SVM Questions

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Hyperplanes

- **Given the hyperplane defined by the line**
  - $y = x_1 - 2x_2$
  - $y = (1,-2)^T \mathbf{x} = \mathbf{w}^T \mathbf{x}$
- **Is this point correctly predicted?**

  1) $y = 1$, $\mathbf{x} = (1,0)$?

  2) $y = 1$, $\mathbf{x} = (1,1)$?

A. Yes
B. No

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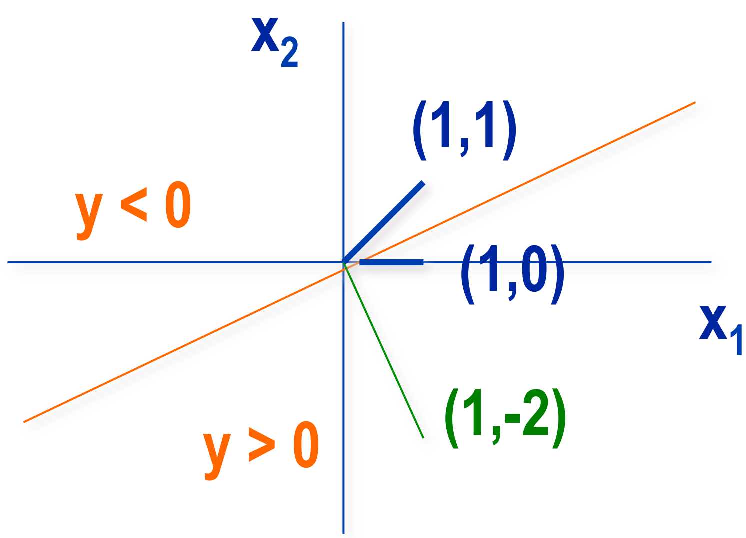


So (1,0) having y=1 is correct and (1,1) having y=1 is not correct

Projections

- The projection of a point x onto a line w is

  $x^T w/|w|_2$

- The distance of a point x to a hyperplane defined by the line w is

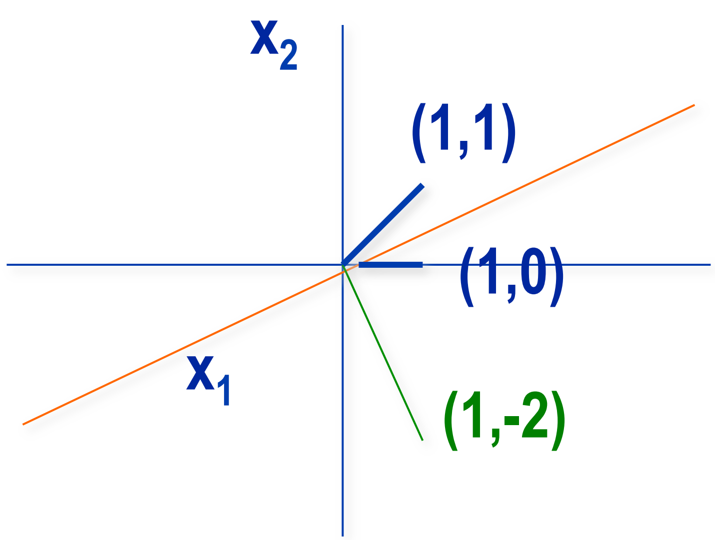
  $x^T w/|w|_2$

Why?

Hyperplanes

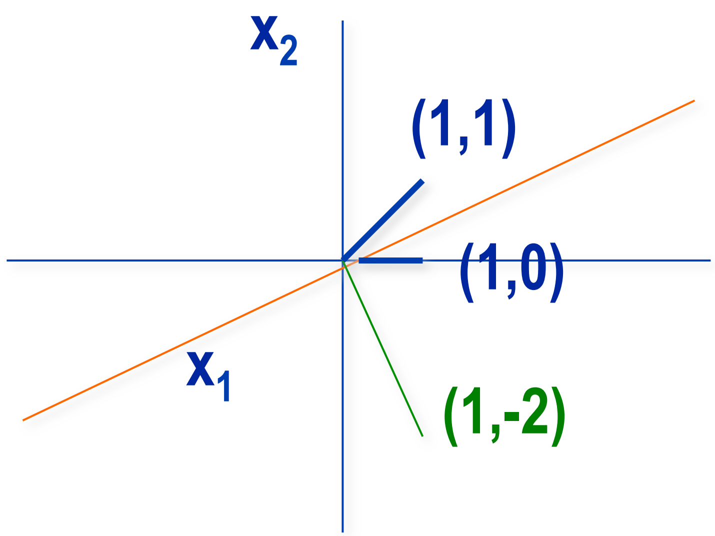
- The projection of a point x on a plane defined by a line w is $x^T w/|w|_2$
- The distance of x from the hyperplane defined by (1, -2) is what
  - for x = (-1,2)
  - for x = (1,0)
  - for x = (1,1)

