CIS 540 Spring 2015: Preparing for the Final Exam

The final exam will be held on Wednesday, May 6, 12 – 2 pm in Moore 216. The final will be out of 150 pts (30% of the total grade). It is open book: you can consult the textbook, lecture slides, and your own personal notes during the exam. All topics covered during the lectures are relevant for the final. From the lecture notes, you can skip: Section 3.3, the second half of Section 3.4.3 (implementation and operations on ROBDDs), Section 5.1.4, Section 5.2.3, Section 5.3, Section 6.4, Section 7.2.2, Section 7.2.3, Section 7.3.2, Chapter 8, Section 9.2, and Section 9.3.

Attached is the final exam from 2014. We do not have written solutions to these questions. If you email me your answers to any of the questions, I will be happy to provide feedback.

CIS 540 Spring 2014: Final Exam, May 9, Noon–2pm

You are allowed to consult the textbook, your class notes, and class handouts. You are not allowed to use laptops and/or Internet access during the exam.

1. Construct a Büchi automaton that accepts traces that satisfy the LTL-formula

   \[ \square (e \rightarrow e \mathcal{U} f) \].

   Note that it is not necessary to use the tableau construction, and it suffices to draw the desired automaton directly.

2. For the leader election protocol of Section 4.3.1, consider a ring with 8 processes, where the identifiers of the processes in order are: 14, 9, 16, 7, 8, 2, 29, and 13. Which processes will proceed to the successive phases (second, third etc)? Which process will be elected as the leader?

3. For each of the statements below, state whether the statement is True or False. Justify your answer with a brief (one or two sentences) explanation.

   (a) For a synchronous reactive component described using tasks, if an output variable y awaits an input variables x, then the component must have a task that reads x and writes y.

   (b) If a property \( \phi \) is an inductive invariant of a transition system \( T \), then every state that satisfies \( \phi \) is a reachable state of \( T \).

   (c) The two LTL-formulas \( \Box (e \rightarrow \Diamond \Box f) \) and \( \Box \neg e \lor \Diamond \Box f \) are equivalent.

   (d) The single dimensional continuous-time component with the dynamics given by \( \dot{x} = x^2 - x \) is Lipschitz-continuous.

4. Consider a transition system \( T \) with two integer variables \( x \) and \( y \). The transitions of the system correspond to executing the statement:

   \[ \text{if } (x > y) \text{ then } x := 2y + 1. \]

   Consider a region \( A \) of the above transition system described by the formula

   \[ (0 \leq x \leq 5) \land (1 \leq y \leq 3). \]

   Compute the formula describing the pre-image of \( A \).

5. Consider a linear system with 2 state variables, 1 input variable, and the dynamics given by:

   \[ \dot{s}_1 = -2s_1 + s_2 + 2u; \quad \dot{s}_2 = -s_1 + 2s_2 - u. \]

   Is the system stable? Is the system controllable? Justify your answers showing all your calculations.
6. Consider an asynchronous process $P$ shown above with the input task $A_z$ and internal tasks $A_x$ and $A_y$. Answer each of the questions below with a clear justification. When adding fairness assumptions, clearly specify whether you are using strong fairness, or weak fairness, and for which tasks.

(a) Does the process $P$ satisfy the LTL-specification $\Diamond (x > 1)$? If not, is there a suitable fairness assumption regarding execution of tasks under which the process satisfies this specification?

(b) Same question as part (a), but now for the specification $\Diamond (y > 1)$.

(c) Same question as part (a), but now for the specification $\square \Diamond (z = 1) \rightarrow \Diamond (y > 1)$.

7. Suppose a timed process has 2 clocks $x_1$ and $x_2$. Before entering a mode $A$, suppose we know that $3 \leq x_1 \leq 4$ and $1 \leq x_1 - x_2 \leq 6$ and $x_2 \geq 0$.

(a) Show the DBM corresponding to the given constraints.

(b) Is the DBM in part (a) canonical? If not, obtain an equivalent canonical form.

(c) Suppose the clock-invariant of mode $A$ is $x_2 \leq 5$. Compute the canonical DBM that captures the set of clock values that can be reached as the process waits in mode $A$.

(d) Consider a mode-switch out of mode $A$ with guard $x_1 \geq 7$ and update $x_1 := 0$. Compute the canonical DBM that captures the set of clock values that are possible after taking this transition.

8. Consider a mobile robot that moves in a two-dimensional world corresponding to the positive quadrant of the X-Y plane. The robot is initially at the origin and is stationary. The input command to the robot consists of a target location to go to. Assume that there are no obstacles. The robot can move in the horizontal direction at speed 6 m/s, in the vertical direction at speed 8 m/s, or along any other arbitrary direction at speed 5 m/s. The robot plans its trajectory to the target to minimize the time taken. Once it reaches the target, it waits there to receive another input command to move to a new target, and repeats the same behavior. Construct a hybrid process (using the extended-state-machine notation) to model the behavior of the robot. Clearly specify input and state variables along with their types. For the purpose of this question, you can assume that the time needed to change the velocity is negligible (that is, the robot can change its speed from, say, 0 to 5, instantaneously).