MIDTERM -REVIEW

1. Definition of linear and shift-invariant systems

$$f(t) \to \boxed{\qquad} \to g(t)$$

$g(t) = f(t-1) + f(t)$

$g(t) = \alpha f(t) + (1-\alpha)\, g(t-1)$

$\Big\}$ linear and shift invariant

$g(t) = f(t)^2$ X $a_1 f_1(t) + a_2 f_2(t) \not\to a_1 f_1(t)^2 + a_2 f_2(t)^2$

Shift-invariance $f(t) \to \boxed{\quad} \to g(t)$

$f(t-t_0) \to \boxed{\quad} \to g(t-t_0)$

$g(t) = t f(t)$ not shift-invariant

2. $g(t) = \int_{-\infty}^{\infty} h(t-t')f(t')\,dt'$

reflection

$h(\cancel{t'-t})$

3. $F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi st}\, dt$

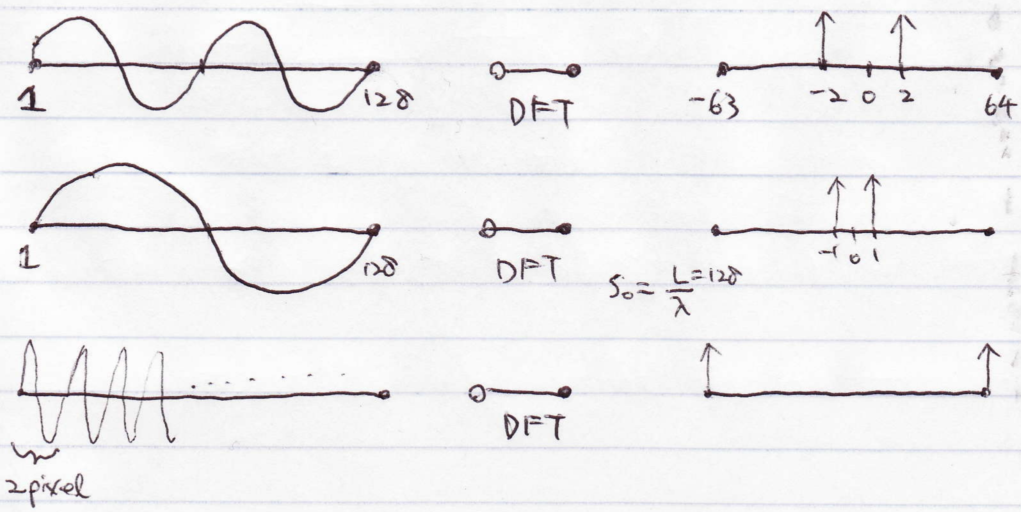
Convolution theorem $(f * h) \circ\!\!-\!\!\bullet F \cdot H$

4. $g(t;\sigma_1) = g_1(t) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma_1^2}} \circ\!\!-\!\!\bullet e^{-2\pi^2 s^2 \sigma_1^2}$

$g(t;\sigma_2) = g_2(t) \circ\!\!-\!\!\bullet e^{-2\pi^2 s^2 \sigma_2^2}$

what is $(g_1 * g_2)(t) = g(t;\sqrt{\sigma_1^2+\sigma_2^2}) \circ\!\!-\!\!\bullet e^{-2\pi^2 s^2(\sigma_1^2+\sigma_2^2)}$

5. $\cos 2\pi s_0 t \quad \circ\!\!-\!\!\bullet \quad \frac{1}{2}\{\delta(s+s_0) + \delta(s-s_0)\}$

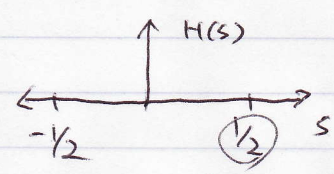


$$DFT(k) = F(s)\Big|_{s=\frac{k}{L}} = \sum_{n=0}^{L-1} f[n]\, e^{-2\pi \frac{n}{L}\cdot k}$$

6. Discrete signal has also a continuous Fourier transform

$$H(s) = \sum_{k=0}^{L} h[k]\, e^{-j2\pi s k}$$

no division with the length

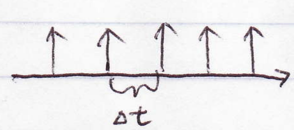


min. wavelength is 2 pixels

max. frequency is $\frac{1}{2}$

7. $\text{Ш}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t)$

comb



$\circ\!\!-\!\!\bullet \quad \text{Ш}(s) = \sum_{k=-\infty}^{\infty} \delta(s - \frac{k}{\Delta t})$

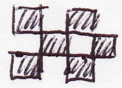
$f(t) \cdot \text{Ш}(t)$ means <u>sampling</u>

$f(t) * \text{Ш}(t)$ means <u>xeroxing</u>

$\Rightarrow$ A discrete signal always has a <u>periodic</u> Fourier transform

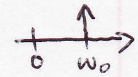
A periodic signal always has a <u>discrete</u> Fourier transform

Try 2D fourier transform of

1)  2) 

Q : What is negative frequency?

