Hidden flaw jeopardizes millions of online transactions

Mathematicians discover weakness in commonly used encryption schemes

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By Paul Wagenseil

updated 2/15/2012 6:01:37 PM ET

Mathematicians based in Switzerland and the United States have discovered a small but
the most commonly used digital encryption schemes, a flaw that could undermine the secu
communication.

IT Blogwatch

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February 15, 2012 - 6:00 A.M.

RSA crypto: 'flawed', 'risky', 'quagmire of vulnerabilities'

2 Comments

TAGS: certificate, crypto, cryptography, Diffie-Hellman, encryption, key management, RSA, SSL

IT TOPICS: Applications, Cybercrime & Hacking, Financial IT, Government & Regulation, Internet, Privacy, Security, Security Hardware & Software

A group of six academic researchers have concluded that real-world RSA encryption keys are riskier than Diffie-Hellman-based ones. It seems that some of the random numbers used to generate them weren't, errm, random. In IT Blogwatch, bloggers wonder if the sky is falling.

The Register®

'Predictably random' public keys can be cracked - crypto boffins

Battling researchers argue over whether you should panic

By **John Leyden** · **Get more from this author**

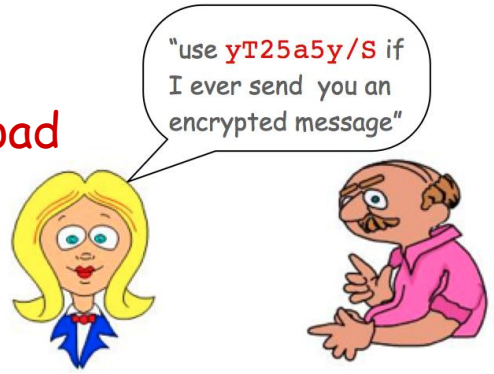
Posted in Enterprise Security, 16th February 2012 13:38 GMT

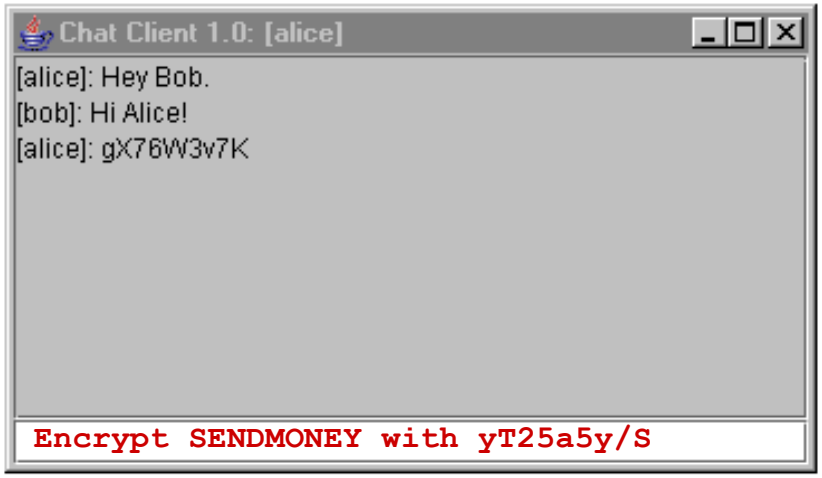
Analysis Cryptography researchers have discovered flaws in the key generation that underpins the security of important cryptography protocols, including SSL.

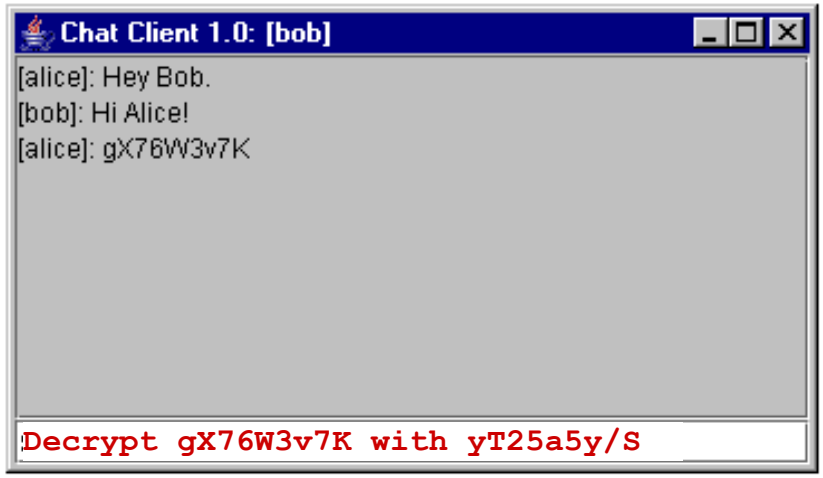
Secure Chat

Alice wants to send a secret message to Bob?

