Name (printed): ____________________________

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Signature: ____________________________ Date: ____________________________

SOLUTIONS
1. True or False (10 points)

a. T F In X86 assembly, the instruction Jmp eax will cause the program counter register (eip) to be set to the contents of the eax register.

b. T F We could remove the Call and Ret instructions from the X86lite subset we’ve been using for class projects without changing the expressiveness of the language.

c. T F It is possible to write a regular expression that matches that set of strings consisting of well-balanced parentheses.

d. T F The regular expression $(\epsilon + b + bb)(a + ab + abb)^*$ accepts all strings of a’s and b’s with at most two consecutive occurrences of b. (Here + indicates alternative choice.)

e. T F LR(k) grammars cannot be right recursive.

f. T F There is no such thing as a shift/shift conflict for a LR parser.

g. T F In an LR(0) parser state, the item $S \longrightarrow (L)$ indicates that nonterminal $L$ is at the top of the stack because $L$ is to the right of the dot.

h. T F Choosing integer tags as a representation for OCaml datatype variants or C and Java-style enums to be sequential and dense (for example just the integers $0 \ldots N$) allows match and switch compilation to use efficient jump tables.

i. T F In C, the compiler lays out a 2D array with statically known dimensions (like int $M[3][4]$) so that all of the elements are contiguous in memory.

j. T F When compiling C for 32-bit x86 and word aligned fields,

\[
\text{sizeof(struct\{char a; char b; int c\})}
\]

returns 6 (bytes).
2. Lexing, Parsing, and Grammars (20 points)

a. Consider the following ambiguous context-free grammar for the language of regular expressions, where $R$ is the only nonterminal and the terminals are taken from the set \{‘$c$’, *, +, (, ), $\epsilon$\}. Here ‘$c$’ stands for a collection of character-carrying tokens, where the character is $c$. (We use + rather than | for alternation to avoid confusion with the | symbol used on the grammar.)

$$R ::= \epsilon | \ 'c' | R + R | R^* | RR | (R)$$

We might implement the datatype of abstract syntax trees for this grammar using the following OCaml code:

```ocaml
type rexp =
| Eps (* $\epsilon$ *)
| Char of char (* ‘$c$’ *)
| Alt of rexp * rexp (* $R + R$ *)
| Star of rexp (* $R^*$ *)
| Seq of rexp * rexp (* $RR$ *)
```

i. (6 points) Demonstrate that this grammar is ambiguous by giving two different abstract syntax trees (OCaml values of type rexp) that might be generated by parsing the input sequence:

‘a’+(‘b’‘c’‘d’)

Alt(Char ‘a’, Seq(Char ‘b’, Seq(Char ‘c’, Char ‘d’)))
or
Alt(Char ‘a’, Seq(Seq(Char ‘b’, Char ‘c’), Char ‘d’))

ii. (8 points) Write down the context-free grammar obtained by disambiguating the language above so that all operators associate to the left and * has higher precedence than sequence, which has higher precedence than +.

$$R ::= R + S | S$$
$$S ::= ST | T$$
$$T ::= \epsilon | \ 'c' | (R) | T^*$$
b. (6 points) Recall that a *palindrome* is a sequence that reads the same forward and backward: A and ABBA and BABAB are all palindromes but AB is not. Write down a context free grammar that recognizes all (and only) the palindromes over the two tokens \{A, B\}. Your grammar is allowed to be ambiguous. Use the nonterminal \( S \) as the start symbol.

\[
S ::= \\
\quad | \epsilon \\
\quad | A \\
\quad | B \\
\quad | ASA \\
\quad | BSB
\]
3. LLVM IR & Intermediate code generation (25 points)

In this problem we consider the LLVM intermediate representation. For your reference, the Appendix at the end of the exam defines the grammar for the subset of the LLVM IR we have been using (so far) in the course.

a. (4 points) A collection of LLVM Lite IR blocks with an entry point entry must satisfy a number of invariants for it to be well formed (i.e. “make sense”) as a program. For example, each %uid that is used as an operand in an instruction must be defined before it is used.

