1. Prove or disprove: You are given a connected undirected graph $G = (V, E)$ with a weight function $w$ defined over its edges. Let $s \in V$ be an arbitrary vertex in $G$. Starting at vertex $s$, if you do a depth-first search (DFS) in $G$ such that the edges going out of any vertex are always explored in increasing order of weight, then the resulting DFS tree is also a minimum spanning tree.

2. You are given an input stream which will display $n$ integers, and you only get to view each element once. Design an efficient algorithm which will find the $k$ largest elements in the stream, using at most $O(k)$ space (assume $k << n$).

3. Prove that an edge $e$ is contained in every spanning tree for a connected graph $G$ if and only if removal of $e$ disconnects $G$.

4. Let $G = (V, E)$ be a strongly connected directed graph and let $T$ be a DFS tree in $G$. Prove that if all the forward edges in $G$, with respect to $T$, are removed from $G$, the resulting graph is still strongly connected.

5. Give an example of a weighted connected undirected graph $G = (V, E)$ and a vertex $v$ such that the minimum spanning tree of $G$ is different than the shortest path tree rooted at $v$. If edge weights are both distinct and positive, can the two trees be completely disjoint?