This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

1. **[10 pts]** Prove that, given a graph $G$, if $\chi(G) \geq 6$, then there must exist at least two odd cycles $C_1$ and $C_2$, such that $V(C_1) \cap V(C_2) = \emptyset$.

Recall that $\chi(G)$ denotes the minimum number of colors needed to color the vertices of $G$ such that no two adjacent vertices have the same color, and $V(C_i)$ are the vertices in cycle $C_i$.

2. **[10 pts]** Krishna is very satisfied. On top of successfully cooking his first Thanksgiving turkey, Krishna’s Kandy Emporium is doing very well. There are currently $n$ people enjoying the amusement park, which consists of $k$ thrilling rides. However, Krishna is a little bit concerned about the wait time of the rides. Namely, he wants to make sure that there are no lines for rides, so he decides to put his CIS 160 knowledge to good use. Luckily, the people at the park are not very picky about which rides they like. Specifically, the number of rides that each person likes is at least as large as the number of people who like any one ride. Prove that Krishna can assign people to go on rides such that every person gets to go on a ride that they like, and any given ride has at most one person going on it.

3. **[18 pts]** Matt is sick and tired of all the Christmas cheer, so he forces all the TAs to play dreidel to get in the Hanukkah spirit instead. But, Matt realizes that he doesn’t actually own a dreidel, so he has to use a standard six-sided die instead. On each player’s turn, the player gets to roll the die at most $n$ times, where $n$ is some arbitrary but particular positive integer. If the die is not a six, the player gets to take a piece of gelt (chocolate coins) from the pot and continue rolling the die. If the die is a six, the player still gets to take a piece of gelt, but then his turn is over. Let $X$ be the number of pieces of gelt a player gets on his turn. Find $\mathbb{E}[X]$ and $\text{Var}[X]$.

4. **[12 pts]** This question is not about expectations;

Let $R, S$ be equivalence relations.

Both are relations upon the same set;

But please don’t get too excited just yet.
You will see there are two statements below,
When you respond, please try to be thorough!
Either prove it with a proof that is ample;
Or disprove it with a counterexample.

(a) $R \cup S$ is an equivalence relation.

(b) $R \cap S$ is an equivalence relation.

5. [20 pts] Vinai and Ianiv both had a blast shopping on Black Friday. After stocking up on matching clothes from the outlet mall, the twins went to their final and most anticipated stop: the Pet Store. They were having a Buy-One-Get-One-Free sale on pigeons, so Ianiv purchases a pair of those. Moreover, they were having a 75% off sale on gibbons, so Vinai purchased one for himself; Vinai and his gibbon soon grew very close.

Before long, Vinai noted that his gibbon was gifted with extraordinary intelligence. Vinai trained his gibbon to sit at a typewriter and hit keys. The gibbon would hit keys corresponding to each of the uppercase letters $A - Z$ uniformly at random, one at a time. Excited by this, Vinai waits for his gibbon to eventually type his initials: “VR.” What is the expected number of letters that Vinai’s gibbon must type until Vinai sees the first instance of “VR” written by the typewriter?

Hint: Consider how we applied the memoryless property of geometric R.V.s in recitation last week.