What is the static single assignment invariant relative to the LLVM IR subset given in the appendix? Each %uid must appear on the left hand side of a defining = exactly once in the entire control-flow graph.

b. (6 points) What value will be returned by running the following LLVM code, starting from the label entry?

```
entry:
%t1 = alloca
store 3, %t1
%t2 = load %t1
%t3 = load %t1
%t4 = mul %t2, %t3
store %t4, %t1
%t5 = icmp eq %t2, %t3
br %t5, label %then, label %else

then:
%t6 = alloca
%t7 = load %t1
%t8 = mul %t7, %t7
store %t8, %t6
%t9 = load %t6
store %t9, %t1
br label %else

else:
%t10 = load %t1
ret %t10
```

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c. (15 points) Some languages have a `do { stmt } while(exp)` construct that executes `stmt` once and then repeats it until the guard expression `exp` becomes false. In this problem, we investigate how to compile this statement form to LLVM IR.

Suppose we have (most of the) implementations of compilation judgments:

\[
\begin{align*}
C &\vdash [exp] = (\text{operand} \ast \text{stream}) \\
C &\vdash [stmt] = \text{stream}
\end{align*}
\]

Here, a stream is a sequence of labels, LLVM instructions, and terminators that can be broken up into basic blocks. (You need not distinguish these cases with constructors.)

How do you translate the `do–while` construct to LLVM? That is, give (OCaml-like) pseudo code that says how you would implement the rule below. Indicate which labels and LLVM uids should be freshly generated.

\[
C \vdash [\text{do} \{ \text{stmt} \} \text{ while } (exp)] = \\
\begin{array}{l}
\text{let } (\text{op}, \text{exp_insns}) = C \vdash [exp] \text{ in} \\
\text{let } \text{stmt_insns} = C \vdash [stmt] \text{ in} \\
\text{let } \text{tmp} = \text{mk_uid}() \text{ in} \\
\text{let } \text{lpre} = \text{mk_label}() \text{ in} \\
\text{let } \text{lpost} = \text{mk_label}() \text{ in} \\
\text{lpre:} \\
\quad \text{stmt_insns} \\
\quad \text{exp_insns} \\
\quad \text{\%tmp} = \text{icmp eq op, 0} \\
\quad \text{br \%tmp label \%lpost, label \%lpre} \\
\text{lpost:}
\end{array}
\]
4. **X86 Assembly Programming** (20 points)

Consider the following C function:

```c
void foo(int x, int* y) {
    int a = x + (*y);
    *y = a;
}
```

Recall that `int* y` declares `y` to be an `int` pointer and that `*y` accesses the contents pointed to by `y`. The `gcc` compiler (in 32-bit only mode and without optimizations) produces the following X86 assembly code, which is in our X86lite subset and follows cdecl calling conventions:

```
_foo:
pushl %ebp
movl %esp, %ebp ; %ebp set here
subl $12, %esp
movl 12(%ebp), %eax
movl 8(%ebp), %ecx
movl %ecx, -4(%ebp)
movl %eax, -8(%ebp)
movl -8(%ebp), %eax
movl (%eax), %eax
movl -4(%ebp), %ecx
addl %ecx, %eax
movl %eax, -12(%ebp)
movl -8(%ebp), %eax
movl -12(%ebp), %ecx
movl %ecx, (%eax)
addl $12, %esp
popl %ebp
ret
```

a. (4 points) The caller of this function places argument `x` at which (indirect offset) memory location? (Relative to the value in `%ebp` after the line marked above.)

a. `-12(%ebp)`  b. `-8(%ebp)`  c. `-4(%ebp)`  d. `8(%ebp)`  e. `12(%ebp)`

b. (4 points) The local variable `a` resides at which (indirect offset) memory location? (Relative to the value in `%ebp` after the line marked above.)

a. `12(%ebp)`  b. `-8(%ebp)`  c. `-4(%ebp)`  d. `8(%ebp)`  e. `12(%ebp)`

c. (4 points) How much memory does the stack frame used by `_foo` in this code take up in bytes? Include the saved return address and base pointer, and any stack space allocated for local storage, but *not* the space allocated by the caller for the function arguments.

a. 8 bytes  b. 16 bytes  c. **20 bytes**  d. 24 bytes  e. 28 bytes  f. 32 bytes
d. (8 points) Which of the following optimized versions could replace the body _foo: and still be correct with respect to the C program and cdecl calling conventions? Mark all that are correct—there may be more than one.

