Proposition:

\[ C(N) \leq C(N/2) + C(N/2) + N \]

\[ \text{left half} \quad \text{merge} \quad \text{right half} \]

\[ D(N) \]

Proposition: If \( D(N) \) satisfies

\[ D(N) = 2 \cdot D(N/2) + N \]

for \( N > 1 \)

with \( D(1) = 0 \), the total comparisons are

\[ D(N) = N \log_2 N \]

Proof by picture assuming \( N \) is a power of 2.

Extra cost for merge:

\[ N^2 \]

Total:

\[ D(N) = N \log_2 N \]
Proposition: If \( D(N) \) satisfies \( D(N) = 2 \cdot D(N/2) + N \) for \( N \geq 1 \) with \( D(1) = 0 \) then \( D(N) = N \cdot \log_2 N \).

Proof by telescoping (assuming \( N \) is a power of 2):

For \( N > 1 \):

\[
\frac{D(N)}{N} = \frac{2 \cdot D(N/2)}{N/2} + \frac{N}{N} \]

\[
= \frac{D(N/4)}{N/4} + 1 + 1 \]

\[
= \frac{D(N/8)}{N/8} + 1 + 1 + 1 \]

...