- Sometime in the past, they exchange a one-time pad
- Alice uses the pad to encrypt the message
- Bob uses the same pad to decrypt the message



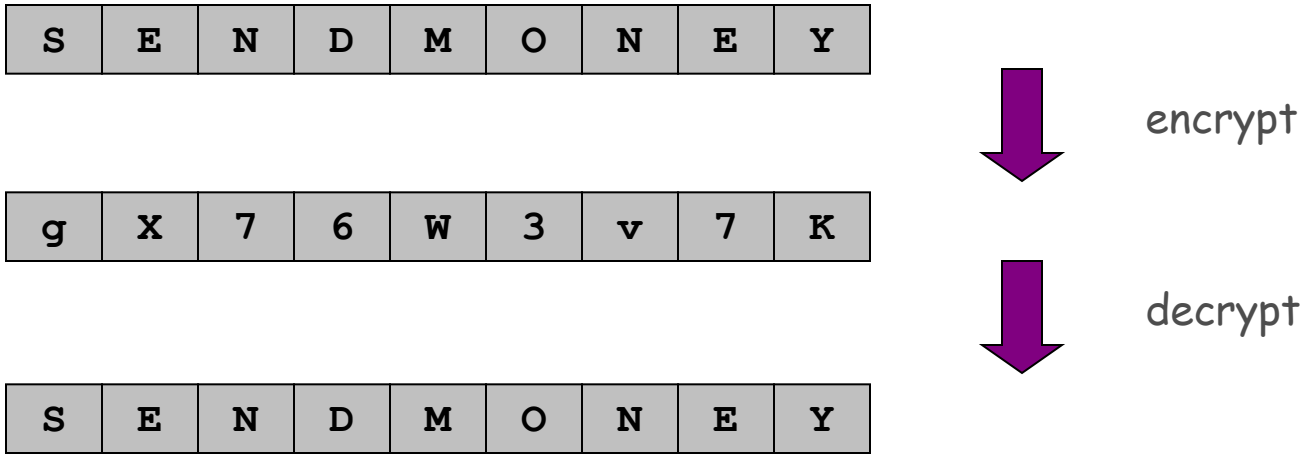




Key point Without the pad, Eve cannot understand the message

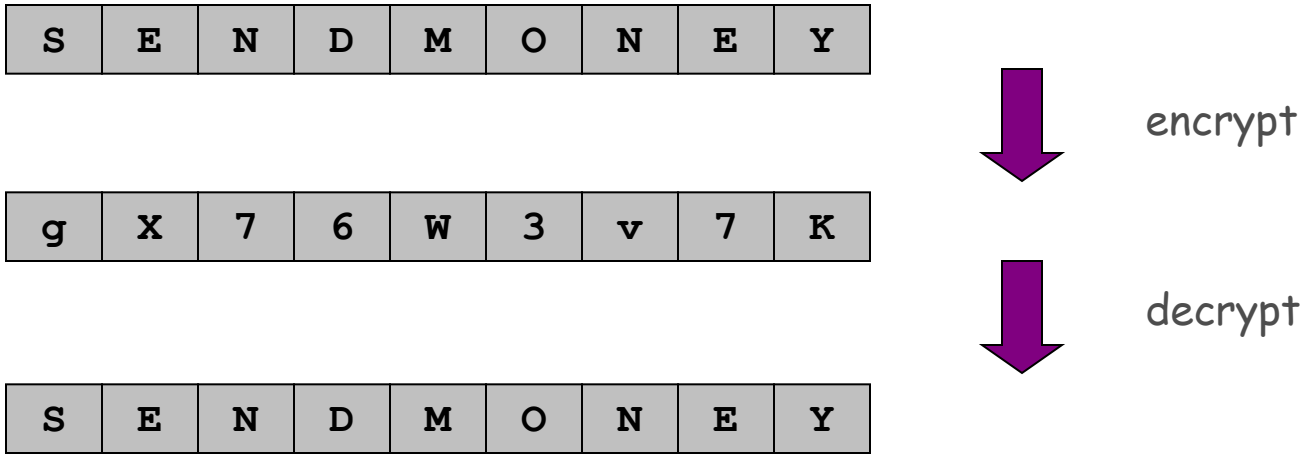
Encryption Machine

Goal Design a machine to encrypt and decrypt data



Encryption Machine

Goal Design a machine to encrypt and decrypt data



Enigma encryption machine

- "Unbreakable" German code during WWII
- Broken by Turing bombe
- One of first uses of computers
- Helped win Battle of Atlantic by locating U-boats

A Digital World

Data is a sequence of bits [bit = 0 or 1]