i. THIS ONE

_foo:

```
pushl %ebp
movl %esp, %ebp
movl 12(%ebp), %eax
movl 8(%ebp), %ecx
addl %ecx, (%eax)
popl %ebp
ret
```

ii. _foo:

```
pushl %ebp
movl %esp, %ebp
movl 12(%ebp), %eax
addl 8(%ebp), %eax
movl %ebp, %esp
popl %ebp
ret
```

iii. _foo:

```
movl 4(%esp), %eax
addl 8(%esp), %eax
ret
```

iv. _foo:

```
movl 4(%esp), %ebx
movl 8(%esp), %eax
addl %ebx, (%eax)
ret
```
5. Scope Checking (25 points)

In this problem we will consider scope checking (a simple subset of) OCaml programs. The grammar for this subset of OCaml is given by the syntactic categories below:

\[
\text{exp} ::= \text{Expressions} \\
| x \mid f \quad \text{variables and function names} \\
| \text{int} \quad \text{integer constants} \\
| \text{exp}_1 + \text{exp}_2 \quad \text{arithmetic} \\
| \text{exp}_2 \text{exp}_2 \quad \text{function application} \\
| \text{let } x = \text{exp}_1 \text{ in } \text{exp}_2 \quad \text{local lets} \\
| (\text{exp})
\]

\[
\text{prog} ::= \text{Programs} \\
| ;; \text{exp} \quad \text{answer expression} \\
| \text{let } x = \text{exp} \text{ prog} \quad \text{top-level declarations} \\
| \text{let } f x = \text{exp} \text{ prog} \quad \text{top-level, one-argument function declarations}
\]

Scoping contexts for this language consist of comma-separated lists of variable and function names:

\[
\text{G} ::= \text{Scoping Contexts} \\
| \cdot \quad \text{empty context} \\
| x, G \quad \text{add x to the context} \\
| f, G \quad \text{add f to the context}
\]

Scope checking for expressions is defined by the following inference rules, which use judgments of the form \(G \vdash \text{exp}\). Recall that the notation \(x \in G\) means that \(x\) occurs in the context list \(G\).

- **[VARX]**: 
  \[
  \frac{x \in G}{G \vdash x}
  \]

- **[VARF]**: 
  \[
  \frac{f \in G}{G \vdash f}
  \]

- **[INT]**: 
  \[
  \frac{G \vdash \text{exp}_1 \quad G \vdash \text{exp}_2}{G \vdash \text{exp}_1 + \text{exp}_2}
  \]

- **[ADD]**: 
  \[
  \frac{G \vdash \text{exp}_1 \quad G \vdash \text{exp}_2}{G \vdash \text{exp}_1 \text{exp}_2}
  \]

- **[APP]**: 
  \[
  \frac{G \vdash \text{exp}_1 \quad G \vdash \text{exp}_2}{G \vdash \text{exp}_1 \text{exp}_2}
  \]

- **[LET]**: 
  \[
  \frac{G \vdash \text{exp}_1 \quad G \vdash \text{exp}_2 \quad \text{let } x = \text{exp}_1 \text{ in } \text{exp}_2}{G \vdash \text{exp}_2}
  \]

Scope checking for programs is defined by these three inference rules, which use judgments of the form \(G \vdash \text{prog}\).

- **[EXP]**: 
  \[
  \frac{G \vdash \text{exp}}{G \vdash ;; \text{exp}}
  \]

- **[LETX]**: 
  \[
  \frac{G \vdash \text{exp} \quad G \vdash \text{prog}}{G \vdash \text{let } x = \text{exp} \text{ prog}}
  \]

- **[LETF]**: 
  \[
  \frac{G \vdash \text{exp} \quad G \vdash \text{prog}}{G \vdash \text{let } f x = \text{exp} \text{ prog}}
  \]

A program \(\text{prog}\) is considered to be well scoped exactly when it is possible to derive a judgment in the empty context: \(\cdot \vdash \text{prog}\).
a. Consider the following program:

```
let x = 3
let f y = let z = 3 + y in z + x
;; f x
```

It is well-scoped, as shown by the following (partial) derivation that has some missing pieces as marked by names \( G? \), \( D1? \), and \( D2? \).

