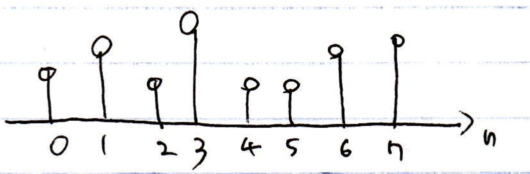
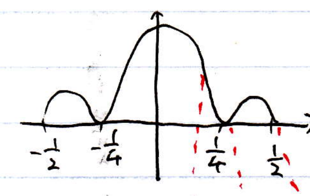
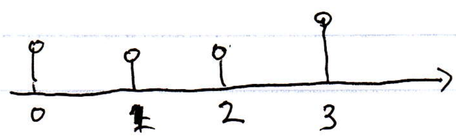
2/3

* Subsampling of digital signal

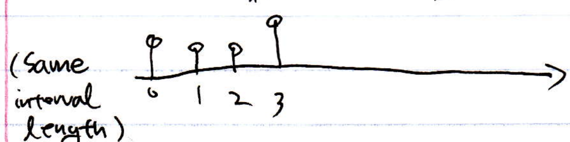


$f(n)$ FT $\multimap$



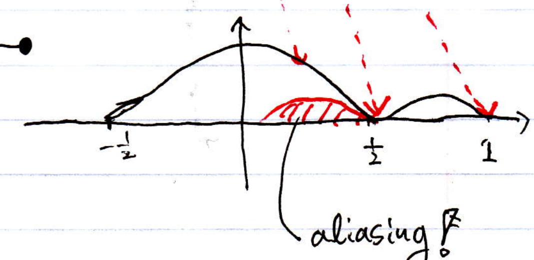


$g(n) = f(2n)$ $\multimap$



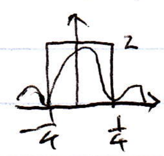
$g(3) = f(6)$

$G(s) = \frac{1}{2} F(\frac{s}{2})$



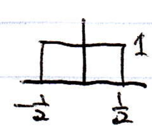
Remedy. Apply a smoothing filter that will eliminate as much as possible from the frequency contribution in $[\frac{1}{4}, \frac{1}{2}]$ and preserve as close as possible the content from $[0, \frac{1}{4}]$

The ideal filter would be $2\,\text{rect}(\cancel{\frac{s}{2}})$ $2s$ =



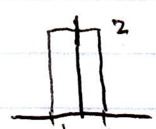
The inverse of $2\,\text{rect}(2s)$ is $\text{sinc}(\frac{n}{2})$

<u>Note</u>: $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$ $\multimap$



$\text{rect}(s)$

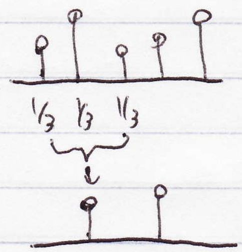
$\text{sinc}(\frac{n}{2}) = \frac{\sin \frac{\pi n}{2}}{\frac{\pi n}{2}}$ $\multimap$



$2\,\text{rect}(2s)$

<u>infinite sequence</u>

Amateur Solution : Averaging

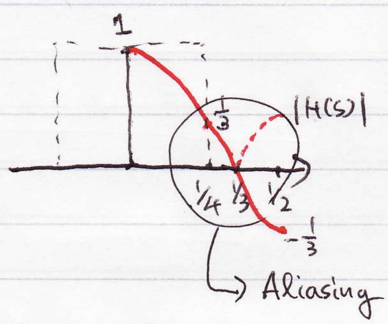


$$h(-1) = h(0) = h(1) = \frac{1}{3}$$

$$H(s) = \sum_{n=-1}^{1} \frac{1}{3} e^{-j2\pi s n}$$

$$= \frac{1}{3}\left(e^{j2\pi s} + 1 + e^{-j2\pi s}\right)$$

$$= \frac{1}{3}\left(1 + 2\cos 2\pi s\right)$$



$[\frac{1}{2}\ \frac{1}{2}] \circ\!\!-\!\!\bullet\ G(s)$? does not work niether.

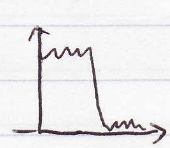
Design of an optimal filter : optimization problem

unknowns : filter coefficians $h(n)$

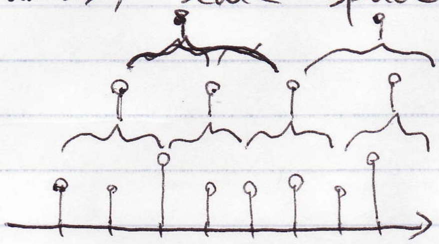
$$H(s) = \sum_{n=-\infty}^{\infty} h(n) e^{-j2\pi s n}$$

1) minimize aliasing $\quad \int_{1/4}^{1/2} |H(s)|^2 ds \Rightarrow \min h(n)$

2) minimize distortion $\quad \int_{0}^{1/4} |1 - H(s)|^2 ds \Rightarrow \min h(n)$

Solution : equiripple 

* Pyramids, scale space



*SLIDES

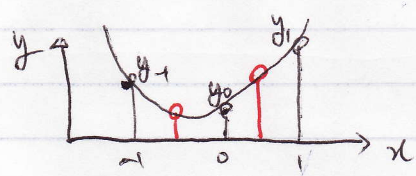
original signal is 2^M

How large is the Pyramid $2^M + 2^{M-1} + \cdots + 1 = 2^{M+1} - 1$

Resampling : Doubling the size and interpolating

ex) HD (1920 x 1080) shows 640 x 480

Any polynomial fitting can be expressed as a convolution



$y = ax^2 + bx + c$: fit it

$$y_{-1} = a(-1)^2 + b(-1) + c$$
$$y_0 = c$$
$$y_1 = a(1)^2 + b(1) + c$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

solve it: $y_{-1} + y_1 - 2y_0 = 2a$

$y_1 - y_{-1} = 2b$.

$$y_{\frac{1}{2}} = a\frac{1}{4} + b\frac{1}{2} + c$$

$$= \frac{1}{2}(y_{-1} + y_1 - 2y_0)\frac{1}{4} + \frac{1}{2}(y_1 - y_{-1})\frac{1}{2} + y_0$$

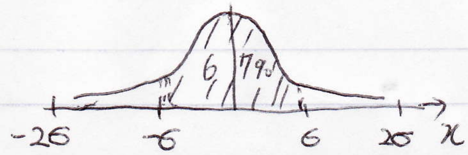
$$= \underline{\underline{h_{-1}}}\,y_{-1} + \underline{\underline{h_0}}\,y_0 + \underline{\underline{h_1}}\,y_1$$

use them as a kernel for convolution

: constants, not a function of data!

(→ shift-invariant)

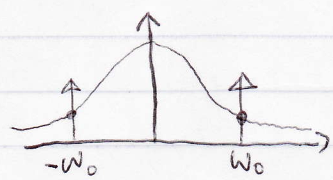
* Gaussian : $\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$



Fourier Transform : $e^{-2\pi^2\sigma^2 s^2}$ $\left(s = \frac{w_0}{2\pi}\right)$

what is $\cos(w_0 x) * \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$?

in frequency domain :



$\mathcal{F}(\cos w_0 x) \left(= \frac{\delta(s-w_0) + \delta(s+w_0)}{2}\right)$

$\times \mathcal{F}(\text{gaussian})$

another cosine !

$= e^{\frac{-\sigma^2 w_0^2}{2}} \cdot \cos(w_0 x)$