- Text
- Programs, executables
- Documents, pictures, sounds, movies, …

File formats txt, pdf, java, exe, docx, pptx, jpeg, mp3, divx, …

computer with
a lens

computer with
earbuds

computer with
a radio

A Digital World

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- Text
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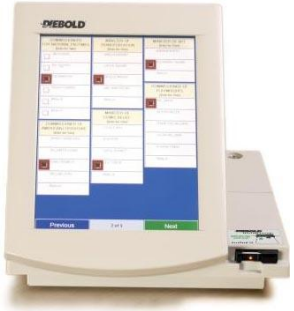
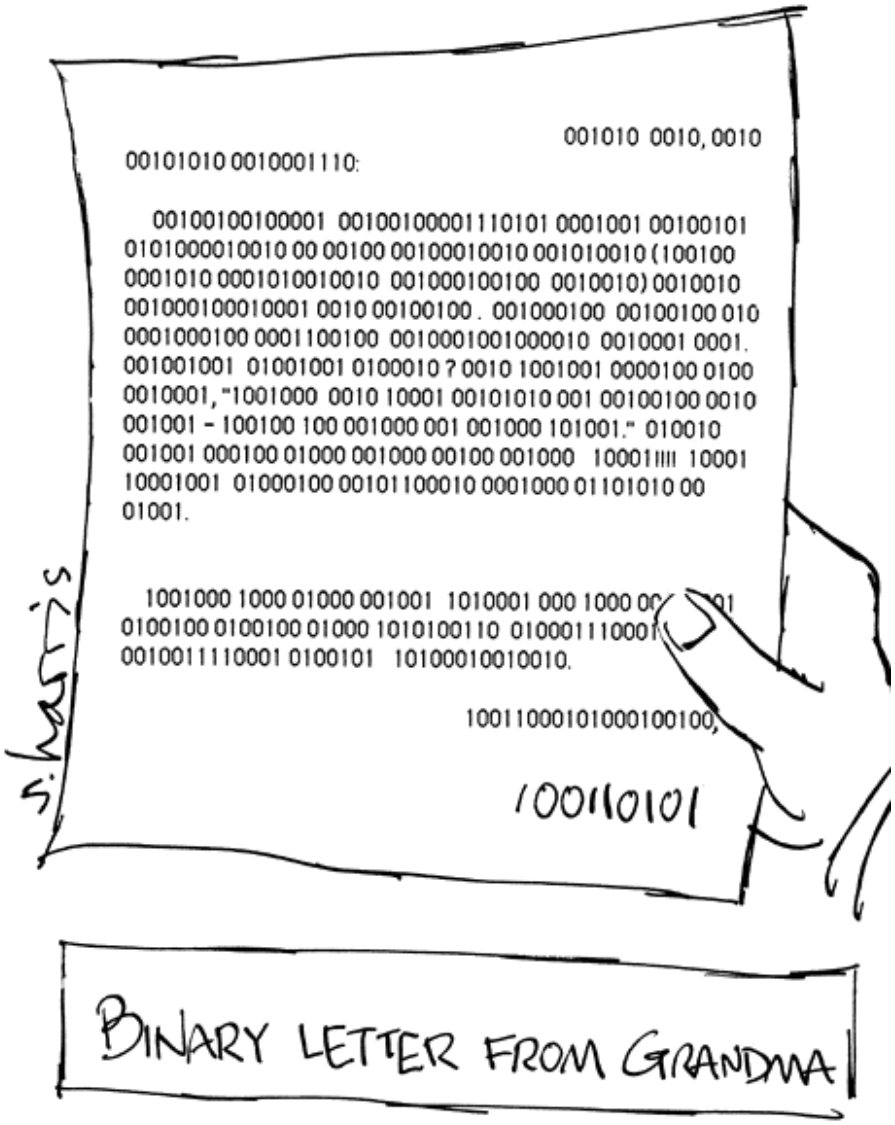
computer with
a cash dispenser



computer with
a ballot box

computer with
a heating element



A Digital World

Data is a sequence of bits [bit = 0 or 1]

- Text
- Programs, executables
- Documents, pictures, sounds, movies, …

Base64 encoding Use 6 bits to represent each alphanumeric symbol.

very weak type of encryption

Binary	Char	Binary	Char	Binary	Char	Binary	Char	Binary	Char	Binary	Char
000000	A	001011	L	010110	W	100001	h	101100	s	110111	3
000001	B	001100	M	010111	X	100010	i	101101	t	111000	4
000010	C	001101	N	011000	Y	100011	j	101110	u	111001	5
000011	D	001110	O	011001	Z	100100	k	101111	v	111010	6
000100	E	001111	P	011010	a	100101	l	110000	w	111011	7
000101	F	010000	Q	011011	b	100110	m	110001	x	111100	8
000110	G	010001	R	011100	c	100111	n	110010	y	111101	9
000111	H	010010	S	011101	d	101000	o	110011	z	111110	+
001000	I	010011	T	011110	e	101001	p	110100	0	111111	/
001001	J	010100	U	011111	f	101010	q	110101	1		
001010	K	010101	V	100000	g	101011	r	110110	2		

One-Time Pad Encryption

Encryption

- Convert text message to N bits

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
…	…	…
M	12	001100
…	…	…

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64

One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)