\[
\begin{array}{cccccccccccc}
\hline
y, x, \vdash 3 & \text{[INT]} & y, x, \vdash y & \text{[VARX]} & z \in G? & \text{[VARX]} & x \in G? & \text{[VARX]} \\
\hline
y, x, \vdash 3 + y & \text{[ADD]} & G? \vdash z & \text{[ADD]} & G? \vdash z + x & \text{[LET]} & D2?
\end{array}
\]

\[
\begin{array}{cccccccccccc}
\hline
y, x, \vdash \text{let } z = 3 + y \text{ in } z + x & \text{[LET]} \\
\hline
x, \vdash \text{let } f \ y = \text{let } z = 3 + y \text{ in } z + x & \text{[LETF]} \\
\hline
\end{array}
\]

\[
\begin{array}{cccccccccccc}
\hline
\vdash \text{let } x = 3 & \text{[LETX]} \\
\hline
\text{let } f \ y = \text{let } z = 3 + y \text{ in } z + x \\
\hline
;; f x
\end{array}
\]

i. (3 points) Which of the following derivation trees belongs in the spot marked \( D1? \)

- a. \( x \in x, \cdot \) \text{ [VARX]}
- b. \( \vdash 3 \) \text{ [INT]}
- c. \( x, \cdot \vdash 3 \) \text{ [INT]}
- d. \( x \in \cdot \vdash x \) \text{ [VARX]}

ii. (3 points) What is the missing context \( G? \)

- a. \( z, x, \cdot \)
- b. \( y, x, \cdot \)
- c. \( x, y, z, \cdot \)
- d. \( z, y, x, \cdot \)

iii. (8 points) In the space below, draw the missing derivation tree marked by \( D2? \). Be sure to include the names of the rules as in the tree above.

\[
\begin{array}{cccccccccccc}
\hline
f \in f, x, \cdot & \text{[VARF]} & x \in f, x, \cdot & \text{[VARX]} & x \in f, x, \cdot & \text{[VARX]} \\
\hline
f, x, \cdot \vdash f & \text{[APP]} & f, x, \cdot \vdash x & \text{[EXP]} \\
\hline
\end{array}
\]
b. (5 points) The following program is syntactically well-formed but ill-scoped—one of the $x \in G$ checks fails for some variable $x$ and some context $G$. What are they? (Be careful with the ordering of variables in the context!)

```plaintext
let a = 3
let f1 b = b + a
let f2 c = (let d = f1 3 in f1 a) + (f1 d)
;; f2 (f1 a)
```

\[ x = d \]

\[ G = c, f1, a, \cdot \]

c. (6 points) The LETF rule above does not allow the function to be recursive. Write down the premises of the rule needed to allow a well-scoped function body to mention the function name itself:

\[
\frac{\frac{f, x, G \vdash \text{exp} \quad f, G \vdash \text{prog}}{G \vdash \text{let } f \ x = \text{exp prog}}}{[\text{LETREC}]}
\]
Appendix

LLVM Lite IR

A subset of the LLVM IR that we have used in Project 3:

\[
\begin{align*}
\text{op} & ::= \%\text{uid} \mid \text{constant} \\
\text{bop} & ::= \text{add} \mid \text{sub} \mid \text{mul} \mid \text{shl} \mid \ldots \\
\text{cmpop} & ::= \text{eq} \mid \text{ne} \mid \text{slt} \mid \text{sle} \mid \ldots \\
\text{insn} & ::= \\
& \quad | \%\text{uid} = \text{bop op1,op2} \\
& \quad | \%\text{uid} = \text{alloca} \\
& \quad | \%\text{uid} = \text{load op1} \\
& \quad | \text{store op1,op2} \\
& \quad | \%\text{uid} = \text{icmp cmpop op1,op2} \\
\text{terminator} & ::= \\
& \quad | \text{ret op} \\
& \quad | \text{br op label \%lbl1,label \%lbl2} \\
& \quad | \text{br label \%lbl} \\
\text{block} & ::= \%\text{lbl}: \\
& \quad \text{insn}_1 \\
& \quad \ldots \\
& \quad \text{insn}_n \\
& \quad \text{terminator}
\end{align*}
\]

An LLVM Lite IR program is a collection of blocks such that all labels mentioned in the terminators of those blocks are included among the labels of the blocks themselves. One label called entry is the designated entry point of the program. A \%uid is an operand identifier and \%lbl is a label identifier.