Administrivia

Recitations start this week:

- Wednesday, 3:30-5:00PM Levine 612 Advanced
- Wednesday, 6:00-7:30PM Towne 309 Advanced
- Thursday, 6:00-7:30PM Towne 309 Review
- Thursday, 5:00-6:30PM Towne 305 Review
- Thursday, 6:30-8:00PM Towne 305 Advanced
- Thursday, 6:00-7:30PM Towne 303 Review
- Friday, 9:00-10:30AM Moore 212 Review

Permanent assignments to recitations will be made next week. For this week, attend any recitation you like.

Study Groups

How many have already organized into study groups??
We will help organize study groups for the rest, beginning Wednesday.

Homework 2

Homework 1 was due at noon today.
Homework 2 will be due next Monday at noon.

- Read Chapter 6 of Jason Hickey’s “Introduction to the Objective Caml Programming Language” before starting
- Homework submission process is different this week!
Questions from last time...

Are there any?

Basic Pattern Matching

Recursive functions on lists tend to have a standard shape: we test whether the list is empty, and if it is not we do something involving the head element and the tail.

```
# let rec listSum (l:int list) =
  if l = [] then 0
  else List.hd l + listSum (List.tl l);
```

OCaml provides a convenient pattern-matching construct that bundles the emptiness test and the extraction of the head and tail into a single syntactic form:

```
# let rec listSum (l: int list) =
  match l with
  []   -> 0
| x::y -> x + listSum y;
```

Pattern matching can be used with types other than lists. For example, here it is used on integers:

```
# let rec fact (n:int) =
  match n with
  0   -> 1
| _  -> n * fact(n-1);
```

The _ pattern here is a wildcard that matches any value.

Complex Patterns

The basic elements (constants, variable binders, wildcards, [], ::, etc.) may be combined in arbitrarily complex ways in match expressions:

```
# let silly l =
  match l with
  [|;_;_] -> "three elements long"
| _::x::y::z_]::rest -> if x>y then "foo" else "bar"
| _  -> "dunno";
val silly : int list -> string = <fun>
```

# silly [1;2;3];
- : string = "three elements long"

# silly [1;2;3;4];
- : string = "dunno"

# silly [1;2;3;4;5];
- : string = "bar"
Type Inference

One pleasant feature of OCaml is a powerful type inference mechanism that allows the compiler to calculate the types of variables from the way in which they are used.

```
# let rec fact n =  
  match n with  
    0 -> 1  
  | _ -> n * fact(n-1);  
val fact : int -> int = <fun>
```

The compiler can tell that `fact` takes an integer argument because `n` is used as an argument to the integer `*` and `-` functions.

```
# let rec listSum l =  
  match l with  
    [] -> 0  
  | x::y -> x + listSum y;  
val listSum : int list -> int = <fun>
```

Similarly:

Polymorphism

Suppose we ask the OCaml system to infer the type of the following very similar definition:

```
# let rec length l =  
  match l with  
    [] -> 0  
  | _::y -> 1 + length y;  
val length : 'a list -> int = <fun>
```

The `'a` here, pronounced “alpha,” is a type variable, standing for an arbitrary type.

The inferred type tells us that the function can take a list with elements of any type (i.e., a list with elements of type alpha, for any choice of alpha).

We’ll have more to say about polymorphism later.

Tuples

Items connected by commas are “tuples”

```
# "age", 44;  
- : string * int = "age", 44  
# ("professor",("age", 33));  
- : string * (string * int) = "professor", ("age", 33)  
# ("children", ["bob";"ted";"alice)];  
- : string * string list = "children", ["bob"; "ted"; "alice"]
```

```
# let g (x,y) = x*y;  
val g : int * int -> int = <fun>
```

How many arguments does `g` take?
Tuples are not lists

Please do not confuse them!

```haskell
# let tuple = "cow", "dog", "sheep";
val tuple : string * string * string = "cow", "dog", "sheep"
# let list = ["cow"; "dog"; "sheep"];
val list : string list = ["cow"; "dog"; "sheep"]
# List.hd tuple;;
This expression has type string * string * string but is here used
with type 'a list
# List.hd list;;
- : string = "cow"
# let tup2 = 1, "cow";;
val tup2 : int * string = 1, "cow"
# let l12 = [1; "cow"];;
This expression has type string but is here used with type int
```

