Abstract Types: Sets
Announcements

• Homework 3 is available on the web
  – Due **MONDAY**, February 4\textsuperscript{th} at 11:59:59pm
  – Practice with BSTs, generic functions, *abstract datatypes*

• If you added CIS 120 recently, make sure that you can see your scores online.
  – If you get feedback about your scores, you are in our database.
  – If not, please send mail to \texttt{tas120@lists.seas.upenn.edu}
Abstract Collections
Design Process

1. Understand the problem
2. Formalize the interface
3. Write test cases
4. Implement the required behavior

- How to formalize the interface?
  - Determine data representation + relevant operations
  - So far, we have used built-in structures for representation (int, list, tree)
  - What if the built-in structures don’t fit exactly?
A design problem

As a high-school student, Stephanie had the job of reading books and finding which words, out of a list of the 1000-most common SAT vocabulary words, appeared in a particular book. She enjoyed being paid to read, but she would have enjoyed being paid to program more. How could she have automated this task?

1. What are the important concepts or abstractions for this problem?
   • The collection of words that appear in a book
   • The collection of 1000-most common SAT words
   • The collection of words from the first collection that are contained in the second collection
A set is an abstract type

• Use a set for this collection
  – In math, we typically write sets like this: \{1,2,3\} \{true,false\} with operations like: \( S \cup T \) or \( S \cap T \) for union and intersection
  we write \( x \in S \) to mean that “x is a member of the set S”

• A set is a lot like a list, except:
  – Order doesn't matter
  – Duplicates don't matter
  – *It isn't built into OCaml*

• Sets show up frequently in applications
  – Examples: set of students in a class, set of coordinates in a graph, set of answers to a survey, set of data samples from an experiment, …
2. Formalize the interface: myset.ml file

```ml
type 'a set = ...

(* Need a way to create sets... *)
let empty : 'a set = ...
let add (x:'a) (s:'a set) : 'a set = ...
let union (s1:'a set)(s2:'a set) : 'a set = ...
let remove (x:'a) (s:'a set) : 'a set = ...
let list_to_set (l:'a list) : 'a set = ...

(* ...and a way to work with them *)
let is_empty  (x:'a set) : bool = ...
let member  (x:'a) (s:'a set) : bool = ...
let equal  (s1:‘a set) (s2:‘a set) : bool = ...
let elements  (s:'a set) : 'a list = ...
```
2a. Look at the interface: myset.mli file

```ocaml
type 'a set = ...
val empty : 'a set
val add : 'a -> 'a set -> 'a set
val union : 'a set -> 'a set -> 'a set
val remove : 'a -> 'a set -> 'a set
val list_to_set : 'a list -> 'a set
val is_empty : 'a set -> bool
val member : 'a -> 'a set -> bool
val equal : 'a set -> 'a set -> bool
val elements : 'a set -> 'a list
```

- OCaml puts *interfaces* (as above) in a .mli file
- The corresponding *implementation* goes in the .ml file
Aside: Function Types

• In OCaml, the type of functions from input \( t \) to output \( u \) is written:
  \[ t \rightarrow u \]

• Functions with multiple arguments use multiple arrows

• Here are some examples we have already seen:

\[
\begin{align*}
\text{size} & : \text{tree} \rightarrow \text{int} \\
\text{hamming_distance} & : \text{helix} \rightarrow \text{helix} \rightarrow \text{int} \\
\text{acids_of_helix} & : \text{helix} \rightarrow \text{acids list} \\
\text{length} & : \text{'a list} \rightarrow \text{int} \\
\text{zip} & : \text{'a list} \rightarrow \text{'b list} \rightarrow (\text{'a}'\text{'b}) \text{ list} \\
\text{lookup} & : \text{tree} \rightarrow \text{int} \rightarrow \text{bool} \\
\text{insert} & : \text{'a tree} \rightarrow \text{'a} \rightarrow \text{'a tree}
\end{align*}
\]
3. Write tests (in another file)

```ocaml
let s1 = Myset.add 3 Myset.empty
let s2 = Myset.add 4 Myset.empty
let s3 = Myset.union s1 s2

let test () : bool = (Myset.member 3 s1) = true
;; run_test "Myset.member 3 s1" test
let test () : bool = (Myset.member 4 s3) = true
;; run_test "Myset.member 4 s3" test
```

- To use the values defined in the set module use the “dot” syntax:
  
  `Myset.<member>`

- Alternatively, use “`;; open Myset`” at the top of a file to bring all of the names defined in the interface into scope.