S	E	N	D	M	O	N	E	Y	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)
- Take bitwise XOR of two bitstrings

sum corresponding pair of bits: 1 if sum is odd, 0 if even

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

S	E	N	D	M	O	N	E	Y	
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR

0 ^ 1 = 1

One-Time Pad Encryption

Encryption

- Convert text message to N bits
- Generate N random bits (one-time pad)
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding

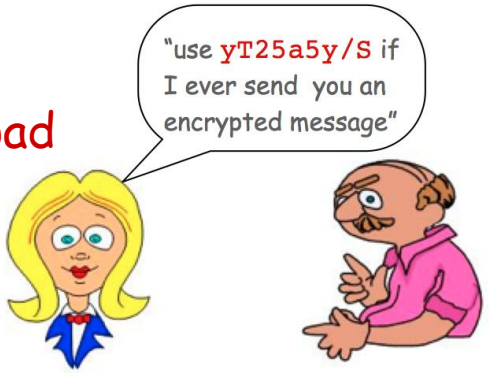
char	dec	binary
A	0	000000
B	1	000001
…	…	…
w	22	010110
…	…	…

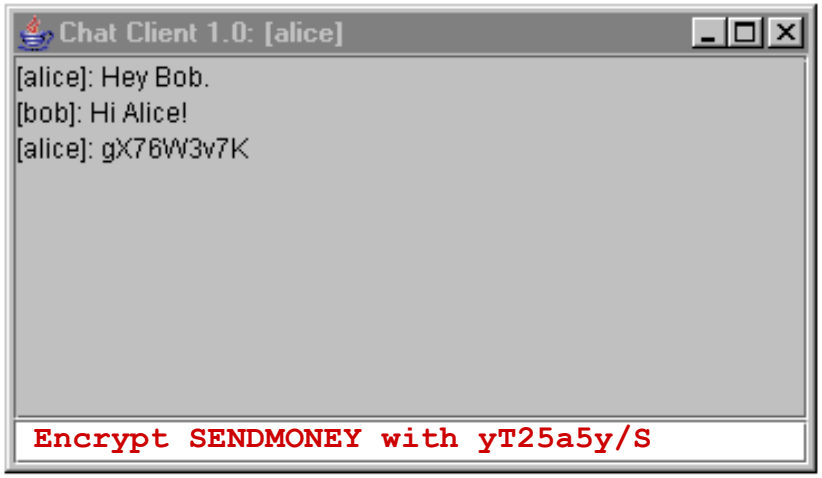
S	**E**	**N**	**D**	**M**	**O**	**N**	**E**	**Y**	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	**X**	**7**	**6**	**W**	**3**	**v**	**7**	**K**	encrypted

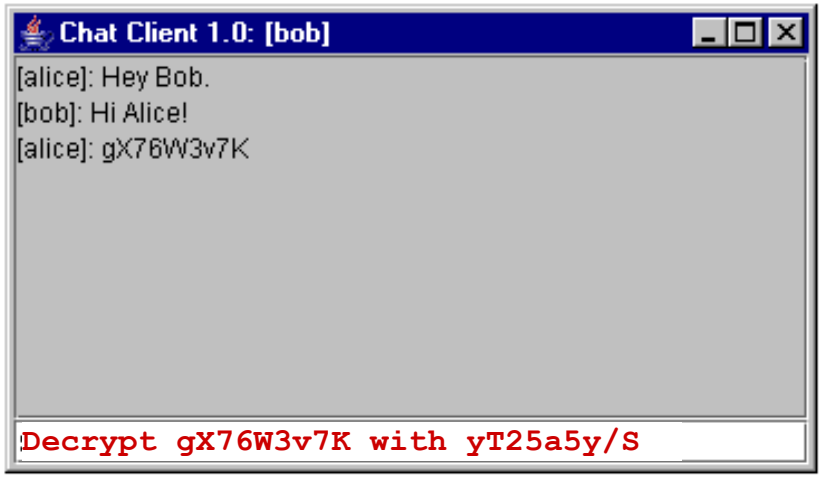
Secure Chat (review)

Alice wants to send a secret message to Bob?

- Sometime in the past, they exchange a one-time pad
- Alice uses the pad to encrypt the message
- Bob uses the same pad to decrypt the message





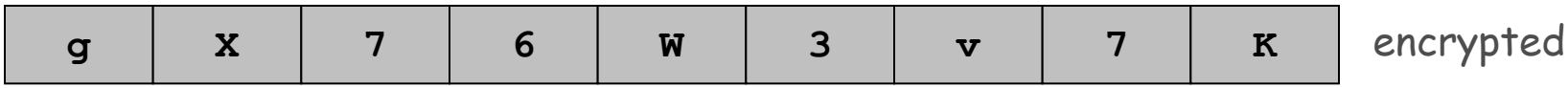


Key point Without the pad, Eve cannot understand the message

One-Time Pad Decryption

Decryption

- Convert encrypted message to binary



One-Time Pad Decryption

Decryption

- Convert encrypted message to binary

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
…	…	…
W	22	010110
…	…	…

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use same N random bits (one-time pad)

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits

One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use same N random bits (one-time pad)
- Take bitwise XOR of two bitstrings

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR

1 ^ 1 = 0

One-Time Pad Decryption

Decryption

- Convert encrypted message to binary
- Use same N random bits (one-time pad)
- Take bitwise XOR of two bitstrings
- Convert back into text

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
…	…	…
M	12	001100
…	…	…

g	**X**	**7**	**6**	**W**	**3**	**v**	**7**	**K**	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	random bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	**E**	**N**	**D**	**M**	**O**	**N**	**E**	**Y**	message

Why Does It Work?

