Lecture 20

CIS 341: COMPILERS
Announcements

• HW5: Full OAT: Objects & Typechecking
  – Implement (parts of) the typechecker and compiler for an OO-language
• DUE: Monday April 6th
A high-level tour of a variety of optimizations.
When to apply optimization

- Inlining
- Function specialization
- Constant folding
- Constant propagation
- Value numbering
- Dead code elimination
- Loop-invariant code motion
- Common sub-expression elimination
- Strength Reduction
- Constant folding & propagation
- Branch prediction / optimization
- Register allocation
- Loop unrolling
- Cache optimization
Inlining

• Replace a call to a function with the body of the function itself with arguments rewritten to be local variables:

• Example in OAT code:

```c
int g(int x) { return x + pow(x); }
int pow(int a) { int b = 1; int n = 0;
    while (n < a) {b = 2 * b}; return b; }
```

→

```c
int g(int x) { int a = x; int b = 1; int n = 0;
    while (n < a) {b = 2 * b}; tmp = b; return x + tmp; }
```

• May need to rename variable names to avoid name capture
  – Example of what can go wrong?

• Best done at the AST or relatively high-level IR.

• When is it profitable?
  – Eliminates the stack manipulation, jump, etc.
  – Can increase code size.
  – Enables further optimizations
Code Specialization

• Idea: create specialized versions of a function that is called from different places with different arguments.

• Example: specialize function \( f \) in:

```java
class A implements I { int m() {...} }
class B implements I { int m() {...} }
int f(I x) { x.m(); } // don’t know which m
A a = new A(); f(a); // know it’s A.m
B b = new B(); f(b); // know it’s B.m
```

• \( f_A \) would have code specialized to dispatch to \( A.m \)
• \( f_B \) would have code specialized to dispatch to \( B.m \)
• You can also inline methods when the run-time type is known statically
  – Often just one class implements a method.
Common Subexpression Elimination

- In some sense it’s the opposite of inlining: fold redundant computations together
- Example:

\[ a[i] = a[i] + 1 \] compiles to:
\[ [a + i*4] = [a + i*4] + 1 \]

Common subexpression elimination removes the redundant add and multiply:
\[ t = a + i*4; \quad [t] = [t] + 1 \]

- For safety, you must be sure that the shared expression always has the same value in both places!
Unsafe Common Subexpression Elimination

• Example: consider this OAT function:

```c
unit f(int[] a, int[] b, int[] c) {
    int j = ...; int i = ...; int k = ...;
    b[j] = a[i] + 1; c[k] = a[i]; return;
}
```

• The following optimization that shares the expression `a[i]` is unsafe... why?

```c
unit f(int[] a, int[] b, int[] c) {
    int j = ...; int i = ...; int k = ...;
    t = a[i];
    b[j] = t + 1; c[k] = t; return;
}
```
LOOP OPTIMIZATIONS
Loop Optimizations

• Program hot spots often occur in loops.
  – Especially inner loops
  – Not always: consider operating systems code or compilers vs. a computer game or word processor

• Most program execution time occurs in loops.
  – The 90/10 rule of thumb holds here too. (90% of the execution time is spent in 10% of the code)

• Loop optimizations are very important, effective, and numerous
  – Also, concentrating effort to improve loop body code is usually a win
Loop Invariant Code Motion (revisited)

- Another form of redundancy elimination.
- If the result of a statement or expression does not change during the loop and it’s pure, it can be hoisted outside the loop body.
- Often useful for array element addressing code
  - Invariant code not visible at the source level

```java
for (i = 0; i < a.length; i++) {
    /* a not modified in the body */
}

for (i = 0; i < a.length; i++) {
    /* same body as above */
}
```

For clarity, the expression `t = a.length;` is hoisted out of the loop to be evaluated once, making the loop invariant code visible at the source level.
Strength Reduction (revisited)

• Strength reduction can work for loops too
• Idea: replace expensive operations (multiplies, divides) by cheap ones (adds and subtracts)
• For loops, create a dependent induction variable:

• Example:
  for (int i = 0; i<n; i++) { a[i*3] = 1; }  // stride by 3

  int j = 0;
  for (int i = 0; i<n; i++) {
      a[j] = 1;
      j = j + 3;  // replace multiply by add
  }
Loop Unrolling (revisited)

• Branches can be expensive, unroll loops to avoid them.
  
