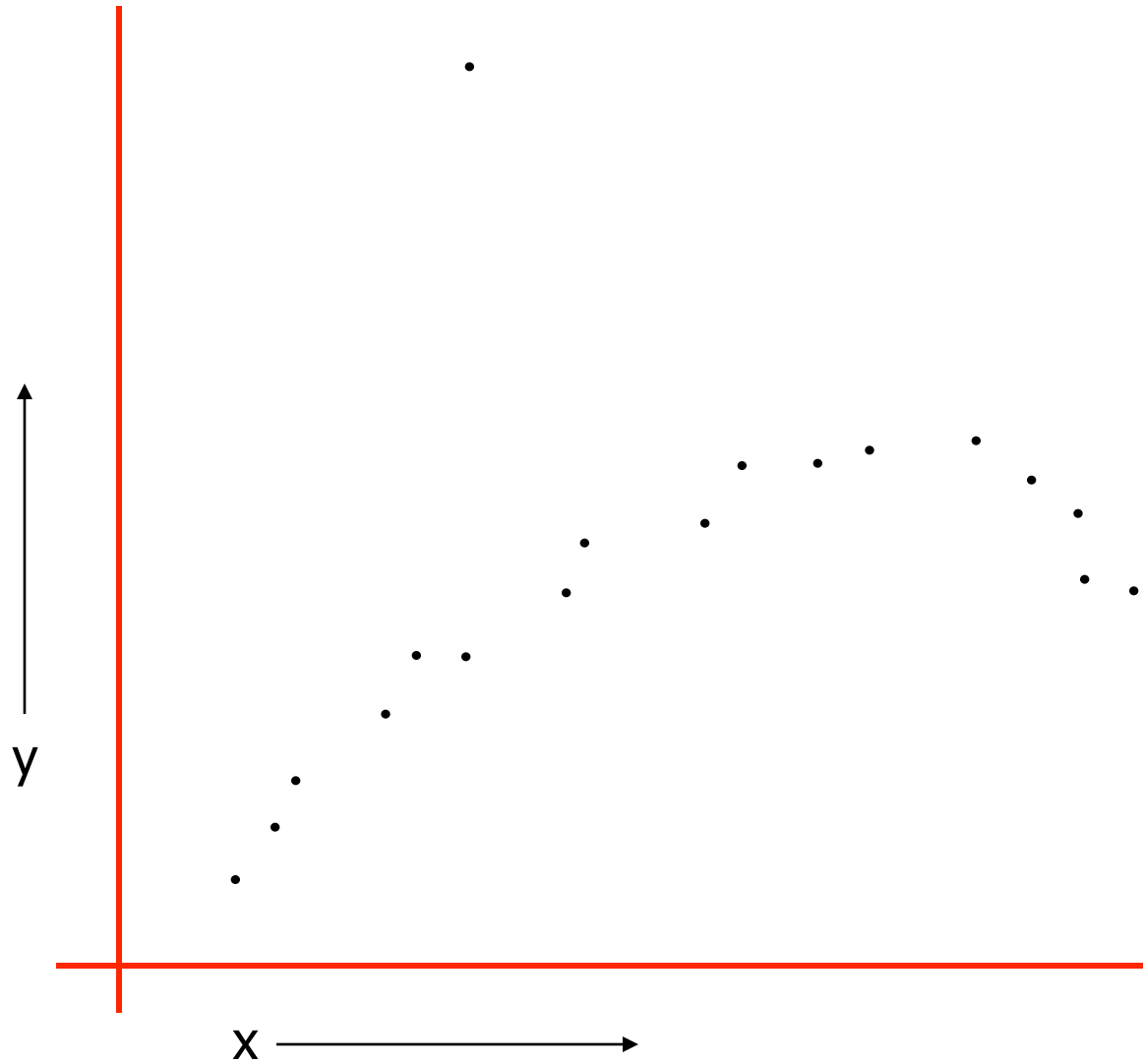