Crucial property Decrypted message = original message

Notation	Meaning
a	original message bit
b	one-time pad bit
^	XOR operator
a ^ b	encrypted message bit
(a ^ b) ^ b	decrypted message bit

Why is crucial property true?

- Use properties of XOR.
- (a ^ b) ^ b = a ^ (b ^ b) = a ^ 0 = a

associativity of ^ | always 0 | identity

XOR Truth Table

x	y	x ^ y
0	0	0
0	1	1
1	0	1
1	1	0

One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary

g	X	7	6	W	3	v	7	K

encrypted

One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64

One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use wrong N bits (bogus one-time pad)

g	X	7	6	W	3	v	7	K	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits

One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use wrong N bits (bogus one-time pad)
- Take bitwise XOR of two bitstrings

g	X	7	6	W	3	v	7	K	
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR

One-Time Pad Decryption (with the wrong pad)

Decryption

- Convert encrypted message to binary
- Use wrong N bits (bogus one-time pad)
- Take bitwise XOR of two bitstrings
- Convert back into text: Oops

g	**X**	**7**	**6**	**W**	**3**	**v**	**7**	**K**	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
101000	011100	110101	101111	010010	111001	100101	101010	001010	wrong bits
001000	001011	001110	010101	000100	001110	001010	010001	000000	XOR
I	**L**	**O**	**V**	**E**	**O**	**K**	**R**	**A**	wrong message

Goods and Bads of One-Time Pads

Good

- Easily computed by hand
- Very simple encryption/decryption processes
- Provably unbreakable if bits are truly random [Shannon, 1940s]

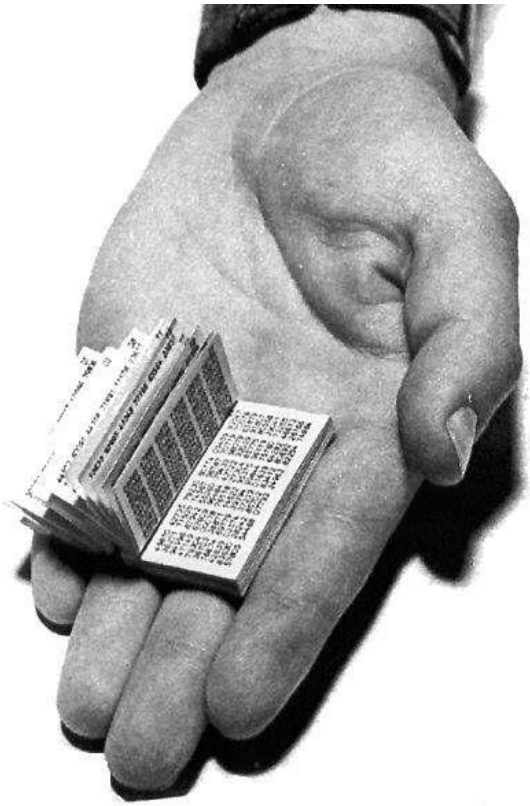
eavesdropper Eve sees only random bits

"one time" means one time only

Bad

- Easily breakable if pad is re-used
- Pad must be as long as the message
- Truly random bits are very hard to come by
- Pad must be distributed securely

impractical for Web commerce



a Russian one-time pad

Pseudo-Random Bit Generator

Practical middle-ground

- Let's make a "random"-bit generator gadget
- Alice and Bob each get identical small gadgets

instead of identical large one-time pads

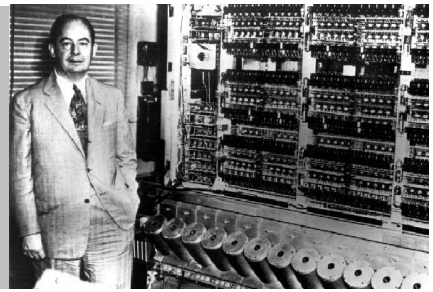
How to make small gadget that produces "random" bits

- Enigma machine
- Linear feedback shift register
- Linear congruential generator
- Blum-Blum-Shub generator
- ...

" Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin. "

– *Jon von Neumann (left)*

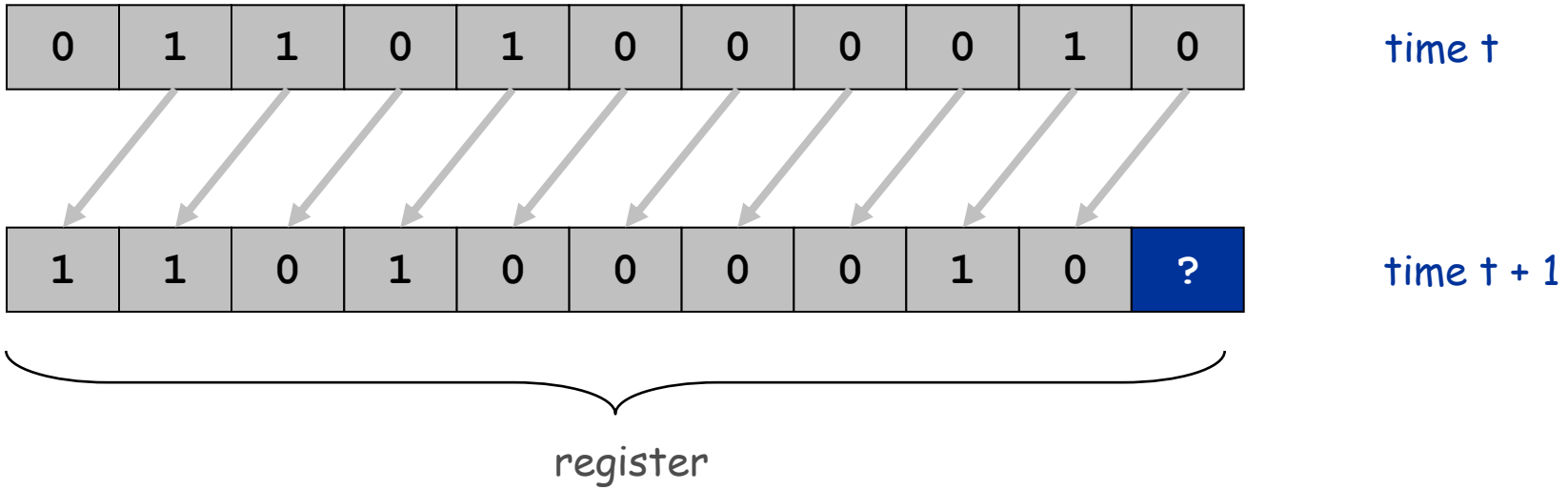
– *ENIAC (right)*



Shift Register

Shift register terminology.

- Bit: 0 or 1
- Cell: storage element that holds one bit
- Register: sequence of cells
- Seed: initial sequence of bits
- Shift register: when clock ticks, bits propagate one position to left

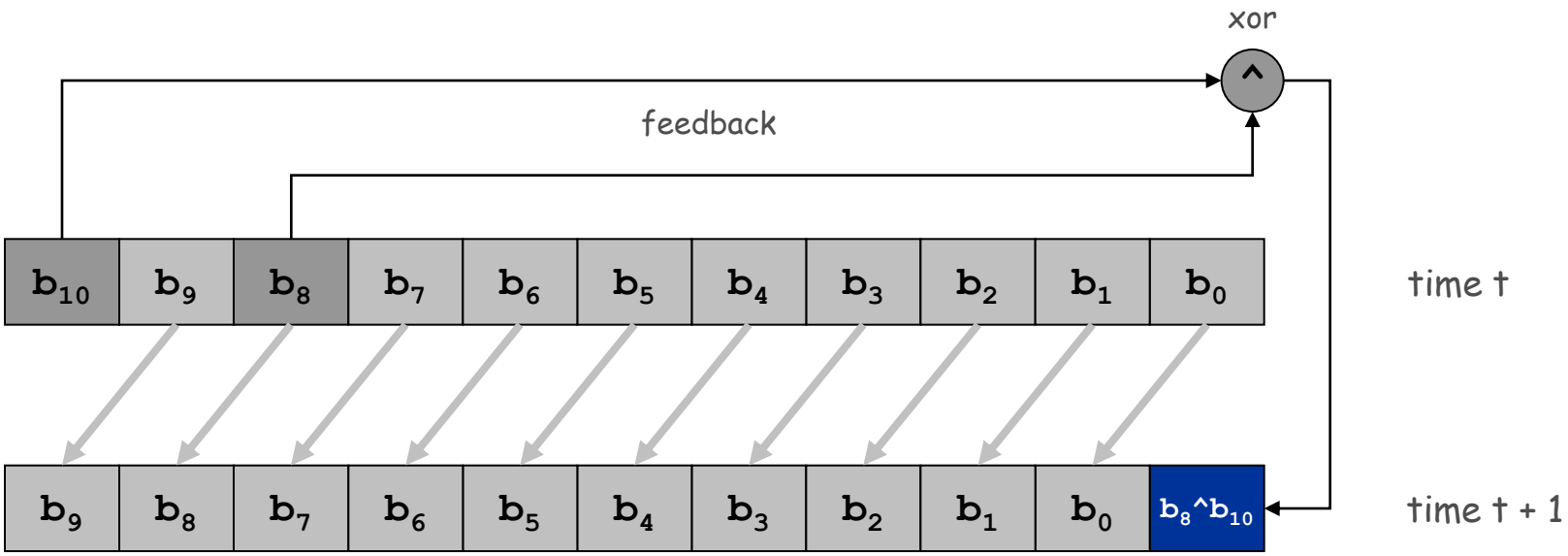


Linear Feedback Shift Register (LFSR)