  ```c
  for (int i=0; i<n; i++) { S }
  ```

  ```c
  for (int i=0; i<n-3; i+=4) {S;S;S;S};
  for (       ; i<n; i++) { S } // left over iterations
  ```

• With k unrollings, eliminates (k-1)/k conditional branches
  – So for the above program, it eliminates ¾ of the branches

• Space-time tradeoff:
  – Not a good idea for large S or small n

• Interacts with instruction caching, branch prediction
EFFECTIVENESS?
Optimization Effectiveness?

\[
\text{%speedup} = \left( \frac{\text{base time}}{\text{optimized time}} - 1 \right) \times 100\%
\]

Example:
base time = 2s  
optimized time = 1s  \Rightarrow  100\% speedup

Example:
base time = 1.2s  
optimized time = 0.87s  \Rightarrow  38\% speedup

Graph taken from:
Jianzhou Zhao, Santosh Nagarakatte, Milo M. K. Martin, and Steve Zdancewic.  
Formal Verification of SSA-Based Optimizations for LLVM.  
In Proc. 2013 ACM SIGPLAN Conference on Programming Languages Design and Implementation (PLDI), 2013
Optimization Effectiveness?

- **mem2reg**: promotes alloca’ed stack slots to temporaries to enable register allocation
- **Analysis:**
  - mem2reg alone (+ back-end optimizations like register allocation) yields ~78% speedup on average
  - -O1 yields ~100% speedup (so all the rest of the optimizations
  - -O3 yields ~120% speedup
- **Hypothetical program that takes 10 sec. (base time):**
  - Mem2reg alone: expect ~5.6 sec
  - -O1: expect ~5 sec
  - -O3: expect ~4.5 sec
CODE ANALYSIS
Motivating Code Analyses

- There are lots of things that might influence the safety/applicability of an optimization
  - What algorithms and data structures can help?

- How do you know what is a loop?
- How do you know an expression is invariant?
- How do you know if an expression has no side effects?
- How do you keep track of where a variable is defined?
- How do you know where a variable is used?
- How do you know if two reference values may be aliases of one another?
Moving Towards Register Allocation

• The OAT compiler currently generates as many temporary variables as it needs
  – These are the %uids you should be very familiar with by now.

• Current compilation strategy:
  – Each %uid maps to a stack location.
  – This yields programs with many loads/stores to memory.
  – Very inefficient.

• Ideally, we’d like to map as many %uid’s as possible into registers.
  – Eliminate the use of the *alloca* instruction?
  – Only 16 max registers available on 64-bit X86
  – %rsp and %rbp are reserved and some have special semantics, so really only 10 or 12 available
  – This means that a register must hold more than one slot

• When is this safe?
Liveness

- Observation: `%uid1` and `%uid2` can be assigned to the same register if their values will not be needed at the same time.
  - What does it mean for an `%uid` to be “needed”?
  - Ans: its contents will be used as a source operand in a later instruction.
- Such a variable is called “live”
- Two variables can share the same register if they are not live at the same time.
We can already get some coarse liveness information from variable scoping.

Consider the following OAT program:

```c
int f(int x) {
    int a = 0;
    for (int b = 0; b < 10; b=b+1;) {
        a = a+b*x;
    }
    int c = a * x; return c;
}
```

Note that due to OAT’s scoping rules, variables `b` and `c` can never be live at the same time.

- `c`’s scope is disjoint from `b`’s scope

So, we could assign `b` and `c` to the same allocated slot and potentially to the same register.
But Scope is too Coarse

- Consider this program:

```c
int f(int x) {
    int a = x + 2;
    int b = a * a;
    int c = b + x;
    return c;
}
```

- The scopes of `a`, `b`, `c`, `x` all overlap – they’re all in scope at the end of the block.
- But, `a`, `b`, `c` are never live at the same time.
  - So they can share the same stack slot / register
Live Variable Analysis

• A variable $v$ is live at a program point if $v$ is defined before the program point and used after it.

• Liveness is defined in terms of where variables are defined and where variables are used.

• Liveness analysis: Compute the live variables between each statement.
  – May be conservative (i.e. it may claim a variable is live when it isn’t) so because that’s a safe approximation.
  – To be useful, it should be more precise than simple scoping rules.

• Liveness analysis is one example of dataflow analysis.
  – Other examples: Available Expressions, Reaching Definitions, Constant-Propagation Analysis, …
Control-flow Graphs Revisited

• For the purposes of dataflow analysis, we use the control-flow graph (CFG) intermediate form.

• Recall that a basic block is a sequence of instructions such that:
  – There is a distinguished, labeled entry point (no jumps into the middle of a basic block)
  – There is a (possibly empty) sequence of non-control-flow instructions
  – The block ends with a single control-flow instruction (jump, conditional branch, return, etc.)

• A control flow graph
  – Nodes are blocks
  – There is an edge from B1 to B2 if the control-flow instruction of B1 might jump to the entry label of B2
  – There are no “dangling” edges – there is a block for every jump target.