Tuples and Pattern Matching

Tuples can be “deconstructed” by pattern matching:

```haskell
# let lastName name =
match name with
  (ni,..) -> ni;;
# lastName ("Pierce", "Benjamin", "Penn");
- : string = "Pierce"
```

Example: Finding words

Suppose we want to take a list of characters and return a list of lists of
characters, where each element of the final list is a “word” from the
original list.

```haskell
# split [t'; h'; e'; ' '; 'b'; 'r'; 'o'; 'w'; 'n'; ' ' ; 'd'; 'o'; 'g'];;
- : char list list =
  [[t'; h'; e']; [b'; r'; o'; w'; n'; ' ' ; d'; o'; g']]
```

(Note that character constants are written with single quotes.)

An implementation of `split`

```haskell
# let rec loop w l =
match l with
  [] -> [w]
| (c:ls) -> loop (w @ [c]) ls;;
val loop : char list -> char list -> char list list = <fun>
# let split l = loop [] l;;
val split : char list -> char list list = <fun>
```

Note the use of both tuple patterns and nested patterns (as well as
wildcards).

[N.b.: this version is cleaner than the one in the photocopied slides!]

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Aside: Local function definitions

The `loop` function is completely local to `split`: there is no reason for anybody else to use it — or even, for anybody else to be able to see it! It is good style in OCaml to write such definitions as local bindings:

```ocaml
# let split l =
  let rec loop w l =
    match l with
    [] -> [w]
    | (':':::ls) -> w :: (loop [] ls)
    | (c::ls) -> loop (w @ [c]) ls
  in
  loop [] l;;
```

In general, any let definition that can appear at the top level can also appear in a `let...in...` form.

```ocaml
# let ... in e;;
```

A Better Split

Our `split` function worked fine for the example we tried it on. But here are some other tests:

```ocaml
# split ['a'; ';' ';'; 'b'];;
- : char list list = [['a']; []; ['b']]
# split ['a'; '];
- : char list list = [['a']; []]
```

Could we refine `split` so that it would leave out these spurious empty lists in the result?

Sure. First rewrite the pattern match a little (without changing its behavior):

```ocaml
# let split l =
  let rec loop w l =
    match w,l with
    w, [] -> [w]
    | w, (':':ls) -> w :: (loop [] ls)
    | w, (c::ls) -> loop (w @ [c]) ls
  in
  loop [] l;;
```
Then add a couple of clauses:

```ocaml
# let better_split l =
  let rec loop w l =
    match w,l with
    | [],[]  -> []
    | _,[]   -> [w]
    | [],(w::ls) -> loop [] ls
    | _,(w::ls) -> w :: (loop [] ls)
    | _,(c::ls) -> loop (w @ [c]) ls in
  loop [] l;

# better_split ['a';'b';' ';'c';'d';']
- : char list list = [[['a';'b';['c';['d'];']]]
# better_split ['a';']
- : char list list = [['a']]
# better_split [' ';'']
- : char list list = []
```

Naturally, exceptions can also be caught within a program (using the `try...with...` form), but let’s leave that for another day.

---

### Basic Exceptions

OCaml’s exception mechanism is roughly similar to that found in, for example, Java.

We begin by defining an exception:

```ocaml
# exception Bad;
```

Now, encountering `raise Bad` will immediately terminate evaluation and return control to the top level:

```ocaml
# let rec fact n =
  if n<0 then raise Bad
  else if n=0 then 1
  else n * fact(n-1);;
# fact (-3);
Exception: Bad.
```

---

### Data Types

We have seen a number of data types:

- `int`
- `bool`
- `string`
- `char`
- `lists`
- `tuples`

OCaml has a few other built-in data types — in particular, `float`, with operations like `+`, `*`, etc.

One can also create completely new data types.
The need for new types

The ability to construct new types is an essential part of most programming languages.

Suppose we are building a (very simple) graphics program that displays circles and squares. We can represent each of these with three real numbers.

