

**CIS 121**  
**Practice Problems for Exam 2**  
**March 20, 2018**

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- 1.** Prove or disprove: You are given a connected undirected graph  $G = (V, E)$  with a weight function  $w$  defined over its edges. Let  $s \in V$  be an arbitrary vertex in  $G$ . Starting at vertex  $s$ , if you do a depth-first search (DFS) in  $G$  such that the edges going out of any vertex are always explored in increasing order of weight, then the resulting DFS tree is also a minimum spanning tree.
  
- 2.** You are given an input stream which will display  $n$  integers, and you only get to view each element once. Design an efficient algorithm which will find the  $k$  largest elements in the stream, using at most  $O(k)$  space (assume  $k \ll n$ ).
  
- 3.** Prove that an edge  $e$  is contained in every spanning tree for a connected graph  $G$  if and only if removal of  $e$  disconnects  $G$ .
  
- 4.** Let  $G = (V, E)$  be a strongly connected directed graph and let  $T$  be a DFS tree in  $G$ . Prove that if all the forward edges in  $G$ , with respect to  $T$ , are removed from  $G$ , the resulting graph is still strongly connected.
  
- 5.** Give an example of a weighted connected undirected graph  $G = (V, E)$  and a vertex  $v$  such that the minimum spanning tree of  $G$  is different than the shortest path tree rooted at  $v$ . Can the two trees be completely disjoint?