• Note: the following slides are intentionally a bit ambiguous about the exact nature of the code in the control flow graphs:
  – at the x86 assembly level
  – an “imperative” C-like source level
  – at the LLVM IR level
  – Same general idea, but the exact details will differ
    • e.g. LLVM IR doesn’t have “imperative” update of %uid temporaries.
    • In fact, the SSA structure of the LLVM IR makes some of these analyses simpler.
Dataflow over CFGs

- For precision, it is helpful to think of the “fall through” between sequential instructions as an edge of the control-flow graph too.
  - Different implementation tradeoffs in practice…
Liveness is Associated with *Edges*

- This is useful so that the same register can be used for different temporaries in the same statement.
- Example: \( a = b + 1 \)

- Compiles to:

  - \( \text{Mov a, b} \)
  - \( \text{Add a, 1} \)
  - \( \text{Mov eax, eax} \)
  - \( \text{Add eax, 1} \)
Uses and Definitions

• Every instruction/statement uses some set of variables
  – i.e. reads from them
• Every instruction/statement defines some set of variables
  – i.e. writes to them

• For a node/statement s define:
  – use[s] : set of variables used by s
  – def[s] : set of variables defined by s

• Examples:
  – \( a = b + c \)  
    use[s] = \{b,c\}  
    def[s] = \{a\}
  – \( a = a + 1 \)  
    use[s] = \{a\}  
    def[s] = \{a\}
Liveness, Formally

- A variable \( v \) is *live* on edge \( e \) if:
  - There is a node \( n \) in the CFG such that \( \text{use}[n] \) contains \( v \), *and*
  - a directed path from \( e \) to \( n \) such that for every statement \( s' \) on the path, \( \text{def}[s'] \) does not contain \( v \)

- The first clause says that \( v \) will be used on some path starting from edge \( e \).
- The second clause says that \( v \) won’t be redefined on that path before the use.

- Questions:
  - How to compute this efficiently?
  - How to use this information (e.g. for register allocation)?
  - How does the choice of IR affect this? (e.g. LLVM IR uses SSA, so it doesn’t allow redefinition \( \Rightarrow \) simplify liveness analysis)
Simple, inefficient algorithm

• “A variable $v$ is live on an edge $e$ if there is a node $n$ in the CFG using it \textit{and} a directed path from $e$ to $n$ passing through no def of $v$.”

• Backtracking Algorithm:
• For each variable $v$…
• Try all paths from each use of $v$, tracing backwards through the control-flow graph until either $v$ is defined or a previously visited node has been reached.
• Mark the variable $v$ live across each edge traversed.

• Inefficient because it explores the same paths many times (for different uses and different variables)
Dataflow Analysis

- **Idea:** compute liveness information for all variables simultaneously.
  - Keep track of sets of information about each node

- **Approach:** define *equations* that must be satisfied by any liveness determination.
  - Equations based on “obvious” constraints.

- **Solve the equations by iteratively converging on a solution.**
  - Start with a “rough” approximation to the answer
  - Refine the answer at each iteration
  - Keep going until no more refinement is possible: a *fixpoint* has been reached

- This is an instance of a general framework for computing program properties: dataflow analysis
Dataflow Value Sets for Liveness

- Nodes are program statements, so:
  - use[n]: set of variables used by n
  - def[n]: set of variables defined by n
  - in[n]: set of variables live on entry to n
  - out[n]: set of variables live on exit from n

- Associate in[n] and out[n] with the “collected” information about incoming/outgoing edges

- For Liveness: what constraints are there among these sets?
  - Clearly:
    \[ \text{in}[n] \supseteq \text{use}[n] \]

- What other constraints?
Other Dataflow Constraints

- We have: \( \text{in}[n] \supseteq \text{use}[n] \)
  - “A variable must be live on entry to \( n \) if it is used by \( n \)”

- Also: \( \text{in}[n] \supseteq \text{out}[n] - \text{def}[n] \)
  - “If a variable is live on exit from \( n \), and \( n \) doesn’t define it, it is live on entry to \( n \)”
  - Note: here ‘-’ means “set difference”

- And: \( \text{out}[n] \supseteq \text{in}[n'] \) if \( n' \in \text{succ}[n] \)
  - “If a variable is live on entry to a successor node of \( n \), it must be live on exit from \( n \).”
Iterative Dataflow Analysis

- Find a solution to those constraints by starting from a rough guess.
- Start with: \( \text{in}[n] = \emptyset \) and \( \text{out}[n] = \emptyset \)
- They don’t satisfy the constraints:
  - \( \text{in}[n] \supseteq \text{use}[n] \)
  - \( \text{in}[n] \supseteq \text{out}[n] - \text{def}[n] \)
  - \( \text{out}[n] \supseteq \text{in}[n'] \text{ if } n' \in \text{succ}[n] \)

