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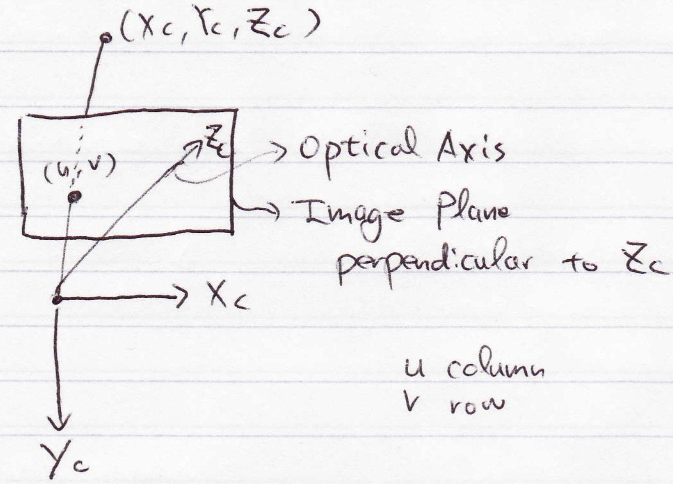
Szeliski: Ch 1 Camera, Geometry , Kostas' Chapter



u column
v row

Projection equation

$$u = f\frac{X_c}{Z_c} + u_0 \qquad v = f\frac{Y_c}{Z_c} + v_0$$

Pin-hole model

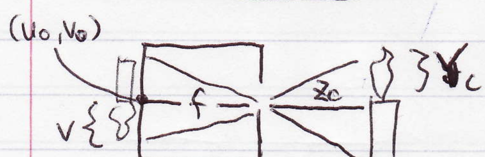
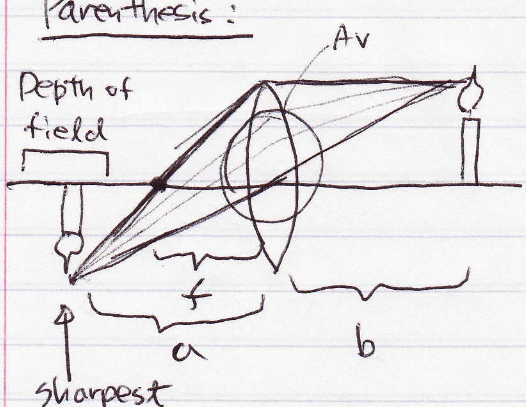


image center = intersection of opt. Axis with image plane

f : focal length [only in the pin-hole model = distance of the image plane from the projection center]

Parenthesis:



$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b} \quad f = \frac{ab}{a+b} = \frac{a}{1+\frac{a}{b}} \approx a$$

$\frac{a}{b} << 1$: pin-hole holds

Tv : time of exposure

Av : 1.8 → 22 — larger DoF

focus : Varying a

Zoom : Varying f

$f_{pixels} = f_{mm} / d\,[mm/pixel]$

(u_0, v_0) center of image

Obtaining (f, u_0, v_0) : <u>Calibrations</u>

Points at infinity are projected into "finite" points

<u>Projective Geometry</u>

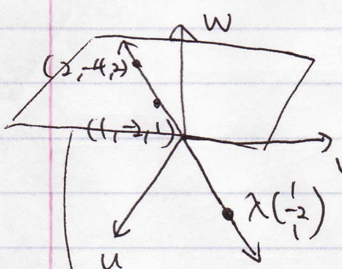
$$x \sim y \qquad \left.\begin{aligned} &x \sim x \\ &x \sim y \iff y \sim x \\ &x \sim y,\ y \sim z \iff x \sim z \end{aligned}\right\} \text{ equivalence}$$

Projective equivalence

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} \sim \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} \text{ iff } \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \lambda \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix},\ \lambda \neq 0$$

$\mathbb{P}^2$ = set of all equiv. classes of $\mathbb{R}^3 \setminus \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \sim \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} \sim \begin{pmatrix} 1/4 \\ 2/4 \\ 3/4 \end{pmatrix} \sim \begin{pmatrix} 10 \\ 20 \\ 30 \end{pmatrix}$$