A) $1/\sqrt{5}$
B) $-1/\sqrt{5}$
C) $1/5$
D) $\sqrt{5}$
E) None of the above



Hyperplanes

- **The projection of a point x on a plane defined by a line w is $x^T w/|w|_2$**
- **The distance of x from the hyperplane defined by (1, -2) is what**
  - for x = (-1,2) $\sqrt{5}$
  - for x = (1,0) $1/\sqrt{5}$
  - for x = (1,1) $1/\sqrt{5}$



project onto the line
$x^T (1\ -2)/\sqrt{(1^2 + (-2)^2)} =$
$x^T (1\ -2)/\sqrt{5}$

Hyperplanes

- *True or False?* **When solving for a hyperplane specified by $\mathbf{w}^\top \mathbf{x} + b = 0$ one can always set the margin to 1;**

$$\mathbf{w}^\top \mathbf{x}_1 + b = -1 \text{ and } \mathbf{w}^\top \mathbf{x}_2 + b = 1$$

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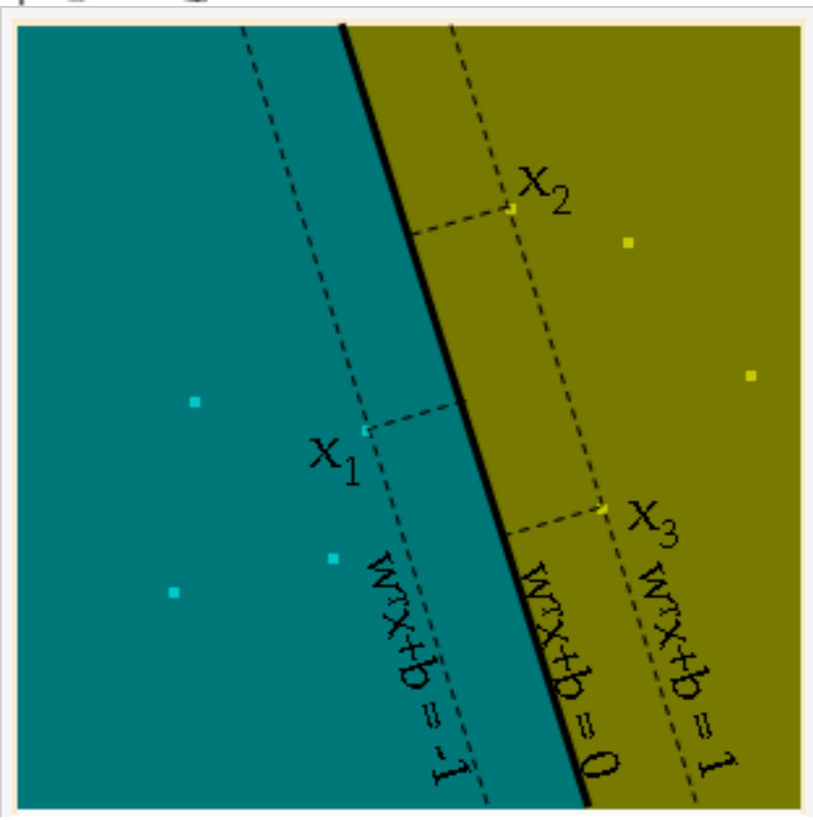
True (as long as it is separable)



Hyperplanes

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- *True or False?* **This then implies that the margin, the distance of the support vectors from the separating hyperplane, is**

$$\frac{\mathbf{w}^\mathsf{T}}{2||\mathbf{w}||_2}(\mathbf{x}_2 - \mathbf{x}_1) = \frac{1}{||\mathbf{w}||_2}$$

Hyperplanes

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True

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True

(non)separable SVMs

- ***True or False?*** In a real problem, you should check to see if the SVM is separable and then include slack variables if it is not separable.

- ***True or False?*** Linear SVMs have **no** hyperparameters that need to be set by cross-validation

(non)separable SVMs

- ***True or False?*** In a real problem, you should check to see if the SVM is separable and then include slack variables if it is not separable.

