1. [10 pts] Somil’s Strawberry Stand decides to compete with Will’s Watermelon World and Krishna’s Kandyland by opening Somil’s Strawberry Sling, Somil’s very own carnival game. Each game consists of throwing nine distinct strawberries into four distinct baskets, with each strawberry equally likely to land in any of the baskets. If at the end of the game, all four baskets contain at least one strawberry, you win a jar of Somil’s Strawberry Sauce. Annie, the owner of Annie’s Amazing Apples, decides to check out Somil’s Strawberry Sling and plays one game. What is the probability that she wins a jar of Somil’s Strawberry Sauce?

2. [10 pts] Rohan presents the following problem in his recitation: Suppose that the sequence \( a_1, a_2, \ldots \) is defined by \( a_1 = 1 \) and \( a_{n+1} = 3a_n^2 \), for \( n \geq 1 \). Prove using induction that \( a_n \leq 3^{2^n} \), for all positive integers \( n \).

   (a) He first lets his induction hypothesis be \( a_k \leq 3^{2^k} \). Does this work? If not, show why.

   (b) Instead, Shawn points out that he could try to prove the statement \( a_n \leq 3^{(2^n-1)} \) using induction. Prove Shawn’s claim and show that it implies what Rohan wanted to prove in part a.

3. [10 pts] In celebration of Vinai’s birthday, the CIS 160 staff bands together to get him the best possible present—two brand-new, fantastic proofs. But, Vinai didn’t really want proofs for his birthday (he actually wanted a bicycle) and is pretty disappointed. The CIS 160 staff feels bad and decides to buy him his bicycle and assign the two proofs to you instead. Can you complete Vinai’s birthday challenge?

   Let \( A \) and \( B \) be two events. For each of these inequalities, do the following:

   (i) Prove the inequality.

   (ii) Under what conditions for \( A \) and \( B \) are these inequalities satisfied as equations?

   A mathematical derivation is required.

   (a) \( \Pr[A] + \Pr[B] \geq \Pr[A \cup B] \geq \max\{\Pr[A], \Pr[B]\} \)
(b) \( \min\{\Pr[A], \Pr[B]\} \geq \Pr[A \cap B] \geq \Pr[A] + \Pr[B] - 1 \)