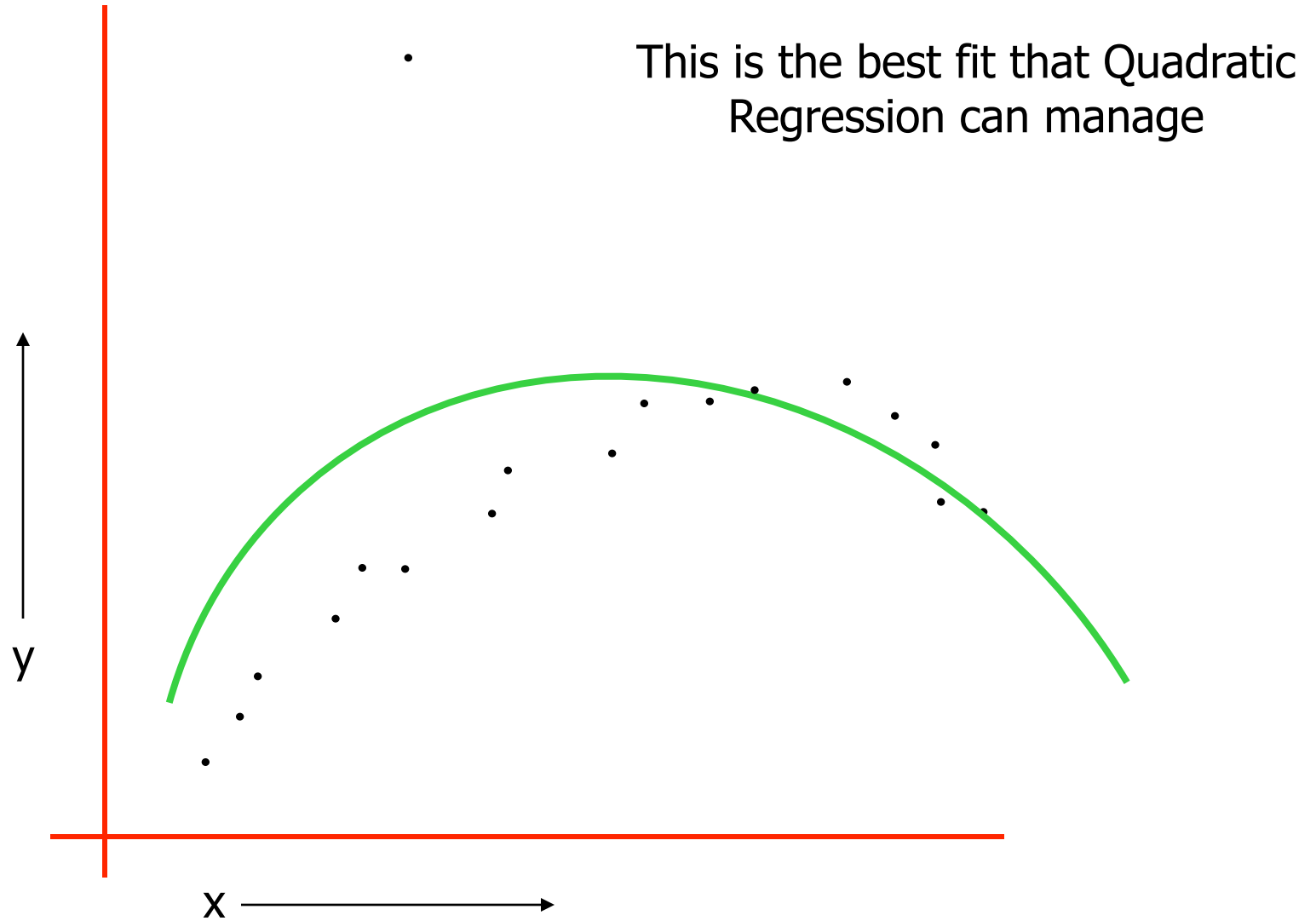


