Uninformed Search Strategies

AIMA 3.4
The Goat, Cabbage, Wolf Problem

Problem: The boat only holds two, but you can't leave the goat with the cabbage or the wolf with the goat.

Solution:
1. Take the goat across.
2. Return alone.
3. Take the cabbage across.
4. Leave the wolf. Why did you have a wolf?

(From xkcd.com)
But First: Missionaries & Cannibals

Three missionaries and three cannibals come to a river. A rowboat that seats two is available. If the cannibals ever outnumber the missionaries on either bank of the river, the missionaries will be eaten. (problem 3.9)

How shall they cross the river?
Formulation: Missionaries & Cannibals

- **States:** (CL, ML, BL)
- **Initial state:** (331)
- **Goal test:** True if all M, C, and boat on other bank (000)
- **Actions:** Travel Across  Travel Back
  - -101  101
  - -201  201
  - -011  011
  - -021  021
  - -111  111
Outline for today’s lecture

- Introduction to Uninformed Search
  - (Review of Breadth-first and Depth-first search)

- Iterative deepening search
  - Strange Subroutine: Depth-limited search
  - Depth-limited search + iteration = WIN!!

- Briefly: Bidirectional search

- If time: Uniform Cost Search
Uninformed search strategies:

• AKA “Blind search”
• Uses only information available in problem definition

Informally:
• *Uninformed search*: All non-goal nodes in frontier look equally good
• *Informed search*: Some non-goal nodes can be ranked above others.
Search Strategies

• **Review: Strategy** = order of tree expansion
  • Implemented by different queue structures (LIFO, FIFO, priority)

• **Dimensions for evaluation**
  • **Completeness** - always find the solution?
  • **Optimality** - finds a least cost solution (lowest path cost) first?
  • **Time complexity** - # of nodes generated (worst case)
  • **Space complexity** - # of nodes in memory (worst case)

• **Time/space complexity variables**
  • $b$, maximum branching factor of search tree
  • $d$, depth of the shallowest goal node
  • $m$, maximum length of any path in the state space (potentially $\infty$)
Introduction to space complexity

- You know about:
  - “Big O” notation
  - *Time complexity*

- *Space complexity* is analogous to time complexity

- Units of space are arbitrary
  - Doesn’t matter because Big O notation ignores constant multiplicative factors
  - Space units:
    - One Memory word
    - Size of any fixed size data structure
      - eg Size of fixed size node in search tree
Review: Breadth-first search

- **Idea:**
  - Expand *shallowest* unexpanded node

- **Implementation:**
  - *frontier* is FIFO (First-In-First-Out) Queue:
    - Put successors at the end of *frontier* successor list.
**Breadth-first search (simplified)**

```python
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure

node <- a node with STATE = problem.INITIAL-STATE, PATH-COST=0

if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)

frontier <- a FIFO queue with node as the only element

loop do
    if EMPTY?(frontier) then return failure

    node <- POP(frontier) // chooses the shallowest node in frontier
    add node.STATE to explored

    for each action in problem.ACTIONS(node.STATE) do
        child <- CHILD-NODE(problem, node, action)
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)

    frontier <- INSERT(child, frontier)
```

From Figure 3.11 Breadth-first search (ignores loops, repeated nodes)

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Properties of breadth-first search

- **Complete?** Yes (if $b$ is finite)
- **Time Complexity?** $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
- **Space Complexity?** $O(b^d)$ (keeps every node in memory)
- **Optimal?** Yes, if cost = 1 per step
  (not optimal in general)

$b$: maximum branching factor of search tree
$d$: depth of the least cost solution
$m$: maximum depth of the state space ($\infty$)
Exponential Space (and time) Not Good...