A circle is represented by the co-ordinates of its center and its radius. A square is represented by the co-ordinates of its bottom left corner and its width. So we can represent both shapes as elements of the type:

\[ \text{float} \times \text{float} \times \text{float} \]

However, there are two problems with using this type to represent circles and squares. First, it is a bit long and unwieldy, both to write and to read. Second, because their types are identical, there is nothing to prevent us from mixing circles and squares. For example, if we write

\[
\text{let areaOfSquare}(\_\_\_, \_d) = d \ast. \_d;
\]

we might accidentally apply the areaOfSquare function to a circle and get a nonsensical result. (Recall that numerical operations on the float type are written differently from the corresponding operations on int — e.g., +, instead of ++. See the OCaml manual for more information.)

Data Types

We can improve matters by defining square as a new type:

\[
\text{# type square = Square of float \times float \times float;};
\]

This does two things:

- It creates a new type called square that is different from any other type in the system.
- It creates a constructor called Square (with a capital S) that can be used to create a square from three floats. For example:

\[
\text{# Square}(1.1, 2.2, 3.3);
\]

Constructors are recognized by being capitalized (the first letter is upper case).

Taking data types apart

We take types apart with (surprise, surprise...) pattern matching.

\[
\text{# let areaOfSquare } s =
match s with
  \text{Square}(\_\_\_, \_d) -> d \ast. \_d;
\text{val areaOfSquare : square -> float = <fun>}
\]

\[
\text{# let bottomLeftCoords } s =
match s with
  \text{Square}(x, y, \_) -> (x,y);
\text{val bottomLeftCoords : square -> float \times float = <fun>}
\]

So we can use constructors like Square both as functions and as patterns.

Constructors are recognized by being capitalized (the first letter is upper case).
These functions can be written a little more concisely by combining the pattern matching with the function header:

```haskell
# let areaOfSquare (Square(_, _, d)) = d * d;;
# let bottomLeftCoords (Square(x, y, _)) = (x,y);;
```

Continuing, we can define a data type for circles in the same way.

```haskell
# type circle = Circle of float * float * float;;
# let c = Circle (1.0, 2.0, 2.0);;
# let areaOfCircle (Circle(_, _, r)) = 3.14159 * r * r;;
# let centerCoords (Circle(x, y, _)) = (x,y);;
# areaOfCircle c;;
- : float = 12.56636
```

We cannot now apply a function intended for type `square` to a value of type `circle`:

```haskell
# areaOfSquare(c);;
This expression has type circle but is here used with type square.
```

**Variant types**

Going back to the idea of a graphics program, we obviously want to have several shapes on the screen at once. For this we’d probably want to keep a list of circles and squares, but such a list would be heterogenous. How do we make such a list?

The solution is to build a type that can be either a circle or a square.

```haskell
# type shape = Circle of float * float * float
  | Square of float * float * float;;
```

Now both constructors `Circle` and `Square` create values of type `shape`. For example:

```haskell
# Square (1.0, 2.0, 3.0);;
- : shape = Square (1.000000, 2.000000, 3.000000)
```

A type that can have more than one form is often called a variant type.

We can also write functions that do the right thing on all forms of a variant type. Again we use pattern matching:

```haskell
# let area s =
  match s with
  | Circle (_, _, r) -> 3.14159 * r * r
  | Square (_, _, d) -> d * d * d;;
# area (Circle (0.0, 0.0, 1.5));;
- : float = 7.0685775
```
A “heterogeneous” list:

```apl
# let l = [Circle (0.0, 0.0, 1.5); Square (1.0, 2.0, 1.0);
  Circle (2.0, 0.0, 1.5); Circle (5.0, 0.0, 2.5)];

# area (List,hd l);;
- : float = 7.0696775
```

Mixed-mode Arithmetic

Many programming languages (Lisp, Basic, Perl, database query languages) use variant types internally to represent numbers that can be either integers or floats. This amounts to “tagging” each numeric value with an indicator that says what kind of number it is.

```apl
# type num = Int of int | Float of float;;

# let add r1 r2 =
  match (r1,r2) with
  (Int i1,  Int i2)  -> Int (i1 + i2)
| (Float r1, Int i2)  -> Float (r1 +. float i2)
| (Int i1,  Float r2)  -> Float (float i1 +. r2)
| (Float r1, Float r2)  -> Float (r1 +. r2);;

# add (Int 3) (Float 4.5);
- : num = Float 7.5
```