Q : Signal with only positive frequency?

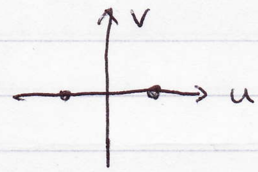
A: $e^{-jw_0 x}$ $\circ\!-\!\bullet$ 

8. 2D Fourier Transform $f(x,y) \circ\!-\!\bullet F(u,v)$

$$F(w_x, w_y)$$
$$w_x = 2\pi x, \quad w_y = 2\pi y$$

$$F(u,v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$f(x-x_0, y-y_0) \circ\!-\!\bullet F(u,v)\, e^{-j2\pi(\underbrace{ux_0+vy_0}_{\text{phase change}})}$$

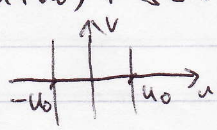
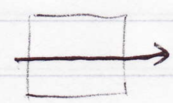
<u>Caution</u> : $f(x,y) = \cos w_0 x$ $\circ\!-\!\bullet$ 

$$F(u,v) = \iint f(x)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty} f(x)\, e^{-2\pi ux}\, dx \cdot \int_{-\infty}^{\infty} 1 \cdot e^{-2\pi vy}\, dy$$

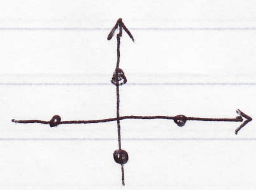
$$w_0 = 2\pi u_0$$

$$= \frac{1}{2}\{\delta(u+u_0) + \delta(u-u_0)\} \cdot \delta(v)$$

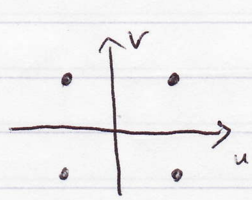
 $\cdot$ 

<u>Separable function</u>

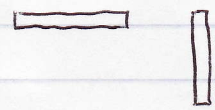
$$f(x,y) = f_1(x)\, f_2(y)$$
$$F_{2D}(u,v) = F_{1D}(u)\, F_{1D}(v)$$

$\cos 2\pi u_0 x + \cos 2\pi v_0 y$ $\circ\!-\!\bullet$ 

$\cos 2\pi u_0 x \cdot \cos 2\pi v_0 y$ $\circ\!-\!\bullet$  $*$ 

$\Rightarrow$ 

9. Speed of convolution with separable functions : $f_1(x)\, f_2(y)$



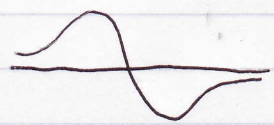
Cost of 2D convolution of $N \times N$ image and $M \times M$ image : $N^2 M^2$ (add + mul)

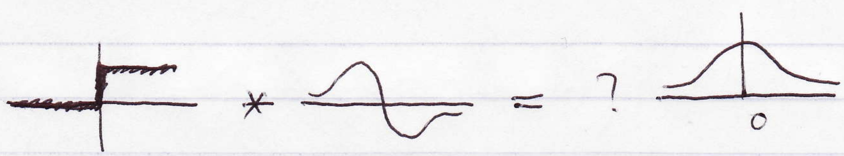
vs

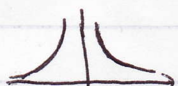
$f_1(x) f_2(y)$: $N^2 M + N^2 M = 2N^2 M$

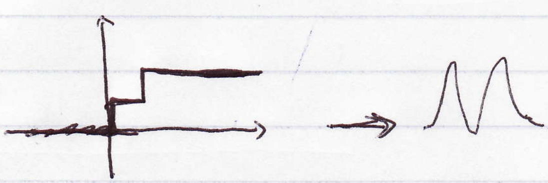
10. Edge detection

$$\frac{\partial}{\partial x} g(x;\sigma) = \left(\frac{-2x}{2\sigma^2}\right) \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}}$$









when does it converge to Λ ?

11. Scale space

$$\frac{\partial g}{\partial \sigma} \overset{?}{=} \frac{1}{2} \frac{\partial^2 g}{\partial x^2}$$

partial differential equation in (σ, x) (PDE)

$\sigma = 0 \quad g(x,0) = g_0(x)$

$g(x,\sigma) = e^{-\frac{1}{2}\frac{x^2}{2\sigma^2}} * g_0(x)$

See notes

Problem : $\cos w_0 x * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$

$* \frac{\partial}{\partial x} (\quad)$

Signal is amplified : we need a normalized derivative of Gaussian.

12. Features → detection (Corner, SIFT)
→ descriptor

Detection : Rotation Invariant $f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$
Scale Invariant $f(sx, sy)$