

CIS121 - Fall 2008

Lab 2 – Monday/Tuesday, September 15/16

1

Show that $8n^2 + 10n + 25$ is not $O(n)$.

2

Suppose that $f(x)$, $g(x)$ and $h(x)$ are functions such that $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Show that $f(x)$ is $O(h(x))$.

3

Prove that the factorial function $n!$ is $O(n^n)$. Remember the factorial function $f(n) = n!$ is defined by

$$n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$$

when n is a positive integer, and $0! = 1$.

4

True or false:

1. 2^{n+1} is $O(2^n)$.
2. 2^{2n} is $O(2^n)$.

5

For each of the following, if true prove using the definition of big-Oh bound, if false give a counterexample.

1. $g(n)$ is $O(nf(n))$, where $g(n) = \sum_{i=1}^n f(i) = f(1) + f(2) + \dots + f(n)$, and $f(n)$ positive.
2. For any positive $f(n)$ and $g(n)$, if $\log(f(n))$ is $O(\log(g(n)))$ then $f(n)$ is $O(g(n))$.

6

Give a big-Oh characterization for the running time of the code fragments below, with a short explanation:

```
a) for(int i = 0; i < n; i++ )
    for( int j = i; j <= n; j++ )
        for( int k = i; k <= j; k++ )
            sum++;
```

```
b) static int foo(int[] a) {
    int s=a[0];
    for(int i=2;i<=n;i=i+2)
        s=s+a[i];
    return s;
}
```

```
c) for( int p = 0; p < n*n; p++ )
    for( int q = 0; q < p; q++ )
        sum++;
```