Problem B1 (40 pts). For any integer, $n \geq 0$, let

$$L_n = \{ w \in \{a, b\}^* \mid |w| \neq n \}.$$

(1) Construct a DFA with $n + 2$ states accepting $L_n$.

(2) Let $D = (Q, \{a, b\}, \delta, q_0, F)$ be any DFA over the alphabet $\Sigma = \{a, b\}$. Describe a construction of a DFA $D_{\neq n}$ such that $L(D_{\neq n}) = \emptyset$ iff the DFA $D$ does not accept strings of length $n$.

Problem B2 (50 pts). Let $R$ be any regular language over some alphabet $\Sigma$. Prove that the language

$$L^{1/2} = \{ u \in \Sigma^* \mid uu \in R \}$$

is regular.

Hint. Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting $R$. Express $L^{1/2}$ as a finite union of intersections $L^1_p \cap L^2_p$ of languages $L^1_p$ and $L^2_p$ (with $p \in Q$) accepted by DFA’s obtained by modifying $D$.

Problem B3 (45 pts). Let $L$ be a regular language. Are the following languages regular, and if so, give a construction and a proof of its correctness.

(a) $\text{Pre}(L) = \{ u \mid u$ is a prefix of some $w \in L \}$

(b) $\text{Suf}(L) = \{ u \mid u$ is a suffix of some $w \in L \}$

(c) $\text{Sub}(L) = \{ u \mid u$ is a substring of some $w \in L \}$

TOTAL: 135 points