- Exponential complexity uninformed search problems cannot be solved for any but the smallest instances.
- *(Memory requirements are a bigger problem than execution time.)*

<table>
<thead>
<tr>
<th>DEPTH</th>
<th>NODES</th>
<th>TIME</th>
<th>MEMORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>110</td>
<td>0.11 milliseconds</td>
<td>107 kilobytes</td>
</tr>
<tr>
<td>4</td>
<td>11110</td>
<td>11 milliseconds</td>
<td>10.6 megabytes</td>
</tr>
<tr>
<td>6</td>
<td>$10^6$</td>
<td>1.1 seconds</td>
<td>1 gigabyte</td>
</tr>
<tr>
<td>8</td>
<td>$10^8$</td>
<td>2 minutes</td>
<td>103 gigabytes</td>
</tr>
<tr>
<td>10</td>
<td>$10^{10}$</td>
<td>3 hours</td>
<td>10 terabytes</td>
</tr>
<tr>
<td>12</td>
<td>$10^{12}$</td>
<td>13 days</td>
<td>1 petabyte</td>
</tr>
<tr>
<td>14</td>
<td>$10^{14}$</td>
<td>3.5 years</td>
<td>99 petabytes</td>
</tr>
</tbody>
</table>

Fig 3.13 Assumes b=10, 1M nodes/sec, 1000 bytes/node
Review: Depth-first search

• Idea:
  • Expand *deepest* unexpanded node

• **Implementation:**
  • *frontier* is LIFO (Last-In-First-Out) Queue:
    — Put successors at the *front* of *frontier* successor list.
Properties of depth-first search

- **Complete?** No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    → complete in finite spaces
- **Time?** $O(b^m)$: terrible if $m$ is much larger than $d$
  - but if solutions are dense, may be much faster than breadth-first
- **Space?** $O(b^*m)$, i.e., linear space!
- **Optimal?** No

$b$: maximum branching factor of search tree
$d$: depth of the least cost solution
$m$: maximum depth of the state space ($\infty$)
Depth-first vs Breadth-first

- **Use depth-first if**
  - *Space is restricted*
  - There are many possible solutions with long paths and wrong paths are usually terminated quickly
  - Search can be fine-tuned quickly

- **Use breadth-first if**
  - *Possible infinite paths*
  - Some solutions have short paths
  - Can quickly discard unlikely paths
Iterative Deepening Search
Search Conundrum

• Breadth-first
  ✔ Complete,
  ✔ Optimal
  ❌ but uses $O(b^d)$ space

• Depth-first
  ❌ Not complete unless $m$ is bounded
  ❌ Not optimal
  ❌ Uses $O(b^m)$ time; terrible if $m >> d$
  ✔ but only uses $O(b*m)$ space

How can we get the best of both?
Depth-limited search: A building block

- Depth-First search *but with depth limit* $\ell$.
  - i.e. nodes at depth $\ell$ *have no successors*.
  - No infinite-path problem!

- If $\ell = d$ (by luck!), then optimal
  - But:
    - If $\ell < d$ then incomplete 😞
    - If $\ell > d$ then not optimal 😞

- Time complexity: $O(b^\ell)$
- Space complexity: $O(bl)$ 😊
Iterative deepening search

- A general strategy to find best depth limit $\mathcal{L}$
  - Key idea: use Depth-limited search as subroutine, with increasing $\mathcal{L}$.

  For $d = 0$ to $\infty$ do
  
  depth-limited-search to level $d$
  
  if it succeeds
  
  then return solution

- **Complete & optimal**: Goal is always found at depth $d$, the depth of the shallowest goal-node.

  *Could this possibly be efficient?*
Nodes constructed at each deepening

- Depth 0: 0 (Given the node, doesn’t construct it.)

- Depth 1: $b^1$ nodes

- Depth 2: $b$ nodes + $b^2$ nodes

- Depth 3: $b$ nodes + $b^2$ nodes + $b^3$ nodes

- ...
Total nodes constructed:

- Depth 0: 0 (Given the node, doesn’t construct it.)
- Depth 1: \(b^1 = b\) nodes
- Depth 2: \(b\) nodes + \(b^2\) nodes
- Depth 3: \(b\) nodes + \(b^2\) nodes + \(b^3\) nodes
- ...

