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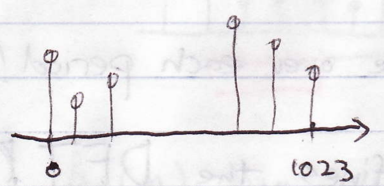
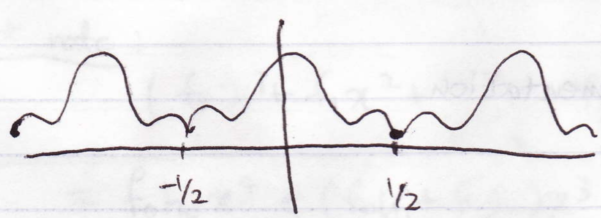
§ 3.3 (No Wiener filtering)
§ 3.4 Pyramid



notation $f_D[\ ]$

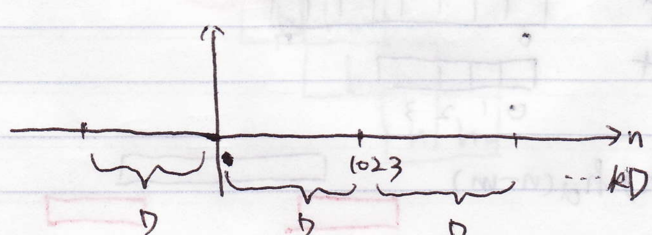
$$f_d(n) \circ\!\!-\!\!\!-\!\!\bullet \sum_{n=0}^{D-1} f_d(n)\, e^{-j2\pi s n}$$

D is the length of the signal, assume T = 1 (sampling interval)



To recover the original $\int_{-\infty}^{\infty} F_d(s)\, e^{j2\pi s t}\, dt$

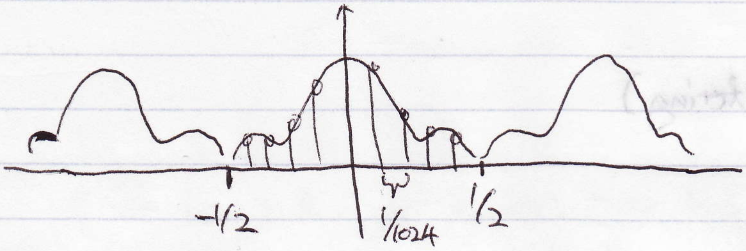
Assume that we virtually "xerox" (replicate) the original signal:



$$f_d(n) * \sum_{k=-\infty}^{\infty} \delta(n-kD)$$

$$F_{dd}(k) = F_d(s) \cdot \sum \delta\left(s-\frac{k}{D}\right)$$

(sampling in Fourier Domain)



Sampling the Fourier Transform D time over each period!

Isolating the interval $[-\frac{1}{2}, \frac{1}{2}]$ defines the DFT!

$$DFT\{f_d(n)\} = F_{dd}(k) = \sum_{n=0}^{D-1} f_d(n)\, e^{-j2\pi n \frac{k}{D}}$$

discrete argument

$\frac{k}{D} = s$ remember $\delta(s - \frac{k}{D})$

FFT is its nlogn implementation

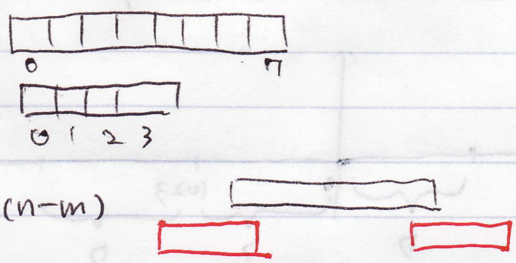
* Convolution of discrete signals

conv (, 'same') : MATLAB

$f_d(n)$ of length $D=8$

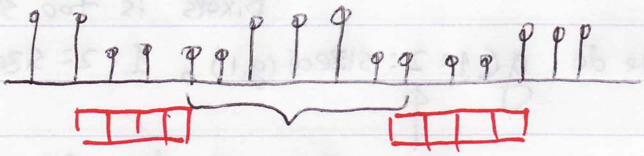
$h_d(n)$ of length $M=4$

$$g_d = f_d * h_d = \sum_{m=0}^{M-1} f_d(m)\, h_d(n-m)$$



if $f_d = 0$ elsewhere g_d has length $D+M-1$

Assuming $f_d = 0$ elsewhere is in contradiction with our assumption that f_d was "zeroxed"



