Spring 2020 CIS 262

Automata, Computability and Complexity
Jean Gallier

Review Session

Released February 19, 2020

Problem 1. (1) Give an NFA with four states (with a single ϵ-transition) accepting $L = \{aa, bb\}^*$.

(2) Convert the NFA of question (1) to a DFA.

Problem 2. (1) Given a DFA $D = (Q, \Sigma, \delta, q_0, F)$, construct an equivalent DFA $D' = (Q', \Sigma, \delta', q'_0, F')$ (i.e. such that $L(D') = L(D)$) and such that there are no incoming transitions into the start state q'_0 of D'.

(2) Prove of disprove (by giving a counter-example) the following statement:

A DFA, D, accepts a finite language iff its underlying directed graph has no cycle.

Problem 3. Given an alphabet Σ, for any language L over Σ, if $L^R = \{w^R \mid w \in L\}$ is the reversal language of L, prove that

$(L_1 L_2)^R = L_2^R L_1^R$;

$(L^*)^R = (L^R)^*$.

Problem 4. (1) Let $\Sigma = \{a, b\}$. Give a DFA accepting

$$L_1 = \{w \in \Sigma^* \mid w \text{ contains an even number of } a\text{'s}\}$$

(There is one with two states and, by the way, $0 =$ zero is even.)

(2) Give a DFA accepting

$$L_2 = \{w \in \Sigma^* \mid w \text{ contains a number of } b\text{'s divisible by } 3\}$$

(There is one with three states and the number of b's is $0, 3, 6, 9, \ldots$.)

(3) Give a DFA accepting $L_3 = L_1 \cap L_2$.

Problem 5.

Let $\Sigma = \{a, b\}$. Describe a method taking as input any DFA D (over $\{a, b\}$) and testing whether

$$L(D) = \{a\}^* b \{a, b\}^*.$$

Hint. Recall that for any two sets, X, Y, we have $X \subseteq Y$ iff $X - Y = \emptyset$.

Problem 6.

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA and assume that Q contains $n \geq 1$ states. Prove that if there is some string $w \in \Sigma^*$ such that $w \in L(D)$ and $|w| \geq n$, then there is some string $u \in \Sigma^*$ such that $u \in L(D)$ and $|u| < n$.