Spring 2020 CIS 262

Automata, Computability and Complexity
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Homework 4

February 6, 2020; Due February 13, 2020, beginning of class

"B problems" must be turned in.

Problem B1 (50 pts). Let $\Sigma = \{a_1, \ldots, a_n\}$ be an alphabet of n symbols, with $n \geq 2$.

(1) Construct an NFA with $2n+1$ states accepting the set L_n of strings over Σ such that, every string in L_n has an odd number of a_i, for some $a_i \in \Sigma$. Equivalently, if L_n^i is the set of all strings over Σ with an odd number of a_i, then $L_n = L_n^1 \cup \cdots \cup L_n^n$.

(2) Prove that there is a DFA with 2^n states accepting the language L_n.

(3) Prove that every DFA accepting L_n has at least 2^n states.

Hint. If a DFA D with $k < 2^n$ states accepts L_n, show that there are two strings u, v with the property that, for some $a_i \in \Sigma$, u contains an odd number of a_i's, v contains an even number of a_i's, and D ends in the same state after processing u and v. From this, conclude that D accepts incorrect strings.

Problem B2 (40 pts). For any integer, $n \geq 0$, let

$$L_n = \{w \in \{a, b\}^* \mid |w| \neq n\}.$$

(1) Construct a DFA D_n with $n + 2$ states accepting L_n.

(2) Let $D = (Q, \{a, b\}, \delta, q_0, F)$ be any DFA over the alphabet $\Sigma = \{a, b\}$. Describe a construction of a DFA $D_{\neq n}$ (obtained from D) such that $L(D_{\neq n}) = \emptyset$ iff the DFA D does not accept strings of length n.

Problem B3 (50 pts). Let R be any regular language over some alphabet Σ. Prove that the language

$$L^{1/2} = \{u \in \Sigma^* \mid uu \in R\}$$

is regular.

Hint. Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA accepting R. Express $L^{1/2}$ as a finite union of intersections $L_p^1 \cap L_p^2$ of languages L_p^1 and L_p^2 (with $p \in Q$) accepted by DFA's obtained by modifying D.

TOTAL: 140 **points**