$h_d(n)$ has infinite length conv(, 'cyclic') (still periodic)

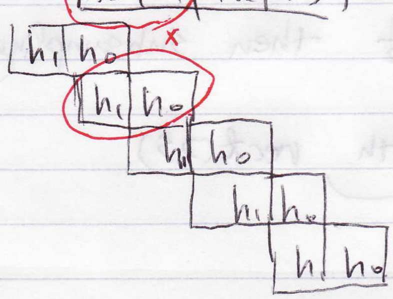
only then $G_{dd}(k) = F_{DD}(k) \cdot H_{DD}(k)$
convolution theorem both in discrete!

Short note:

$$(f_0 x^3 + f_1 x^2 + f_2 x + f_3)(h_0 x + h_1)$$

$$= f_0 h_0 x^4 + (f_0 h_1 + f_1 h_0) x^3 + (f_1 h_1 + f_2 h_0) x^2 + (f_2 h_1 + f_3 h_0) x + f_3 h_1$$

$0 \ \boxed{f_0}\boxed{f_1}\boxed{f_2}\boxed{f_3} \ 0 \ * \ \boxed{h_0}\boxed{h_1}$



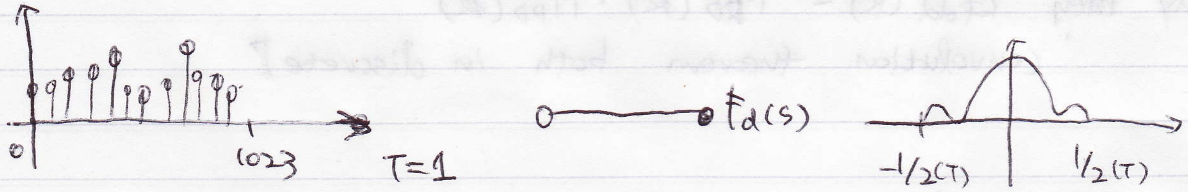
* Pyramids

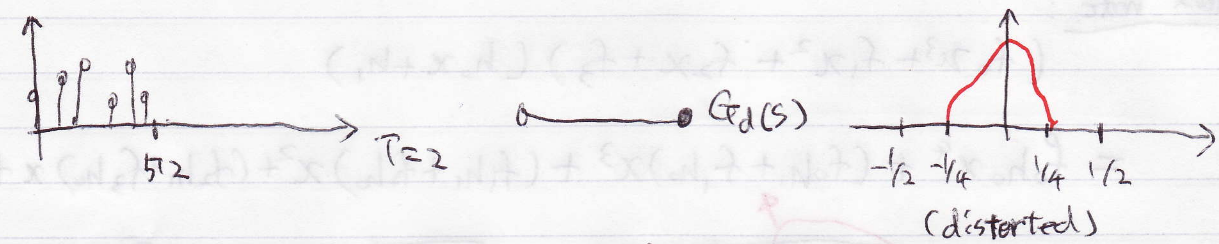
Typical mistake : 640 x 480 = 300k. Algorithm designed for 300k pixels is too slow for 12M pixel.

So we do g(1:2:sizeof(g,1), 1:2:sizeof(g,2))

↑

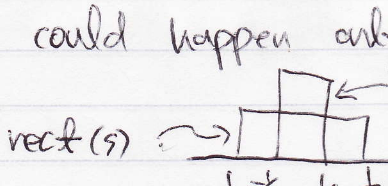
Subsampling of discrete signals ! (Wrong)

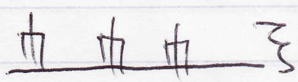


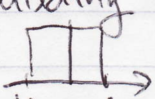


If we apply a filter which eliminates all frequencies from $\frac{1}{4}$ to $\frac{1}{2}$ then subsampling is lossless !

This could happen only with rect(2S)



but $F^{-1}\{$  $\}$ is the discrete sinc with infinite length

The art of subsampling is building filters (smoothing, low pass) which are as close as 

in the frequency domain

The bad news is that the closer to rect the longer the mask