# Robust Regression

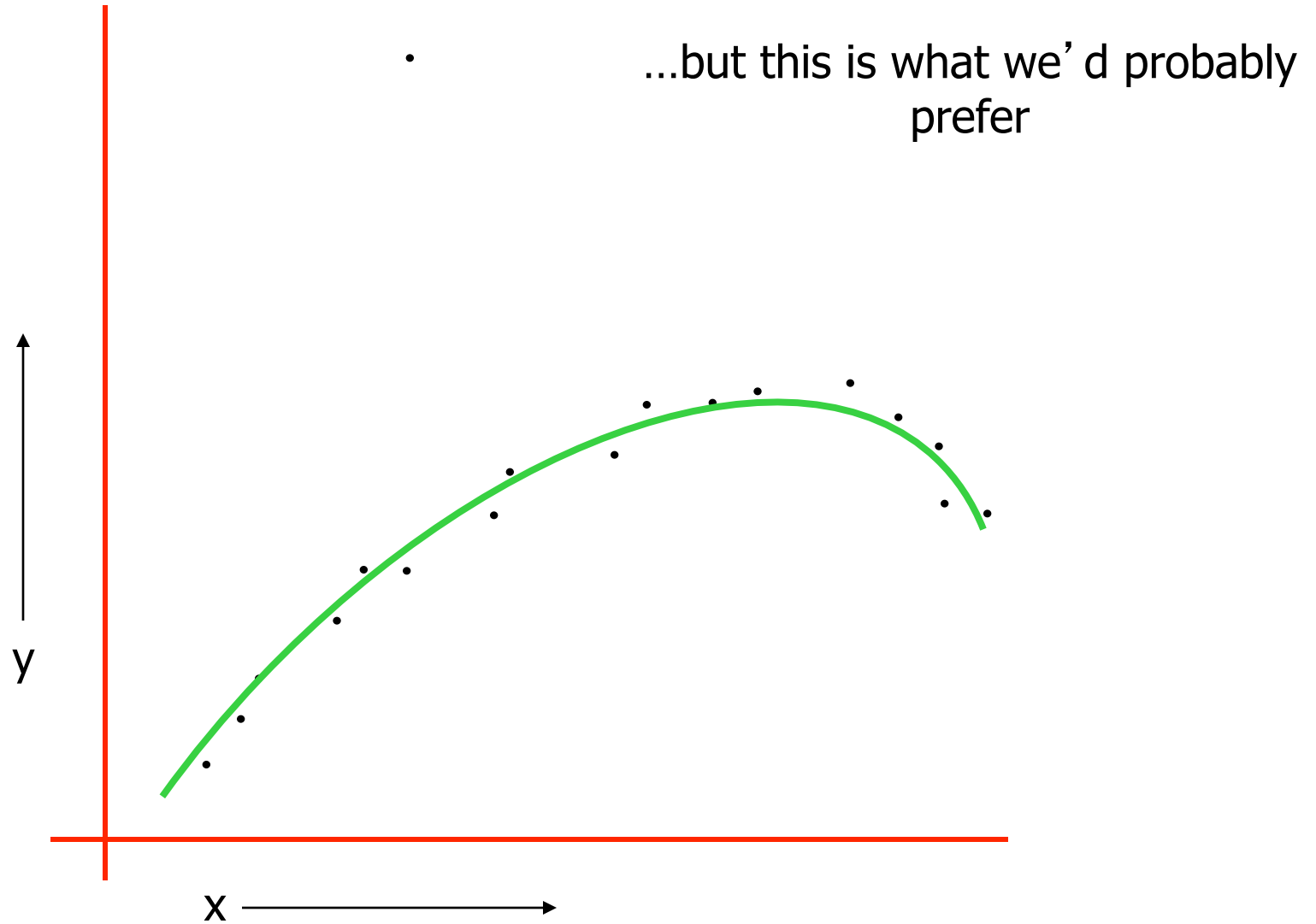
# Robust Regression



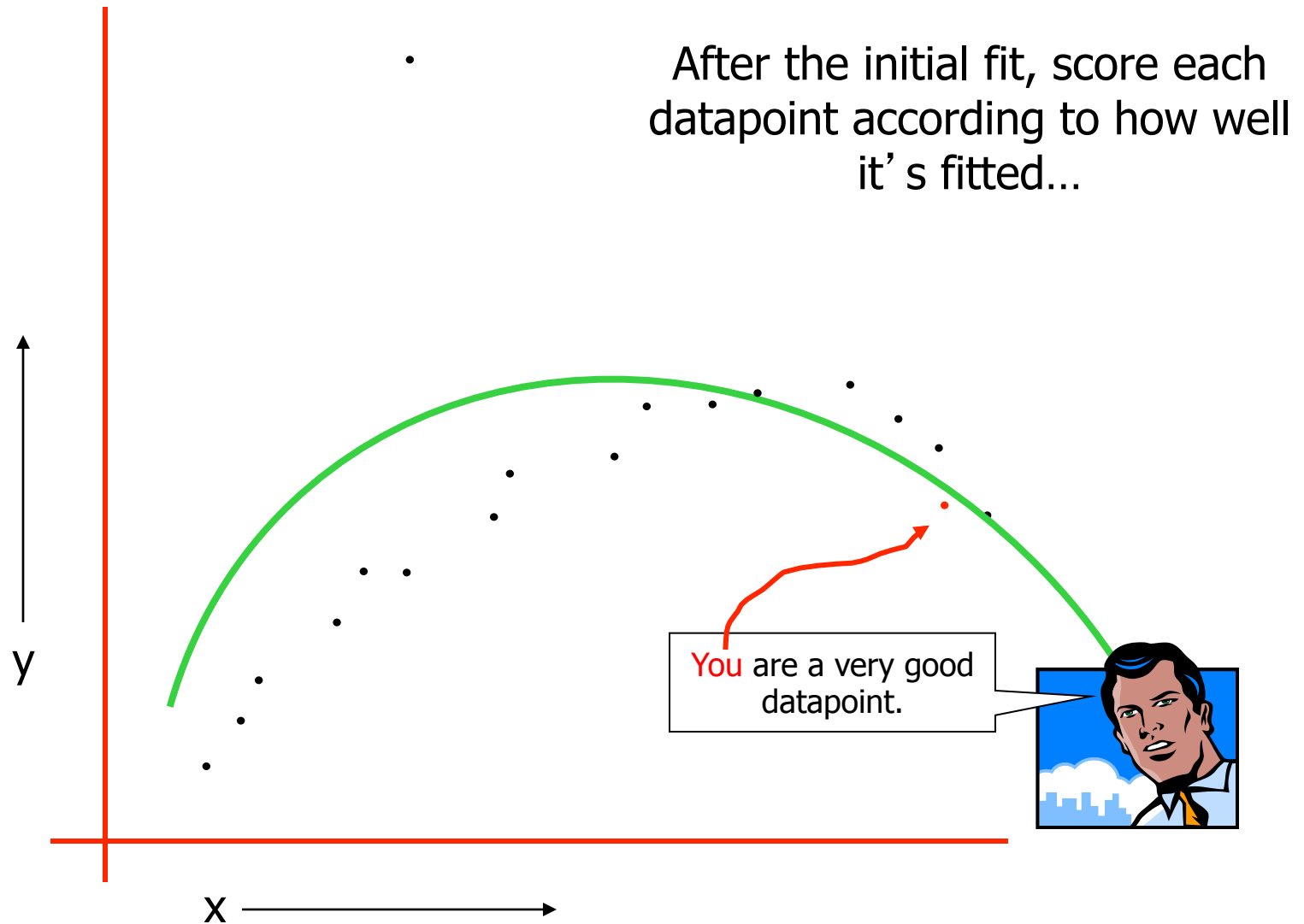
# Robust Regression



# Robust Regression

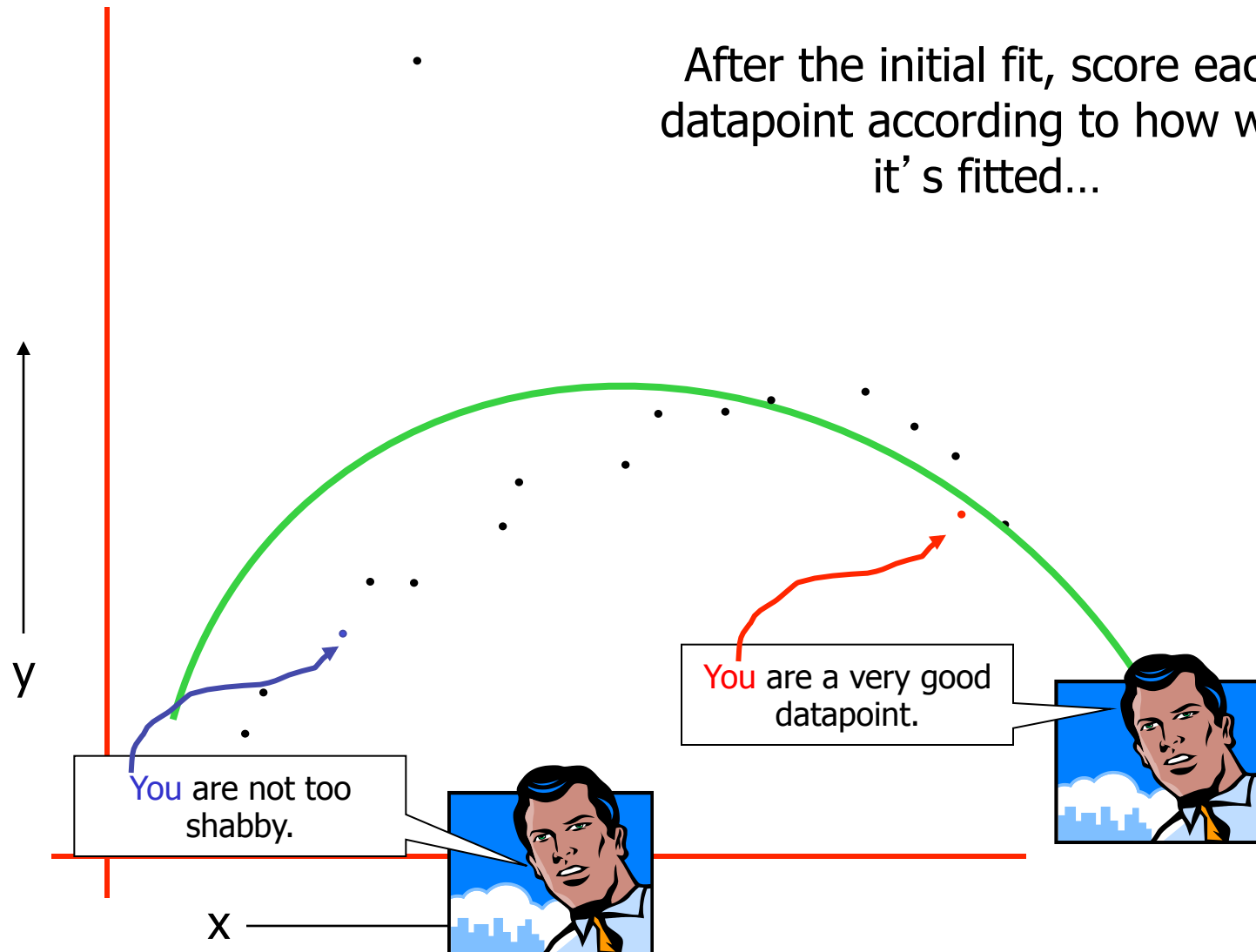


