \( F_n \) satisfy
  \[ \|P(F_{n+1} - F_n)\| \leq \alpha \|F_n\| \]
- Convergence behavior depends on \( \alpha \)
- When \( F_n = \alpha M/2 \), rationality dominates.
- When \( F_n = \alpha M/2 \), randomizes dominates.
- Theorem (Convergence to Near Optimality): Define the best optimality gap by time \( \epsilon \), as \( F_t = \min_{F_{\epsilon+t}} F_t \). If assumptions (A1)-(A4) hold, then
  \[ \lim_{k \to \infty} F_k = 0 \] for \( m \alpha \)

Explanations

- What is the process' behavior between visits to near optimality?
- Starting at \( F_0 = (1 - 1/2^3) \), let \( L \) be the final time interval.
- Worst optimality gap \( F_L = \min_{F_{\epsilon+t}} F_t \).
- When \( F_L = \alpha M^2/2 \), rationality dominates.
- There exists a \( \epsilon \) such that
  \[ P(F_{\epsilon+t} - F_{\epsilon+t} \leq \alpha < \epsilon < 1/\alpha) \]
- Theorem (Excursion Bound): If assumptions (A1)-(A5) hold, then, for arbitrary constant \( \epsilon > 0 \)
  \[ P(F_{\epsilon+t} - F_{\epsilon+t} < \epsilon) \geq 2 - \epsilon M^2/\beta \]
  with \( \alpha = 2 - \epsilon M^2/\beta \) for all \( t \).

Conclusions

- Introduced a network optimization formulation
- Defined distributed update rule heuristic rational optimization
- On the average optimal
- Random activation rule
- Each agent has a positive activation probability
- Exemplified heuristic rational behavior in opinion propagation and field estimation with WSN.
- Convergence to near optimality almost surely.
- Excursion probability follows exponential bound.

Simulations

- Opinion Propagation in Social Networks
  - \( N = 100 \) agents on a \( 100 \times 100 \) rectangular field.
  - Agents' activation times are independent with \( \exp(1) \).
  - Coordinates of agent \( i \) is chosen uniformly at random.
  - \( \epsilon = 1/\alpha < \epsilon < 1/\alpha \) for thresholded \( \alpha \) = 0.2 units.
  - \( \beta = 1 < \beta < 1 \). with \( \epsilon = 1 \) and \( \epsilon = 1 \) for all \( t \).
- Heuristic Rationality: Rational action is added to \( \beta \).