Some Higher-Level Mixed-Mode Functions

```apl
# let mult r1 r2 =
  match (r1,r2) with
  (Int i1,  Int i2)  -> Int (i1 * i2)
| (Float r1, Int i2)  -> Float (r1 *. float i2)
| (Int i1,  Float r2)  -> Float (float i1 *. r2)
| (Float r1, Float r2)  -> Float (r1 *. r2);;

# let unaryMinus n =
  match n with Int i -> Int (- i) | Float r -> Float (-. r);;

# let minus n1 n2 = add n1 (unaryMinus n2);;

# let rec fact n =
  if n = Int 0 then Int 1
  else mult n (fact (Int 1));;

# fact (Int 7);
- : num = Int 5040
```
A Data Type for Optional Values

Suppose we are implementing a simple lookup function for a telephone directory. We want to give it a string and get back a number (say an integer). We expect to have a function `lookup` whose type is

\[
\text{lookup: string \to directory \to int}
\]

where `directory` is a (yet to be decided) type that we'll use to represent the directory.

However, this isn't quite enough. What happens if a given string isn't in the directory? What should `lookup` return?

There are several ways to deal with this issue. One is to raise an exception. Another is based on the following data type:

\[
\text{# type maybe = Absent | Present of int};
\]
A Boolean Data Type

A simple data type can be used to replace the built-in booleans. We use the constant constructors `True` and `False` to represent `true` and `false`. We'll use different names as needed to avoid confusion between our booleans and the built-in ones:

```haskell
# type myBool = False | True;
# let myNot b = match b with False -> True | True -> False;;
# let myAnd b1 b2 =
  match (b1,b2) with
  | (True, True) -> True
  | (True, False) -> False
  | (False, True) -> False
  | (False, False) -> False;;
```

Note that the behavior of `myAnd` is not quite the same as the built-in `/\`!

We can translate this grammar directly into a datatype definition:

```haskell
type ast =
  ANum of int
  | APlus of ast * ast
  | AMinus of ast * ast
  | ATimes of ast * ast;;
```

Notes:
- This datatype (like the original grammar) is recursive.
- The type `ast` represents abstract syntax trees, which capture the underlying tree structure of expressions, suppressing surface details such as parentheses.

Recursive Types

Consider the tiny language of arithmetic expressions defined by the following (BNF-like) grammar:

```
exp ::= number
      | ( exp + exp )
      | ( exp * exp )
      | ( exp - exp )
```

(We'll come back to these grammars in more detail next week...)

An evaluator for expressions

Goal: write an evaluator for these expressions.

```haskell
val eval : ast -> int = <func>
# eval (ATimes (APlus (ANum 12, ANum 340), ANum 5));;
  - : int = 1760
```
The solution uses a recursive function plus a pattern match.

```ml
let rec eval e =  
match e with  
  ANum i -> i  
  | APlus (e1,e2) -> eval e1 + eval e2  
  | AMinus (e1,e2) -> eval e1 - eval e2  
  | ATimes (e1,e2) -> eval e1 * eval e2;;
```

A final example

Goal: write a function that takes two lists of equal length and interleaves their elements in alternating fashion:

```ml
# let rec interleave l1 l2 =  
match l1,l2 with  
  [],[] -> []  
  | x::xs, y::ys -> x::y::(interleave xs ys)  
  _ -> raise Bad;;
```

Review

Throughout the course, we will rely on standard (and hopefully familiar!) concepts from compilers, e.g.:

- Lexing
- Parsing
Now suppose that we want to calculate all the possible interleavings of two lists — i.e., all the lists that can be formed by interleaving elements of the input lists in an arbitrary fashion (but maintaining the ordering from the original lists).

For example:

```ocaml
# let rec cons_all e l =  
  match l with  
  | [] -> []  
  | h::t -> (e::h) :: (cons_all e t);  
val cons_all : 'a -> 'a list list -> 'a list list = <fun>

# let rec interleave ll1 ll2 =  
  match ll1,ll2 with  
  | [],_ -> [ll2]  
  | _,[] -> [ll1]  
  | x::xs, y::ys ->  
    List.append  
    (cons_all x (interleave xs ll2))  
    (cons_all y (interleave ll1 ys));  
val interleave : 'a list -> 'a list list -> 'a list list list = <fun>
```

Harder version

```ocaml
# interleave [1;3] [2;4];
- : int list list =  
  [ [1; 3; 2; 4]; [1; 2; 3; 4]; [2; 1; 3; 4]; [2; 1; 4; 3]; [2; 4; 1; 3]]
```