- Idea: iteratively re-compute \( \text{in}[n] \) and \( \text{out}[n] \) where forced to by the constraints.
  - Each iteration will add variables to the sets \( \text{in}[n] \) and \( \text{out}[n] \) (i.e. the live variable sets will increase monotonically)
- We stop when \( \text{in}[n] \) and \( \text{out}[n] \) satisfy these equations:
  (which are derived from the constraints above)
  - \( \text{in}[n] = \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \)
  - \( \text{out}[n] = \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \)
Complete Liveness Analysis Algorithm

for all n, in[n] := Ø, out[n] := Ø
repeat until no change in ‘in’ and ‘out’
    for all n
        out[n] := \( \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \)
        in[n] := use[n] \( \cup (\text{out}[n] - \text{def}[n]) \)
    end
end

• Finds a fixpoint of the in and out equations.
  – The algorithm is guaranteed to terminate… Why?
• Why do we start with Ø?
Example Liveness Analysis

- Example flow graph:

e = 1;
while(x>0) {
    z = e * e;
    y = e * x;
    x = x - 1;
    if (x & 1) {
        e = z;
    } else {
        e = y;
    }
}
return x;
Example Liveness Analysis

Each iteration update:
\[ \text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \]
\[ \text{in}[n] := \text{use}[n] \cup (\text{out}[n] - \text{def}[n]) \]

- **Iteration 1:**
  \[
  \begin{align*}
  \text{in}[2] &= x \\
  \text{in}[3] &= e \\
  \text{in}[4] &= x \\
  \text{in}[5] &= e,x \\
  \text{in}[6] &= x \\
  \text{in}[7] &= x \\
  \text{in}[8] &= z \\
  \text{in}[9] &= y
  \end{align*}
  \]

(showing only updates that make a change)
Example Liveness Analysis

Each iteration update:
out[n] := ∪_{n'∈succ[n]} in[n']
in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 2:
  out[1] = x
  in[1] = x
  out[2] = e,x
  in[2] = e,x
  out[3] = e,x
  in[3] = e,x
  out[5] = x
  out[6] = x
  out[7] = z,y
  in[7] = x,z,y
  out[8] = x
  in[8] = x,z
  out[9] = x
  in[9] = x,y
Example Liveness Analysis

Each iteration update:
\[
\text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n']
\]
\[
\text{in}[n] := \text{use}[n] \cup (\text{out}[n] - \text{def}[n])
\]

- Iteration 3:
  
  out[1] = e, x
  
  out[6] = x, y, z
  
  in[6] = x, y, z
  
  out[7] = x, y, z
  
  out[8] = e, x
  
  out[9] = e, x
Example Liveness Analysis

Each iteration update:

\[
\begin{align*}
\text{out}[n] & := \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \\
\text{in}[n] & := \text{use}[n] \cup (\text{out}[n] - \text{def}[n])
\end{align*}
\]

- Iteration 4:
  \[
  \begin{align*}
  \text{out}[5] &= x, y, z \\
  \text{in}[5] &= e, x, z
  \end{align*}
  \]
Example Liveness Analysis

Each iteration update:
\[
\begin{align*}
out[n] &:= \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \\
\text{in}[n] &:= \text{use}[n] \cup (\text{out}[n] - \text{def}[n])
\end{align*}
\]

- Iteration 5:
  \(\text{out}[3] = e, x, z\)

Done!
Improving the Algorithm

• Can we do better?

• Observe: the only way information propagates from one node to another is using: \( \text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \)
  – This is the only rule that involves more than one node

• If a node’s successors haven’t changed, then the node itself won’t change.

• Idea for an improved version of the algorithm:
  – Keep track of which node’s successors have changed
A Worklist Algorithm

- Use a FIFO queue of nodes that might need to be updated.

for all \( n \), \( \text{in}[n] := \emptyset \), \( \text{out}[n] := \emptyset \)
w = new queue with all nodes
repeat until \( w \) is empty
    let \( n = w.\text{pop}() \)  // pull a node off the queue
    \( \text{old_in} = \text{in}[n] \)  // remember old \( \text{in}[n] \)
    \( \text{out}[n] := \bigcup_{n' \in \text{succ}[n]} \text{in}[n'] \)
in[n] := use[n] \( \cup (\text{out}[n] - \text{def}[n]) \)
if (\( \text{old_in} != \text{in}[n] \)),  // if \( \text{in}[n] \) has changed
    for all \( m \) in \( \text{pred}[n] \), \( w.\text{push}(m) \)  // add to worklist
end