1. [18 pts] Give answers to the following questions.

(a) Write the following sets explicitly, i.e. list the members of these sets.
   i. \( \{ x \mid x \text{ is a square of an integer, } x \text{ is a cube of an integer, and } x < 105 \} \)
   ii. \( \{ x \mid x \text{ is an integer such that } x^2 = 5 \} \)
   iii. \( \{ x \mid x \text{ is a real number such that } x^4 - 80 = 1 \} \)
   iv. \( \{ x \mid 3x \text{ is a positive integer less than 22 and } 4|x \} \)

(b) Use the set builder notation to give a description of each of these sets.
   i. \( \{ 7, 9, 14, 18, 21, 27, 28, 35, 36 \} \)
   ii. \( \{ 1, 5, 17, 37, 65, 101 \} \)
   iii. \( \{ -2, -1, 0, 1, 2, 3, 4, 5 \} \)

(c) What is the cardinality of each of the following sets?
   i. \( \{ 4 \} \)
   ii. \( \{ a, \{ a \} \} \)
   iii. \( \{ \{ p, m \} \} \)
   iv. \( \{ d, \{ d \}, \{ d, \{ d \} \} \} \)

(d) Determine whether each of the following is true or false.
   i. \( \emptyset \subseteq \{ x \} \)
   ii. \( \emptyset \in \{ x \} \)
   iii. \( x \in \{ x \} \)
   iv. \( x \subseteq \{ x \} \)
   v. \( \{ x \} \in \{ x \} \)
Homework 2w

vi. \( \{x\} \subseteq \{x\} \)

vii. \( \{x\} \in \{\{x\}\} \)

(e) What is the power set of \( \{x, y\} \), where \( x \) and \( y \) are distinct elements? What is its cardinality?

(f) Find two sets \( A \) and \( B \) such that \( A \in B \) and \( A \subseteq B \).

2. [8 pts] In a chess tournament the top five players play matches as follows: First #5 plays #4. The loser receives 5th prize and the winner plays #3 in another match. The loser of this match receives 4th prize and the winner plays #2. The loser of this match receives 3rd prize and the winner plays #1. The winner of this match gets 1st prize and the loser gets 2nd prize. In how many orders can players ranked #1 through #5 receive the prizes?

3. [10 pts] After Katie introduced Krishna to “Exploding Kittens,” Krishna discovered that he loves board games. For his new extracurricular project, he has decided to open a life-size Candy Land amusement park.

First, Krishna must create the Candy Land board. A Candy Land board consists of consecutive solid colored segments that curve throughout the board, passing the various candy-themed characters. Each rectangle in the line can be any one of 28 different colors.

Krishna thinks the board will look best if the colors of the segments in the line are symmetric about the middle of the line. Since he is a very curious CIS 160 TA, Krishna wonders how many ways he could create such a line. However, after a while trying to solve the problem, he realized he was late for “Board Game Wednesdays” and ran home.

Can you help Krishna out? Namely, given that the line has \( n \) segments, how many ways can he create the line such that it is symmetric about the middle?

4. [7 pts] Yonah has a very large, boisterous family. At Yonah’s family reunion, where there are \( n \geq 2 \) people, there is at least one person who has not hugged everyone else at the reunion. What is the maximum number of family members at the reunion who could have hugged everyone else?

5. [10 pts] Arnab and Rishab got in a huge fight because they couldn’t decide whether to use \( \overline{p} \) or \( \neg p \) as the preferred symbol for negation. For weeks, the two refused to talk to each other and their relationship went sour. Eventually, Arnab realized that this argument didn’t really matter because the two expressions were equivalent; in order to remedy the situation, Arnab decides to create a fruit basket as a peace offering for Rishab.
Arnab goes to the gourmet fruit store with his basket in hand. At the store they have 10 persimmons, 15 jackfruits, and 40 lychees. Arnab’s basket can hold up to 65 fruits and he wants to put at least 1 fruit in the basket. Assuming that fruits of the same type are indistinguishable, how many different ways can Arnab create a basket? (Note that there is no notion of ordering in Arnab’s basket and that you do not have to use all of the fruit in the store.)

6. [7 pts] To reward his students for finishing their homework, Rajiv takes his students and TAs out on a camping trip. Everyone gets their own tent, which is either red or blue, and the tents are set up in a circle. Rajiv notices that for each tent in the circle, its neighbor to the left is the same color as its neighbor to the right. He wakes up Jeffrey, who is sleeping in a red tent, and Simran, who is sleeping in a blue tent, to tell them this. If there are 36 red tents in the circle, how many people are on the camping trip?

7. [10 pts]

Krishna decides to shame Rohan for not reading the lecture notes before staff meeting. He devises a game to test whether Rohan has truly mastered the material he has to teach.

At the start of the game, 2017 balls numbered distinctly from 1 to 2017 are placed into a giant bingo cage. In addition, there is a scoreboard which starts at 0. On each turn, a ball is drawn from the cage and the number is read aloud. Rohan then has a choice of either adding or subtracting the number from his current score. If his final score is 0 after all the balls are drawn, he wins!

Help Rohan find a winning strategy if possible. If it’s not possible, explain why.