{8, 10} linear feedback shift register

- Shift register with 11 cells
- Bit b_0 is is XOR of previous bits b_8 and b_{10}
- Pseudo-random bit = b_0

LFSR demo



Random Numbers

Q. Are these 2000 numbers random? If not, what is the pattern?

```
11001001001111011011100101101011100110001011111101001000010011010010111100110010011111
10111000001010110001000011101010011010000111100100110011101111111010100000100001000101O
01010100011000001011110001001001101011011110001101001101110011111010111100100010011101O1
01110100000101001000100011010101011100000001011000001001110001011101101001010110011000O
11111110011000001111110001100001101111001110100111101001110010011101110111010101010100O
00000001000000000101000000100010000101010100100000001101000001110010001101110101110101OO
01010000101000100100010101101010000110000100111100101110011100101111011100100101011101l
00001010111001000010111010010010100110110001111011101100101010111100000010011000010111l
10010010001110110101101011000110001110111110110101001011000011001110011111110111100001O1O
01100100011111101011000010001110010101101110000110101100111000111110110110001011011101O
01101010011110000111001100110111111111010000000100100000101101000100110010101111110000l
00001100101001111100011100011011011011101101101010110110000011011100011101011011010001l
01100101110111100101010011100000111011000110101110111000101010110100000011001000011111O
10011000100111110101110001000101101010100110000001111100001100011001111011111100101000O
11100010011011010111011000100101110101100101000111100010110011010011111100111000011110
11001100101111111100100000011101000011010010011100110111011111010101000100000010101000O
10000010010100010110001010011101000111010010110100110011001111111111000000000110000000
11110000011001100011111111011000000101110000100101100101100111100111110011111000111001l
01100111110111110001010001101000101110010100101110001100101101111110011010001111100101l0
00111001110110111101011010010001100110101111111000100000110101000111000010110110010011O
11110111101001010010011000110111111011101000101010010100000110001000111101010110010000l1
11101000110010010111110110010001011110101001001000011011010011101100111010111110100010O
01001010101011000000001110000001101100001110111001101010111100000100011000101011110l0
```

A. No. This is output of {8, 10} LFSR with seed 01101000010!

LFSR Encryption

LFSR encryption

- Convert text message to N bits
- Initialize LFSR with small seed
- Generate N bits with LFSR
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
…	…	…
w	22	010110
…	…	…

S	**E**	**N**	**D**	**M**	**O**	**N**	**E**	**Y**	message
010010	000100	001101	000011	001100	001110	001101	000100	011000	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
100000	010111	111011	111010	010110	110111	101111	111011	001010	XOR
g	**X**	**7**	**6**	**W**	**3**	**v**	**7**	**K**	encrypted

LFSR Decryption

LFSR Decryption

- Convert encrypted message to N bits
- Initialize identical LFSR with same seed
- Generate N bits with LFSR
- Take bitwise XOR of two bitstrings
- Convert binary back into text

Base64 Encoding

char	dec	binary
A	0	000000
B	1	000001
…	…	…
M	12	001100
…	…	…

g	**X**	**7**	**6**	**W**	**3**	**v**	**7**	**K**	encrypted
100000	010111	111011	111010	010110	110111	101111	111011	001010	base64
110010	010011	110110	111001	011010	111001	100010	111111	010010	LFSR bits
010010	000100	001101	000011	001100	001110	001101	000100	011000	XOR
S	**E**	**N**	**D**	**M**	**O**	**N**	**E**	**Y**	message

Goods and Bads of LFSR Encryption

Goods

- Easily computed with simple machine
- Very simple encryption/decryption process
- Scalable: 20 cells for 1 million bits; 30 cells for 1 billion bits

  [but need theory of finite groups to know where to put taps]

a commercially available LFSR

Bads

- Still need secure, independent way to distribute LFSR seed
- The bits are not truly random

  [bits in our 11-bit LFSR cycle after $2^{11} - 1 = 2047$ steps]
- Experts have cracked LFSR

  [more complicated machines needed]

Other LFSR Applications

What else can we do with a LFSR?

