This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - no collaboration is allowed.

1. [10 pts] Katie is an avid box collector. One day, she came across the best looking box that she had ever seen in her life. Immediately, she pulled out her measuring tape to determine the dimensions of this perfect box. To her pleasant surprise, the box’s dimensions are all distinct positive integers. Furthermore, the volume of the box (in ft$^3$) is equal to half that of the total surface area (in ft$^2$) of the box. If the box measures $a \times b \times c$ where $a < b < c$ (all dimensions are in ft), determine all possible triples for $(a, b, c)$.

2. [10 pts] Let $a, b, c$ be integers satisfying $a^2 + b^2 = c^2$. Prove that $abc$ must be even.

3. [18 pts]
   
   (a) Prove that if $A$ and $B \setminus C$ are disjoint, then $A \cap B \subseteq C$.

   (b) Suppose $A$, $B$, and $C$ are sets with $A \cap B \cap C = \emptyset$. Prove or disprove:
   
   $$|A \cup B \cup C| = |A| + |B| + |C|$$

   (c) Let $A$, $B$, $C$, $D$ be arbitrary sets. Prove that
   
   $$(A \cap C) \cup (B \cap D) \subseteq (A \cup B) \cap (C \cup D)$$

4. [10 pts] Matt is on a mission to find a brownie recipe that is better than Jessica’s. He researches online for new recipes to try and comes up with a set of recipes $R = \{1, 2, \ldots, n\}$. Matt then realizes that since he is a senior, he does not have time to try all of the recipes himself. So, Matt wants to divide the recipes between Stephanie, Vinai, and Waley and have them report back on which recipes are best. Matt wants to ensure that each recipe is tried at least once, but he also does not want to waste ingredients, so decides that any single recipe cannot be given to all three people. Given the previous conditions, in how many ways can Matt give Stephanie, Vinai, and Waley brownie recipes to test?

5. [7 pts] Nicole has two favorite colors: red and blue. As such, her toy box is full of $n$ distinct types of toys such that for each type, she owns a red-colored variant and a blue-colored variant.
She invites some of her friends over to show them some of her 2n toys. However, she doesn’t want to show them both the red and blue variants of any single type of her toys (that would be boring!) and she wants to show them at least one toy (otherwise, she would be wasting their time!). How many different selections of toys can she make?

6. [15 pts] Now that Krishna has created the board for his life-sized Candy Land amusement park, Krishna’s Kandy Emporium, he needs to choose where to place the characters throughout the board. To make things interesting, Krishna creates 10 characters and names them with the first 10 letters of the alphabet (A-J). Before he sends them off to their respective places, he wants to take a picture of all of the characters in a line to use as advertising for the grand opening. However, Krishna believes that the picture will only look good if the characters are arranged such that each character, other than the left-most character, has either the character whose name is alphabetically directly before or directly after somewhere to the left of them (not necessarily their immediate left). For example, either D or F must be somewhere to the left of E.

How many ways can Krishna arrange the ten characters for his photo?

(Hint: Think how to apply the Multiplication Rule)