CIS 121 - Fall 2009

Midterm Review 2

1

Draw a binary tree such that each internal node stores a single character and a preorder traversal reads EXAMFUN while an inorder traversal reads MAFXUEN.

Solution: In parenthetical notation: (E(X(A(MF)U)N)).

2

Let a proper binary tree have height h and n nodes. What are the minimum and maximum number of external nodes? Show that:

$$\log(n+1) - 1 \leq h \leq (n-1)/2$$

Solution: A balanced tree with height h has a maximum of 2^{h-1} external nodes and a minimum of $2^{h-2} + 1$ (for $h > 2$).
Given a binary tree with height h, the maximum possible number of nodes is $n \leq 2^{h+1} - 1$ or $log(n+1) - 1 \leq h$. Note that this tree is proper as every node of depth $< h$ has exactly 2 children and the remaining nodes are all leaves having 0 children. The smallest number of nodes for any tree would be a straight line of exactly $h+1$. However since every internal node needs two children (and there are h internal nodes) we need another h nodes. Therefore $n \geq 2h+1$ or $(n-1)/2 \geq h$. Overall $log(n+1) - 1 \leq h \leq (n-1)/2$.

3

Describe in pseudocode an algorithm for finding all descendants of a node in a binary tree. Assume that external nodes are null and that you have three methods for *getleft*, *getright*, and *getparent*.

Solution:

```
Algorithm getDescendents(Node v):
   List nodeList = new List
   if v is internal
      if v.getleft is internal
```

```
      nodeList.add(v.getleft)
      nodeList.addall(getDescendents(v.getleft))
    endif
    if v.getright is internal
      nodeList.add(v.getright)
      nodeList.addall(getDescendents(v.getright))
    endif
  endif
  return nodeList
```

4

Show the result of inserting 10,12,1,14,6,5,8,15,3,9,7,4,11,13,2 in this order in a binary heap.

Solution: (1(3(6(15 14) 7 (12 9))) 2 (5 (10 11) 4 (13 8)))

5

Use a heap to design an algorithm that finds the k-th smallest element of a set of n distinct integers in O(n+k logn) time.

Solution: Construct a heap, which takes $O(n)$ time. Then call `removeMin` k times, which takes $O(k \log n)$ time.

6

How many different binary search trees can store the keys 1,2,3,4 ?

Solution: 14 as following: 5 with 1 as root, 2 with 2 as root, 2 with 3 as root, and 5 with 4 as root

7

Show the result of inserting 2,1,4,5,9,3,6,7 in this order into an empty AVL-tree.

Solution: For the AVL tree, 4(2(1 3)6(5 9(7))).

8

Given the input

71, 23, 73, 99, 44, 79, 89

and the hash-function $h(x) = x \bmod 11$ and a table of size 11 show the results of separate chaining and of linear probing. Show the results of double hashing with $h(x) = (i + jh_2(x)) \bmod 11$ with $i = x \bmod 11$ and $h_2(x) = 7 - x \bmod 7$.

Solution:

Index	Linear Probing	Separate Chaining	Double Hashing
0	99	• → 99 → 44	99
1	23	• → 23 → 89	23
2	44	• → 79	79
3	79		89
4	89		
5	71	• → 71	71
6			
7	73	• → 73	73
8			
9			
10			44

9

Draw the frequency array, the Huffman tree, the table with the codes, and the code for the following string:

ABAABAACAABAACACAACBAACBAACA

Solution:

Symbol	'A'	'B'	'C'
Frequency	17	5	6
Encoding	1	00	01

Encoded string is: 10011001 10111001 10110111 01001101 0011011

Huffman tree is (Using 0 to denote left child, and 1 for right child): R(0(B C) A)