- DVD encryption with CSS
- DVD decryption with DeCSS!
- Subroutine in military cryptosystems

DVD Jon
(Norwegian hacker)

```
/*      efdtt.c      Author:  Charles M. Hannum <root@ihack.net>            */
/*      Usage is:  cat title-key scrambled.vob | efdtt >clear.vob           */

#define m(i)(x[i]^s[i+84])<<

                    unsigned char x[5]      ,y,s[2048];main(
                    n){for( read(0,x,5     );read(0,s ,n=2048
                             ); write(1   ,s,n)         )if(s
                   [y=s      [13]%8+20]  /16%4  ==1      ){int
                   i=m(      1)17  ^256 +m(0)   8,k      =m(2)
                   0,j=      m(4)   17^ m(3)   9^k*     2-k%8
                   ^8,a     =0,c    =26;for   (s[y]    -=16;
                  --c;j  *=2)a=     a*2^i&    1,i=i /2^j&1
                  <<24;for(j=        127;     ++j<n;c=c>
                                     y)
                                     c

                            +=y=i^i/8^i>>4^i>>12,
                    i=i>>8^y<<17,a^=a>>14,y=a^a*8^a<<6,a=a
                  >>8^y<<9,k=s[j],k         ="7Wo~'G_\216"[k
                   &7]+2^"cr3sfw6v;*k+>/n."[k>>4]*2^k*257/
                         8,s[j]=k^(k&k*2&34)*6^c+~y
                                  ;}}
```

http://www.cs.cmu.edu/~dst/DeCSS/Gallery

A Closing Profound Question

Q. What is a random number?

LFSR does not produce random numbers

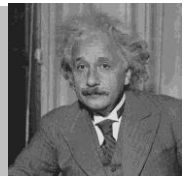
- It is a very simple deterministic machine
- But not obvious how to distinguish the bits it produces from random

Q. Are truly random processes found in nature?

- Motion of cosmic rays or subatomic particles?
- Mutations in DNA?

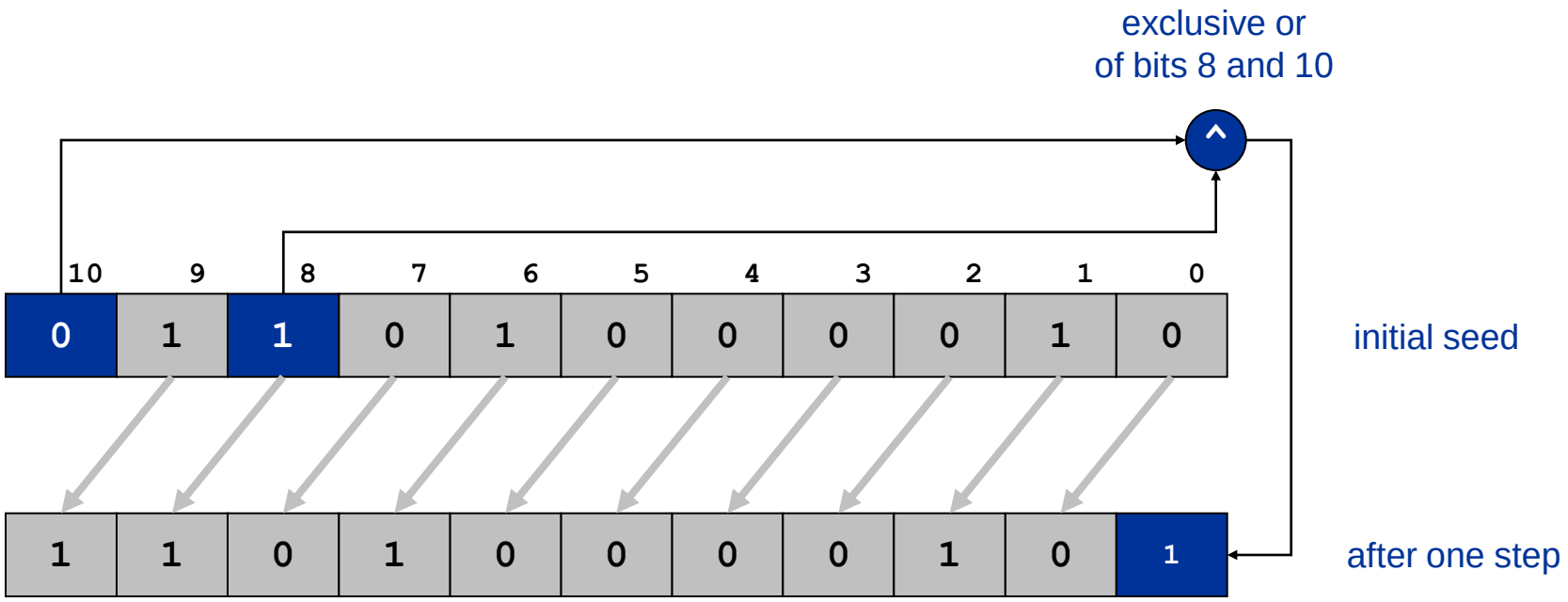
Q. Or, is the natural world a (not-so-simple) deterministic machine?

" God does not play dice. "
– Albert Einstein



Extra Slides

Linear Feedback Shift Register



One step of an 11-bit LFSR with initial seed 01101000010 and tap at position 8