Learning Goals

During this lab, you will:

- Approach problems with consideration for running time complexity
- Apply sorting as a tool for solving some real problems
- Consider tradeoffs between different solutions to problems
- Get a feel for the sort of problem solving that involved in this course

Problem 1: Unique Character Strings

_Given a string of length n, determine whether it contains all unique characters—or, whether any characters are used more than once._

_Tips: Guess the lower-bound! What is the limit on the fastest running time algorithm that anyone can formulate for this problem?_

Problem 2: Anagram Strings

_Given two strings, determine whether they are anagrams of each other._

_TOMMARVOLORIDDLE ←→ IAMLORDVOLDEMORT
The above are anagrams of each other!
(Sorry if I spoiled book 2 of Harry Potter for you…)_

Problem 3: Dutch National Flag

_Given n balls of three colors—say red, white, and blue—arranged randomly in a line, sort them as quickly as possible into contiguous groups of red, white, and blue, with the groups in that order._

_How fast can generic sorts do this?_

Problem 4: Row/Column-sorted Matrix Membership

_Given an m by n matrix of integers, where each row and each column of the matrix is sorted, determine whether an integer k exists somewhere in the matrix._

- _How could we do this if we didn’t know the rows and columns were sorted?_
- _How does knowing the rows and columns are sorted let us do this faster?_
- _What is our optimal running time?_
Problem 5: Stable Matching

Consider a town with $n$ men and $n$ women seeking to get married to one another. Each man has a preference list that ranks all the women, and each woman has a preference list that ranks all the men.

The set of all $2n$ people is divided into two categories: good people and bad people. Suppose that for some number $k$, $1 \leq k \leq n - 1$, there are $k$ good men and $k$ good women; thus there are $n - k$ bad men and $n - k$ bad women.

Everyone would rather marry any good person than any bad person. Formally, each preference list has the property that it ranks each good person of the opposite gender higher than each bad person of the opposite gender: its first $k$ entries are the good people (of the opposite gender) in some order, and its next $n - k$ are the bad people (of opposite gender) in some order.

Show that in every stable matching, every good man is married to a good woman.

Hints:

- What are the properties of a stable matching?
- Try to convince yourself that the claim is true.
- Which proof technique (think 160!) might be best for this type of proof?