1. [10 pts] Prove the following.

(a) Let $A = \{n \in \mathbb{N} | n = 4k + 3, \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} | n = 8k + 3, \text{ for some } k \in \mathbb{N}\}$. Prove that $B \subset A$.

(b) Let $A = \{n \in \mathbb{N} | n = 4k + 2, \text{ for some } k \in \mathbb{N}\}$ and $C = \{m \in \mathbb{N} | m = 4k - 2 \text{ for some } k \in \mathbb{N} \text{ and } k \geq 1\}$. Prove that $A = C$.

(c) Let $A = \{n \in \mathbb{N} | n = 3k + 6, \text{ for some } k \in \mathbb{N}\}$ and $B = \{m \in \mathbb{N} | m = 7k - 1, \text{ for some } k \in \mathbb{Z}^+\}$. Prove that $A \neq B$. 

2. [10 pts] Answer the following.

(a) After an unsuccessful attempt at shaming Rohan, Krishna sets his sights on Bharath. Bharath claims that his favorite number is 6 since $2^6 - 1$ is 63, which he claims is prime. Krishna shakes his head in disappointment and points out that 63 is in fact not prime and moreover that for any positive even integer $k > 2$, $2^k - 1$ is never prime. Can you prove Krishna’s claim?

(b) After being shamed by Krishna, Bharath decides to prove the following statement to Krishna in an effort to impress him:

$$r^{1/t}$$ is irrational.

Unfortunately he is having trouble proving the statement. Can you prove it for him?

3. [10 pts] After getting Arnab’s fruit basket, Rishab decides to come up with his own peace offering and make up with his beloved friend. Knowing that Arnab loves super hard proofs, Rishab slides into Arnab’s Canvas DMs with the following question:

Prove or disprove that $\forall x \in \mathbb{R}, \lfloor -x \rfloor = -\lfloor x \rfloor$. 