# LOESS-based Robust Regression

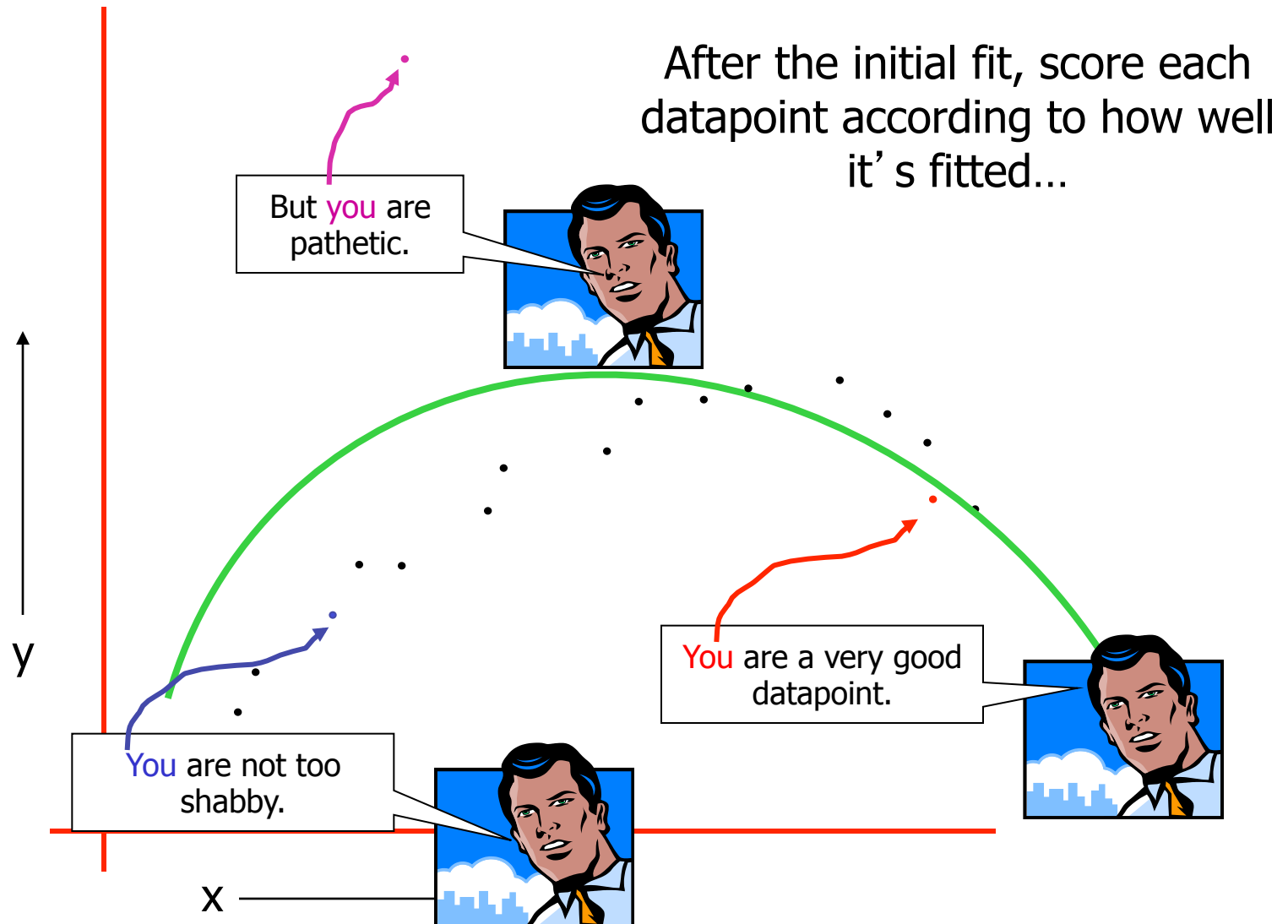


# LOESS-based Robust Regression

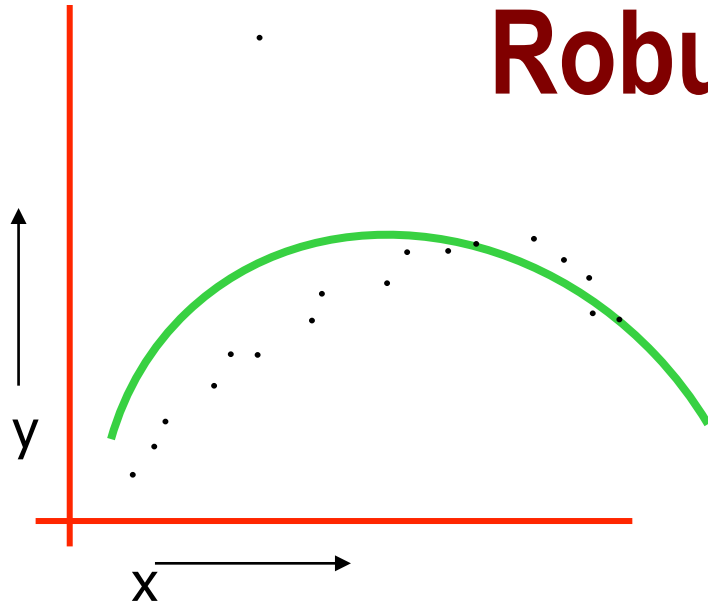
After the initial fit, score each datapoint according to how well it's fitted...