$\mathbb{P}^2$ is the set of all lines through the origin in $\mathbb{R}^3$

but misses all lines $\begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$ (parallel to the image plane)

They corresponds to intersection of parallel lines inside the image plane

A camera selects representatives $w = 1$ //

$$\mathbb{P}^2 = \mathbb{R}^2 \cup \left\{ \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} : \text{points at infinity} \right\}$$

= intersection of ~~par~~ parallel lines

pixels

$(100, 200) \rightarrow (100, 200, 1)$

$\left(\frac{50}{5}, \frac{20}{5}\right) \leftarrow (50, 20, 5)$

$\not\leftarrow (50, 20, 0)$

Projective transformation

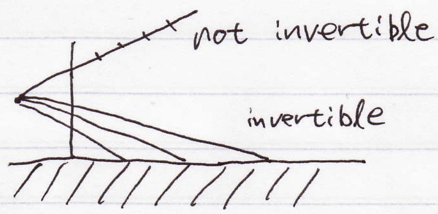
$$p' \sim Ap \qquad \det(A) \neq 0$$

$$\Updownarrow$$

$$\lambda p' = Ap \qquad \lambda \neq 0$$

We knew $u = f\frac{X}{Z}$

$v = f\frac{Y}{Z}$



Line equation in the plane

$$ax + by + c\underset{\overset{\shortparallel}{1}}{w} = 0$$

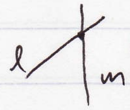
$$(a \ b \ c)\begin{pmatrix} x \\ y \\ w \end{pmatrix} = 0$$

1st line : $l^T \cdot p = 0$

2nd line : $m^T \cdot p = 0$

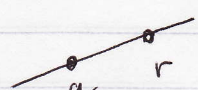
$(d \ e \ f) \quad \begin{pmatrix} x \\ y \\ w \end{pmatrix}$

intersection of l and m



$q \sim l \times m = \lambda\, l \times m$

cross product



$l^T p = 0$

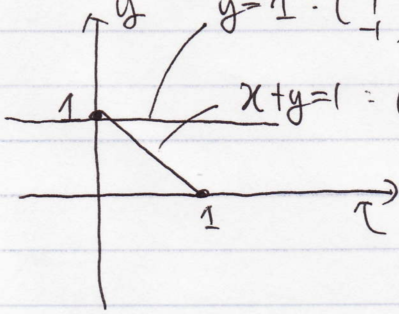
$l^T r = 0$

$l^T q = 0$

$\Rightarrow l \sim q \times r$

line connecting q and r

ex)



$y=1 : \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$x+y=1 : \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$y=0 : \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$0 \cdot x + 1y + 0 \cdot w = 0$

intersection of $x+y=1$ and $y=0$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \sim \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \to (1,0)$$

intersection of $y=0$ with $y=1$

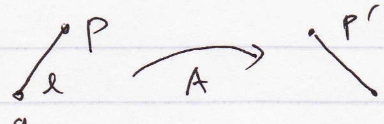
$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \sim \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$ point at infinity

$$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

proj. transformation A

$p' \sim Ap$

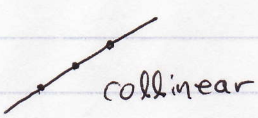
$q' \sim Aq$

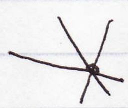


$l' \sim p' \times q' \sim Ap \times Aq$

$\sim A^{-T}(p \times q)$

$l' \sim A^{-T} l$





Geometric Vision

2D → 3D

$(u', v') \rightarrow \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$

$\downarrow$ $\qquad\qquad\qquad \downarrow$

$P^2 \ni (u, v, w) \qquad \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} \in \mathbb{P}^3$

$$\frac{u}{w} = u' = f\frac{X}{Z}$$

$$\frac{v}{w} = v' = f\frac{Y}{Z}$$

$u = fX$
$v = fY$
$w = Z$

$$= \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$= \begin{pmatrix} f & & \\ & f & \\ & & 1 \end{pmatrix} \underset{3\times3 \quad 3\times1}{\begin{pmatrix} R & t \end{pmatrix}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ W_w \end{pmatrix}$$

3x4 : not invertible