Suppose the first solution is the last node at depth 3:
Total nodes constructed:
\[3*b\text{ nodes} + 2*b^2\text{ nodes} + 1*b^3\text{ nodes}\]
ID search, Evaluation II: Time Complexity

- More generally, the time complexity is
  - \( N(\text{IDS}) = (d)b + (d-1)b^2 + \ldots + (1)b^d = O(b^d) \)

- As efficient in terms of \( O(\ldots) \) as Breadth First Search:
  - \( N(\text{BFS}) = b + b^2 + \ldots + b^d = O(b^d) \)
ID search, Evaluation III

- **Complete:** YES (no infinite paths)

- **Time complexity:** $O(b^d)$

- **Space complexity:** $O(bd)$

- **Optimal:** YES if step cost is 1.
## Summary of algorithms

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Depth-First</th>
<th>Depth-limited</th>
<th>Iterative deepening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Time</td>
<td>$b^d$</td>
<td>$b^m$</td>
<td>$b^l$</td>
<td>$b^d$</td>
</tr>
<tr>
<td>Space</td>
<td>$b^d$</td>
<td>$bm$</td>
<td>$bl$</td>
<td>$bd$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>
Very briefly: Bidirectional search

- Two simultaneous searches from start and goal.
  - Motivation: \( b^{d/2} + b^{d/2} < b^d \)
  - Check whether the node belongs to the other frontier before expansion.
  - Space complexity is the most significant weakness.
  - Complete and optimal if both searches are Breadth-First.
How to search backwards?

- The predecessor of each node must be efficiently computable.
  - Works well when actions are easily reversible.
“Uniform Cost” Search

“In computer science, \textit{uniform-cost} search (UCS) is a tree search algorithm used for traversing or searching a \textit{weighted} tree, tree structure, or graph.” - Wikipedia
Motivation: Romanian Map Problem

- All our search methods so far assume $\text{step-cost} = 1$
- *This is only true for some problems*
**g(N): the path cost function**

- **If all moves equal in cost:**
  - Cost = # of nodes in path - 1
  - $g(N) = \text{depth}(N)$ in the search tree
  - Equivalent to what we’ve been assuming so far

- **Assigning a (potentially) unique cost to each step**
  - $N_0, N_1, N_2, N_3 =$ nodes visited on path $p$ from $N_0$ to $N_3$
  - $C(i,j)$: Cost of going from $N_i$ to $N_j$
  - If $N_0$ the root of the search tree,
    
    \[ g(N_3) = C(0,1) + C(1,2) + C(2,3) \]
Uniform-cost search (UCS)

- Extension of BF-search:
  - Expand node with *lowest path cost*

- Implementation:
  \[ \text{frontier} = \text{priority queue ordered by } g(n) \]

- Subtle but significant difference from BFS:
  - Tests if a node is a goal state when it is selected for expansion, *not when it is added to the frontier*.
  - Updates a node on the frontier if a better path to the same state is found.
  - So always enqueues a node *before checking whether it is a goal.*

  **WHY???
Uniform Cost Search

Expand cheapest node first:

Frontier is a priority queue

No longer ply at a time, but follows cost contours

Therefore: Must be optimal
Complexity of UCS

- Complete!
- Optimal!
  - if the cost of each step exceeds some positive bound $\varepsilon$.
- Time complexity: $O(b^1 + C*/\varepsilon)$
- Space complexity: $O(b^1 + C*/\varepsilon)$

where $C*$ is the cost of the optimal solution
(if all step costs are equal, this becomes $O(b^{d+1})$)

NOTE: Dijkstra’s algorithm just UCS without goal
## Summary of algorithms (for notes)

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Breadth-First</th>
<th>Uniform-cost</th>
<th>Depth-First</th>
<th>Depth-limited</th>
<th>Iterative deepening</th>
<th>Bidirectional search</th>
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</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Time</td>
<td>$b^d$</td>
<td>$b^{l+C^*/e}$</td>
<td>$b^m$</td>
<td>$b^l$</td>
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Assumes $b$ is finite