- Note: Module names are always capitalized in OCaml
4. Implement the behavior

• There are many ways to implement sets.
  – lists, trees, arrays, etc.

• **How do we choose which implementation?**

• Many such implementations are of the flavor
  “a set is a ... with some invariants”
  – A set is a *list* with no repeated elements.
  – A set is a *tree* with no repeated elements
  – A set is a *binary* search tree
  – A set is an *array of bits*, where 0 = absent, 1 = present

• **How do we preserve the invariants?**
Abstract types

BIG IDEA: Hide the *concrete representation* of a type behind an *abstract interface*.

- Example:
  - concrete ‘set’ representation – the *implementation* – is a list or a tree
  - abstract interface defines the operations in terms of a ‘set’ type

- The interface restricts how other parts of the program can interact with the data.

- Benefits:
  - **Safety:** The other parts of the program can’t break any invariants
  - **Modularity:** It is possible to change the implementation without changing the rest of the program
Defining Abstract Types

• Different programming languages* have different ways of letting you define abstract types.

• At a minimum, this means providing:
  – A way to specify (write down) an interface
  – A means of hiding implementation details (*encapsulation*)

• In OCaml:
  – Interfaces are specified using a *signature* or ML *interface* file (.mli)
  – Encapsulation is achieved because the interface can *omit* definitions.
  – Clients can’t mention values not named in the interface.

*In Java, interfaces can also be written down explicitly and encapsulation is achieved by public/private modifiers on object fields. (We’ll cover this in detail later.)
Example module interface: .mli file

```ocaml
type 'a set

val empty : 'a set
val add : 'a -> 'a set -> 'a set
val union : 'a set -> 'a set -> 'a set
val remove : 'a -> 'a set -> 'a set
val list_to_set : 'a list -> 'a set
val is_empty : 'a set -> bool
val member : 'a -> 'a set -> bool
val equal : 'a set -> 'a set -> bool
val elements : 'a set -> 'a list
```

- Create a .mli file that omits information on purpose
  - The definition of the set type is hidden
  - Auxiliary functions used in the implementation are hidden
module type Set = sig
  type 'a set

  val empty : 'a set
  val is_empty : 'a set -> bool
  val member : 'a -> 'a set -> bool
  val add : 'a -> 'a set -> 'a set
  val union : 'a set -> 'a set -> 'a set
  val remove : 'a -> 'a set -> 'a set
  val list_to_set : 'a list -> 'a set
  val equal : 'a set -> 'a set -> bool
  val elements : 'a set -> 'a list
end

Name the interface so that it can be reused by multiple implementations.
Module Implementation: trees

module BSet : Set =
struct
    type 'a tree =
    | Empty
    | Node of 'a tree * 'a * 'a tree

type 'a set = 'a tree (* definition hidden by .mli *)

let empty : 'a set = Empty
let is_empty (s:'a set) : bool = ...
...
end

- The implementation has to include all of the interface values
  - It can contain more functions and type definitions (e.g. auxiliary functions) but those cannot be used outside the module
  - The types of the provided implementations must match the interface
Module Implementation: lists

module LSet : Set =
struct
  type 'a set = 'a list
  let empty : 'a set = []
  let is_empty (s:'a set) : bool = ...
  ...
end

Constrains the module to use the given interface.

- To use the values defined in this module later on in the file use the “dot” syntax: \texttt{LSet.<member>}
- In another file, use “\texttt{;; open Hwset.LSet}” at the top of the file to bring all of the names defined in the interface into scope.