1. Tony is playing an online role-playing game with a complicated class system. Each class has multiple more advanced classes that the player can upgrade to by spending a certain number of experience points. It is possible for an advanced class to be upgraded to by multiple less advanced classes, but there is no way to downgrade to a less advanced class. Furthermore, when choosing the beginning class, it is possible for the player to be rewarded (rather than deducted) experience points. A player starts off with a set amount of experience points, and the only time they may earn additional experience is if they choose one of these beginning classes. Given these conditions, is it possible for Tony to use Dijkstra’s algorithm to find the sequence of class upgrades that takes the fewest experience points to end up at any given class? Prove your answer.

2. Ben lives in a town consisting of \( n \) islands connected by at most \( n + 8 \) bridges, such that it is possible to get from any one island to any other island. Each bridge has a cost associated with maintaining it, and all costs are distinct. Deciding that there is no point in maintaining unnecessary bridges, the town council wishes to remove the most expensive bridges possible while maintaining that the town’s islands are still connected. Provide an \( O(n) \) time algorithm to find the correct set of bridges to keep.

3. A common icebreaker activity among teams is to build a tower using marshmallows and uncooked sticks of spaghetti. A general strategy is to stick the ends of the spaghetti pieces into the marshmallows to create a stable structure. Jared, who dreams of being an architect, goes to Fro Gro and purchases \( n \) marshmallows and an abundant amount of
spaghetti. The cashier only judged him slightly. Jared goes home and builds a tower that uses all of the purchased marshmallows. He notices that each marshmallow happens to have at least 2 spaghetti sticks inserted in it. Also, each spaghetti stick has marshmallows attached at both ends.

Let $T$ be the collection of marshmallows that have at least 3 spaghetti sticks stuck in them. Prove that there is a set of marshmallows $M = \{M_1, M_2, \ldots, M_k\}$, $k \leq n$, with the following properties:

a. For $1 \leq i < k$, there is a spaghetti stick that has one end in $M_i$ and the other end in $M_{i+1}$; also, there is a spaghetti stick that has one end in $M_1$ and the other end in $M_k$.

b. $|M \cap T| \leq 2\lceil \log n \rceil$.

You should give a linear time algorithm to find the set $M$.

4. Some time has passed since Problem 2, and Ben has now been elected mayor of the group of islands he lives on. Many more bridges have been built between islands (of which there are $n$ total), such that there are now $m$ total bridges, where we do not know the value of $m$ except that it is still possible to reach an island from any other island. As before, each bridge has a distinct, positive cost associated with maintaining it.

For his first mayoral project, Ben wishes to designate a certain bridge as “Ben’s Bridge” and treat it specially from all other bridges. Once he picks the bridge, he would like to once again eliminate the most expensive bridges while ensuring that the islands stay connected, however keeping “Ben’s Bridge” no matter how expensive it is.

Design an efficient algorithm to find the lowest cost network of bridges to keep, given that some bridge $b$ is designated as “Ben’s Bridge.” Your algorithm should run in $O(m \log n)$ time.

5. Consider a network of $n$ land mines on a battlefield. The mines are placed in a manner such that detonation of a single mine may also causes other mines in a certain vicinity to detonate, which in turn causes more mines to detonate, creating a chain reaction of explosions. More formally, consider all the mines that will detonate as a result of mine $x$ detonating as the neighborhood of $x$. Note that detonating a land mine $y$ that is in the neighborhood of $x$ will only result in $x$ detonating if $x$ is in the neighborhood of $y$ as well.

After sending a probe to survey the battlefield, you have discovered the neighborhood of each land mine. Moreover, you know that the total number of pairwise relations between all the land mines is $m$. We consider a certain mine “volatile” if detonating it creates a chain reaction that causes all other land mines to detonate. Provide an $O(m + n)$ algorithm that takes these land mines and knowledge of the neighborhoods of each land mine as inputs, and outputs the set of all “volatile” mines.