False: you can just run the slack variable problem in either case (but you need to pick C)

- ***True or False?*** Linear SVMs have **no** hyperparameters that need to be set by cross-validation

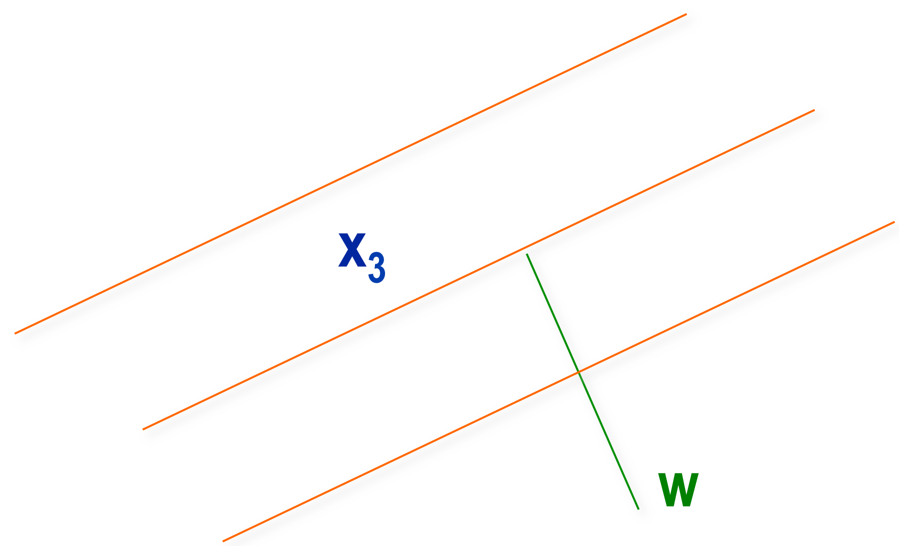
False: you need to pick C

- ***True or False?*** Adding slack variables is equivalent to requiring that all of the α_i are less than a constant in the dual

$$\mathbf{w}^\top \phi(\mathbf{x}) + b = \sum_i \alpha_i y_i k(\mathbf{x}_i, \mathbf{x}) + b$$

Non-separable SVMs

- ***True or False?*** A support vector could be inside the margin.

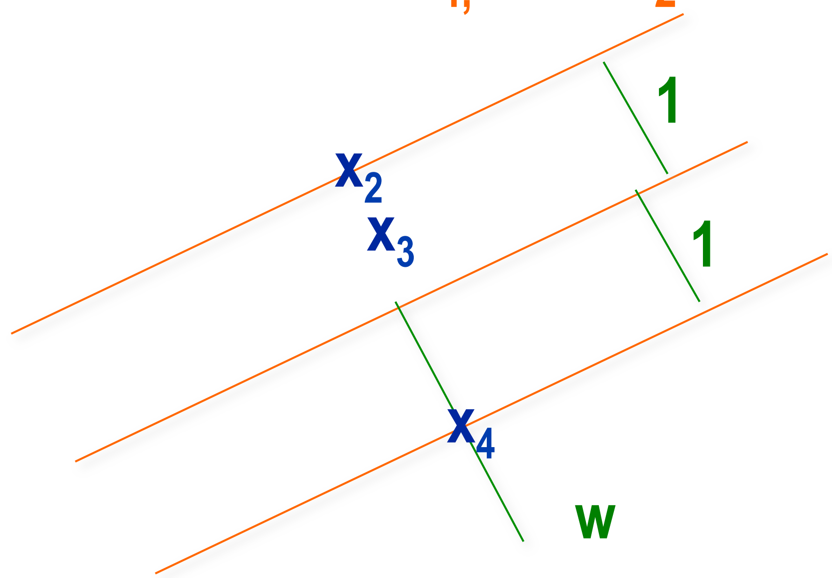


Non-separable SVMs

- $\mathbf{x}_2$, $\mathbf{x}_3$ have y=1, while $\mathbf{x}_4$ has y = -1
- All are co-linear

At what location does x_3 become a support vector?

