Definitions

Definition 1 (Shortest path). A shortest path from vertex \( s \) to vertex \( t \) is a directed path from \( s \) to \( t \) with the property that no other such path has a lower total edge weight.

Dijkstra’s Algorithm

Dijkstra’s algorithm finds the shortest path between two given vertices in a weighted graph, assuming that the graph’s edge weights are non-negative. The running time of the algorithm is \( O(E \log V + V \log V) \) when the graph is implemented using adjacency lists. With a special transformation (use of Fibonacci heaps) this can be reduced to \( O(E + V \log V) \), which is the fastest version of this algorithm. The pseudo-code for the algorithm is given below.

Pseudocode

\[\text{Dijkstra}(G, s)\]
1. for each vertex \( v \in V_G \)
2. \( \text{dist}[v] = \infty \)
3. \( \text{parent}[v] = \text{NIL} \)
4. \( \text{dist}[s] = 0 \)
5. \( Q = V_G \)
6. while \( Q \neq \emptyset \)
7. \( u = \text{Extract-Min}(Q) \)
8. for each vertex \( v \in G.\text{Adj}[u] \)
9. if \( \text{dist}[v] > \text{dist}[u] + w(u, v) \)
10. \( \text{dist}[v] = \text{dist}[u] + w(u, v) \)
11. \( \text{parent}[v] = u \)

Edge-Weighted DAGs (Directed Acyclic Graphs)

The algorithm for shortest path on edge weighted DAGs is simpler and faster than Dijkstra’s algorithm. However, instead of considering vertices by priority of their distance estimates, we consider the vertices of the DAG in a topological order. (Why must a DAG always have a topological order?) Then we just relax each vertex in the topological ordering. Running time: \( O(|V| + |E|) \).

Questions

Problem 1. Explain the running time of Dijkstra’s algorithm.
Problem 2. Dijkstra’s algorithm is a greedy algorithm. What does this mean?
Problem 3. How could you modify the algorithm to find all shortest paths?
Problem 4. How could you modify the algorithm to stop once it’s found the shortest path to a particular node?
Problem 5. Find the shortest path between vertices \( E \) and \( G \).
Problem 6. Find the shortest path between vertices $E$ and $G$.

Problem 7. Does Dijkstra’s Algorithm work with negative weights? Why or why not?

Problem 8. True or false: Dijkstra’s algorithm will not terminate if run on a graph with negative edge weights.

Problem 9. True or false: If we double the weights of all the edges in a graph, then Dijkstra’s algorithm will produce the same shortest path.

Problem 10. True or false: The shortest path algorithm in an edge weighted DAG works even with negative edge weights.