# LOESS-based Robust Regression



# Robust Regression



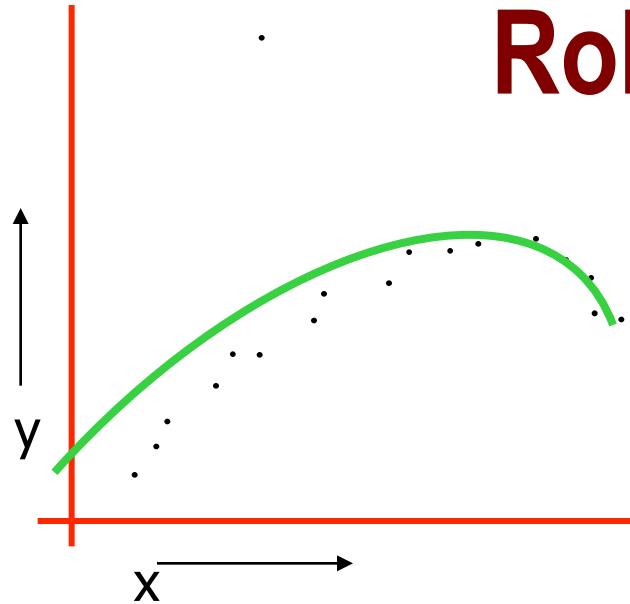
For  $k = 1$  to  $R...$

- Let  $(x_k, y_k)$  be the  $k$ th datapoint
- Let  $y_k^{est}$  be predicted value of  $y_k$
- Let  $w_k$  be a weight for datapoint  $k$  that is large if the datapoint fits well and small if it fits badly:

$$w_k = \text{KernelFn}([y_k - y_k^{est}]^2)$$



# Robust Regression



For  $k = 1$  to  $R...$

- Let  $(x_k, y_k)$  be the  $k$ th datapoint
- Let  $y_k^{est}$  be predicted value of  $y_k$
- Let  $w_k$  be a weight for datapoint  $k$  that is large if the datapoint fits well and small if it fits badly:

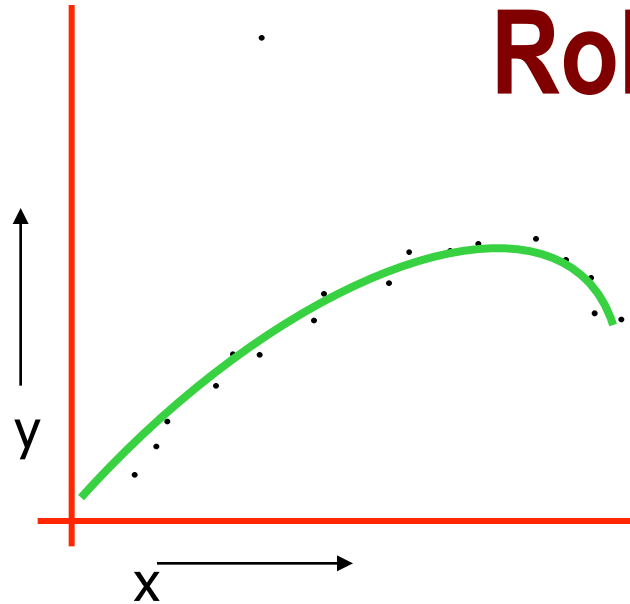
$$w_k = \text{KernelFn}([y_k - y_k^{est}]^2)$$

Then redo the regression using weighted datapoints.

Weighted regression was described earlier in the “vary noise” section, and is also discussed in the “Memory-based Learning” Lecture.

Guess what happens next?

# Robust Regression



For  $k = 1$  to  $R...$

- Let  $(x_k, y_k)$  be the  $k$ th datapoint
- Let  $y_k^{est}$  be predicted value of  $y_k$
- Let  $w_k$  be a weight for datapoint  $k$  that is large if the datapoint fits well and small if it fits badly:

$$w_k = \text{KernelFn}([y_k - y_k^{est}]^2)$$

Then redo the regression using weighted datapoints.

I taught you how to do this in the “Instance-based” lecture (only then the weights depended on distance in input-space)

Repeat whole thing until converged!

# Robust Regression---what we're doing

## What regular regression does:

Assume  $y_k$  was originally generated using the following recipe:

$$y_k = \beta_0 + \beta_1 x_k + \beta_2 x_k^2 + N(0, \sigma^2)$$

Computational task is to find the Maximum Likelihood  $\beta_0$ ,  $\beta_1$  and  $\beta_2$

# Robust Regression---what we're doing

## What LOESS robust regression does:

Assume  $y_k$  was originally generated using the following recipe:

With probability  $p$ :

$$y_k = \beta_0 + \beta_1 x_k + \beta_2 x_k^2 + N(0, \sigma^2)$$

But otherwise

$$y_k \sim N(\mu, \sigma_{huge}^2)$$

Computational task is to find the Maximum Likelihood  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $p$ ,  $\mu$  and  $\sigma_{huge}$

# Robust Regression---what we're doing

## What LOESS robust regression does:

Assume  $y_k$  was originally generated using the following recipe:

With probability  $p$ :

$$y_k = \beta_0 + \beta_1 x_k + \beta_2 x_k^2 + N(0, \sigma^2)$$

But otherwise

$$y_k \sim N(\mu, \sigma_{huge}^2)$$

Computational task is to find the Maximum Likelihood

$\beta_1, \beta_2, p, \mu$  and  $\sigma_{huge}$

Mysteriously, the reweighting procedure does this computation for us.

Your first glimpse of two spectacular letters:

**E.M.**

# Citations

## Radial Basis Functions

T. Poggio and F. Girosi, Regularization Algorithms for Learning That Are Equivalent to Multilayer Networks, Science, 247, 978--982, 1989

## LOESS

W. S. Cleveland, Robust Locally Weighted Regression and Smoothing Scatterplots, Journal of the American Statistical Association, 74, 368, 829-836, December, 1979