True/False? It just needs to be closer to $\mathbf{x}_4$, than $\mathbf{x}_2$ is

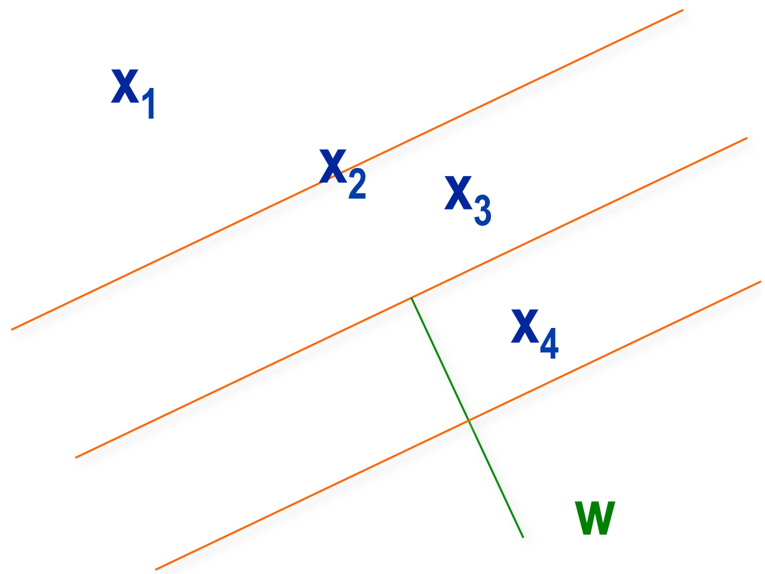


Non-separable SVMs - dual

Consider the following cases

1) a point $\mathbf{x}_1$ is on the correct side of the margin
2) a point $\mathbf{x}_2$ is on the margin
3) a point $\mathbf{x}_3$ is on the wrong side of the margin
4) a point $\mathbf{x}_4$ is on the wrong side of the decision hyperplane

A) $\alpha_i = 0$
B) $\alpha_i < C$
C) $\alpha_i = C$



Hinge dual:

$$\max_{\alpha \geq 0} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{x}_i^\top \mathbf{x}_j$$

$$\text{s.t.} \sum_i \alpha_i y_i = 0, \quad \alpha_i \leq C, \quad i = 1, \ldots, n$$

Non-separable SVMs - dual

Consider the following cases

1) a point $\mathbf{x}_1$ is on the correct side of the margin **$\alpha_i = 0$ nonbinding**
2) a point $\mathbf{x}_2$ is on the margin $\alpha_i < C$
3) a point $\mathbf{x}_3$ is on the wrong side of the margin **$\alpha_i = C$**
4) a point $\mathbf{x}_4$ is on the wrong side of the decision hyperplane **$\alpha_i = C$**

If a point is on the wrong side of the margin (cases 3 and 4), $\xi_i > 0$ and hence $\lambda_i = 0$ (last term of the first equation below; λ_i is the Lagrange multiplier for the *i*th slack variable ξ_i), and hence $\alpha_i = C$

If a point is on the margin (case 2), then $\xi_i = 0$, and λ_i can be greater than zero so in general $\alpha_i < C$.

$$L(\mathbf{w}, b, \xi, \alpha, \lambda) = \frac{1}{2}\mathbf{w}^\top \mathbf{w} + C\sum_i \xi_i + \sum_i \alpha_i(1 - \xi_i - y_i(\mathbf{w}^\top \mathbf{x}_i + b)) - \sum_i \lambda_i \xi_i$$

$$\frac{\partial L}{\partial \xi_i} = 0 \quad \rightarrow \quad C - \alpha_i - \lambda_i = 0$$

Why do SVMs work well?

- Why are SVMs fast?
- Why are SVMs often more accurate than logistic regression?

Why do SVMs work well?

- **Why are SVMs fast?**
  - **Quadratic optimization (convex!)**
  - **They work in the dual, with relatively few points**
  - **The kernel trick**
- **Why are SVMs often more accurate than logistic regression?**
  - SVMs use kernels –but regression can, too
  - **SVMs assume less about the model form**
    - Logistic regression uses all the data points, assuming a probabilistic model, while SVMs ignore the points that are clearly correct, and give less weight to ones that are wrong

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- **What is the minimal adjustment to w to make a new point** $y = 1$, $\mathbf{x} = (1,1)$ **be correctly classified?**

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- **Given the hyperplane defined by the line**
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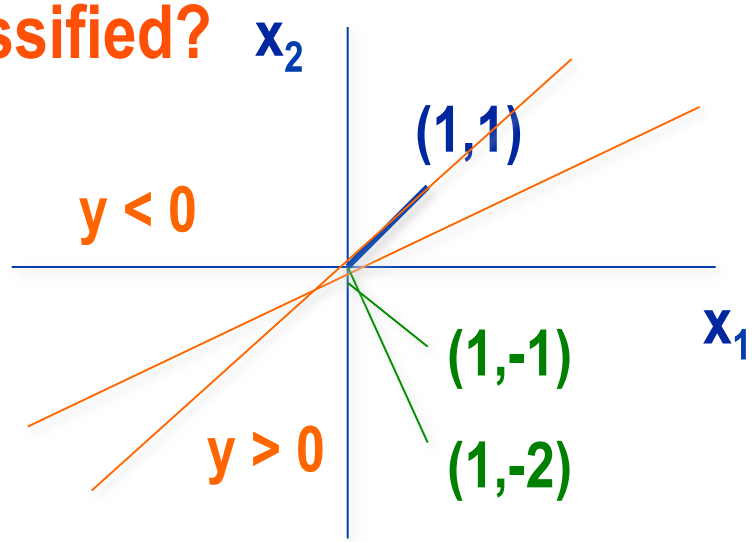

Make **w**=(1,-1)